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Abstract

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A SUPERFLUID MODEL OF THE NUCLEUS

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Mathematical methods developed by N. N. Bogolyubov in constructing the theories of superfluidity and superconductivity have great generality. They make it possible to solve the problem of taking into account residual interactions of nucleons, leading to pair correlations, in the shell and generalized models of the nucleus as a many-body problem. As a result of a number of investigations it has been shown that the residual interactions between nucleons near the energy of the Fermi surface are attractive interactions. Therefore the ground state of any medium or heavy nucleus is a superfluid state of the nucleus, energetically more favorable than a state with successively filled levels.

We shall call superfluid a model which is based on the shell and generalized models and takes into account the residual interactions of nucleons near the energy of the Fermi surface of the nucleus under the following assumptions: 1) the residual interactions between nucleons both in the neutron and in the proton shells are described by a Hamiltonian of the form

$$H = \sum_{s\sigma} \{E(s) - \lambda\} a^+_{s\sigma} a_{s\sigma} - G \sum_{s,s'} a^+_{s+} a^+_{s-} a_{s'-} a_{s'+}; \tag{1}$$

2) calculations are carried out for each definite nucleus, neglecting the fact of a certain averaging connected with conservation of the number of particles on the average. The state of a nucleon is described by a set of quantum numbers $(s\sigma)$, determined by the form of the self-consistent field; $\sigma = \pm 1$ characterizes, for example, the sign of Ω—the projection of the nucleon angular momentum on the symmetry axis of the nucleus. A certain simplification of the physical picture is the assumption that the interaction constant G is constant. The chemical potential λ is determined from the condition

$$n = \sum_{s\sigma} \langle a^+_{s\sigma} a_{s\sigma} \rangle, \tag{2}$$

where n is the number of nucleons. The Hamiltonian (1) should be regarded as part of the complete Hamiltonian, containing, for example, collective interactions. We note that the effects to which the residual interactions (1) lead cannot be obtained by any modifications of the self-consistent field.

In the present note, as the first stage in the investigation of a superfluid model of the nucleus, we shall consider strongly deformed nuclei in the region of the rare-earth elements on the basis of the Nilsson potential in the assumption of the adiabatic approximation.

With the aid of Bogolyubov' s variational principle ([1]), the energy of the ground state of an even shell is obtained in the form

$$\mathcal{E} = \sum_s E(s)\left\{1 - \frac{E(s)-\lambda}{\sqrt{C^2 + \{E(s)-\lambda\}^2}}\right\} - \frac{C^2}{G}, \tag{3}$$

where C and λ are determined from the equations ([2,3])

$$\frac{2}{G} = \sum_s \frac{1}{\sqrt{C^2 + \{E(s)-\lambda\}^2}}; \tag{4}$$

$$n = \sum_s \left\{1 - \frac{E(s)-\lambda}{\sqrt{C^2 + \{E(s)-\lambda\}^2}}\right\}. \tag{5}$$

In the case of an odd shell, if the odd nucleon is in the state s_i, then the energy of the system is

$$\mathcal{E}(s_i) = E(s_i) + \sum_{s \neq s_i} E(s)\left\{1 - \frac{E(s)-\lambda}{\sqrt{C^2 + \{E(s)-\lambda\}^2}}\right\} - \frac{C^2}{G}, \tag{6}$$

and C and λ are found from the equations

$$\frac{2}{G} = \sum_{s \neq s_i} \frac{1}{\sqrt{C^2 + \{E(s)-\lambda\}^2}}; \tag{7}$$

$$n = 1 + \sum_{s \neq s_i} \left\{1 - \frac{E(s)-\lambda}{\sqrt{C^2 + \{E(s)-\lambda\}^2}}\right\}. \tag{8}$$

The equations for C and λ for excited states of an even shell have a similar form. Thus, to find C and λ, it is necessary to solve the corresponding system of equations. We note that the values of C and λ change appreciably both in going from an even shell to an odd one and from the ground state to an excited state; this is demonstrated in Table 1 for the case of a neutron shell with $N = 107$ and $N = 108$ at deformation $\delta = 0.26$, with $E_F = 6.406\,\hbar\omega_0$ ($\hbar\omega_0 = 41A^{-1/3}$ MeV). Since the interaction constant G is unknown, calculations were carried out for 5 values of G in the interval (0.016–0.032) $\hbar\omega_0$ (0.12–0.24 MeV). The behavior of λ characterizes the change in the properties of a many-body system, and, as is seen from Table 1, these changes should not be neglected.

It is known that the levels of Nilsson' s scheme ([4]) do not give the exact order and energy spacings of excited states, which is apparently connected, first, with shortcomings of Nilsson's scheme itself, and, second, with the necessity of taking residual interactions into account.

