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Abstract

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Density Advection and the Intensification of Wind-Driven Currents toward the Western Coast of the Ocean

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In recent years interesting works have appeared on the theory of currents in a baroclinic ocean ([1−4]), one of the principal conclusions of which is a proof of the essential role of density (or thermal) inhomogeneity in oceanic circulation. However, the nonstationary process of adjustment of the mass field to the field of wind-driven currents, qualitatively described by V. B. Shtokman ([5]), has been studied quantitatively only little; the relative contribution