Table 1

G in $\hbar\omega_0$	Even shell $N=108$, ground state C	Even shell $N=108$, ground state λ	Odd shell, $N=107$, ground state $^9/_2+$ [624] C	Odd shell, $N=107$, ground state $^9/_2+$ [624] λ	Odd shell, $N=107$, excited states, particle $^1/_2-$ [510] C	Odd shell, $N=107$, excited states, particle $^1/_2-$ [510] λ	Odd shell, $N=107$, excited states, hole $^5/_2-$ [512] C	Odd shell, $N=107$, excited states, hole $^5/_2-$ [512] λ
0.016	0.075	6.458	0.019	6.430	0.055	6.406	0.063	6.458
0.020	0.137	6.446	0.106	6.423	0.115	6.405	0.118	6.444
0.024	0.204	6.440	0.175	6.415	0.180	6.403	0.181	6.431
0.028	0.271	6.435	0.242	6.409	0.246	6.399	0.246	6.422
0.032	0.338	6.432	0.311	6.405	0.313	6.396	0.312	6.415

The calculation performed of single-particle levels of odd nuclei on the basis of the superfluid model shows that: 1) the energy levels obtained describe excitation spectra more correctly than the levels of Nilsson' s scheme, as is seen from Table 2; 2) the behavior of the levels depends strongly on G, and, as is seen from Table 3, when G is increased the excitation energy decreases; 3) residual interactions

Table 2

Energy of excited states (in MeV)

	Neutron shell	Neutron shell	Proton shell	Proton shell
	$N=101,\ \delta=0{,}28,\ G=0{,}020\,\hbar\omega_0^\circ$	$N=105,\ \delta=0{,}27,\ G=0{,}016\,\hbar\omega_0^\circ$	$Z=67,\ \delta=0{,}30,\ G=0{,}024\,\hbar\omega_0^\circ$	$Z=69,\ \delta=0{,}28,\ G=0{,}028\,\hbar\omega_0^\circ$
Ground state	0 1/2−[521]	0 7/2−[514]	0 7/2−[523]	0 1/2+[411]

	Neutron shell	Neutron shell	Proton shell	Proton shell
Particle levels	0,10 5/2 − [512] 0,42 7/2− [514] 0,67 1/2+ [651]	0,32 9/2 + [624] 0,46 1/2+ [651] 1,18 1/2− [510]	0,16 1/2 + [411] 1,30 7/2+ [404] 1,59 5/2+ [402]	0,55 7/2 + [404] 0,73 5/2+ [402] 1,00 9/2− [514]
Hole levels	0,08 7/2 + [633] 0,27 11/2− [505] 0,84 3/2− [523]	0,25 5/2 − [512] 0,80 1/2− [521] 0,93 7/2+ [633]	0,45 3/2 + [411] 0,52 5/2+ [413] 1,22 5/2− [532]	0,13 7/2 − [523] 0,80 3/2+ [411] 0,86 5/2+ [413]

Table 3

Energy of excited states (in MeV) of odd nuclei with neutron number $N = 107$, $\delta = 0{,}23$; ground state 9/2 + [624]

Level characteristic	G in $\hbar\omega_0^\circ$	G in $\hbar\omega_0^\circ$	G in $\hbar\omega_0^\circ$	G in $\hbar\omega_0^\circ$	G in $\hbar\omega_0^\circ$	G in $\hbar\omega_0^\circ$
	0	0,016	0,020	0,024	0,028	0,032
1/2 − [510] particle	0,821	0,652	0,391	0,288	0,225	0,183
3/2 − 512 particle	0,930	0,718	0,449	0,326	0,261	0,212
7/2 − 503 particle	0,964	0,791	0,507	0,377	0,297	0,241
7/2 − 514 hole	0,547	0,484	0,178	0,080	0,043	0,024
5/2 − 512 hole	0,907	0,848	0,455	0,275	0,181	0,133

as a rule, do not lead to a change of the ground state given by the Nilsson scheme; 4) hole and particle levels behave differently as G increases; however, the residual interactions do not lead to a change in the sequence of hole (particle) levels relative to one another.

Table 4

Energy levels of ${}^{160}_{66}\mathrm{Dy}_{94}$ (in MeV)

Proton shell	Proton shell	Proton shell	Neutron shell	Neutron shell	Neutron shell
Ω, parity	G in $\hbar\omega_0^\circ$	G in $\hbar\omega_0^\circ$	Ω, parity	G in $\hbar\omega_0^\circ$	G in $\hbar\omega_0^\circ$
	0,024	0,028		0,020	0,024
2−; 5−	0,95	1,35	1+; 4+	0,62 (?)	1,04
1−; 6−	1,03	1,43	0+	0,97	1,28
0+	1,30	1,55	0+	1,00	1,37
1+; 2+	1,33	1,67	0−; 5−	1,07	1,50
0+	1,54	1,83	1−; 4−	1,45	1,77
1+; 4+	1,62	1,90	2−; 5−	1,45	1,83
3−; 4−	1,65	1,86	3+; 6+	1,68	1,99
0+	1,68	1,96	1−; 6−	1,76	2,02

The spectrum of excited states of even-even nuclei, calculated on the basis of the superfluid model of the nucleus, shows, at least, which values of the spins and parities of the excited states are most probable. Thus, from the energy levels calculated in Table 4, the ⋯

^{160}Dy it is seen that among the lower levels there should be 0+ states, and that with increasing G the excitation energies increase.

Let us calculate the pairing energy by the formula

$$P_n(Z,N) = 2\mathcal{E}(Z,N-1) - \mathcal{E}(Z,N) - \mathcal{E}(Z,N-2). \tag{9}$$

The neutron pairing energy of ^{174}Yb is found to be 0.39 MeV for $G = 0.016\,\hbar\omega_0^0$; 1.38 MeV for $G = 0.020\,\hbar\omega_0^0$, and 3.38 MeV for $G = 0.028\,\hbar\omega_0^0$, while, for example, the proton pairing energy of ^{176}Hf is 0.11 MeV for $G = 0.016\,\hbar\omega_0^0$; 0.26 MeV for $G = 0.020\,\hbar\omega_0^0$; 0.60 MeV for $G = 0.024\,\hbar\omega_0^0$; 1.32 MeV for $G = 0.028\,\hbar\omega_0^0$, and 2.07 MeV for $G = 0.032\,\hbar\omega_0^0$. We note that in calculating the pairing energy it is necessary to take into account the energy associated with the change in nuclear deformation.

The calculated values of the pairing energy and excitation spectra of a number of nuclei are closest to the experimental values for $G = (0.018 \div 0.024)\hbar\omega_0^0 = (0.13 \div 0.18)$ MeV for the neutron $(93 \leq N \leq 115)$ shell and for $G = (0.022 \div 0.028)\hbar\omega_0^0 = (0.15 \div 0.20)$ MeV for the proton $(63 \leq Z \leq 76)$ shell.

The obtained values of C and λ make it possible to calculate corrections to the probabilities of β- and γ-transitions associated with the superfluidity of the ground and excited states. The corrections enter as factors smaller than unity; for example, for the β-decay of odd nuclei they have the form

$$\frac{1}{4}\left\{1 \mp \frac{E(s_i)-\lambda_i}{\sqrt{C_i^2+\{E(s_i)-\lambda_i\}^2}}\right\}\left\{1 \mp \frac{E(s_k)-\lambda_k}{\sqrt{C_k^2+\{E(s_k)-\lambda_k\}^2}}\right\}. \quad (10)$$

The corrections to the β-decay of ^{163}Er are 0.45 for $G = 0.020\,\hbar\omega_0^0$, 0.14 for $G = 0.028\,\hbar\omega_0^0$; for the β-decay of ^{165}Dy they are of order 0.3 for $G = 0.016\,\hbar\omega_0^0$ and 0.5 for $G = 0.024\,\hbar\omega_0^0$; for the β-decay of ^{167}Ho they are of order 0.17 for $G = 0.016\,\hbar\omega_0^0$ and 0.02 for $G = 0.024\,\hbar\omega_0^0$, etc. The corrections, for example, for an $E1$-transition with energy 0.026 MeV in ^{161}Dy are 0.36 for $G = 0.020\,\hbar\omega_0^0$, 0.12 for $G = 0.024\,\hbar\omega_0^0$, and 0.04 for $G = 0.028\,\hbar\omega_1$, etc.

The examples given show that there is a real possibility of carrying out comprehensive calculations of a number of properties of strongly deformed nuclei that are connected with their internal structure. For this it is necessary to improve Nilsson' s level scheme in such a way that the single-particle levels of odd nuclei calculated on the basis of the superfluid model satisfy the experimental values. This will make it possible to calculate the excitation spectrum of even-even nuclei, the pairing energy, the moments of inertia of the ground and excited states, corrections to the probabilities of β- and γ-transitions, and so on.

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REFERENCES

1. N. N. Bogolyubov, DAN, **119**, 224 (1958).
2. W. Solovyov, Nucl. Phys., **9**, 655 (1959); JETP, **36**, 1869 (1959).
3. S. Belyaev, Kong. Danske Vidensk. Selsk. Mat.-fys. Medd., **31**, No. 11 (1959).
4. S. G. Nilsson, Kong. Danske Vidensk. Selsk. Mat.-fys. Medd., **29**, No. 16 (1955); B. Mottelson, S. G. Nilsson, Mat. Fys. Skr. Den. Vid. Selsk., **1**, No. 8 (1959).

Note: Figure translations are in progress. See original paper for figures.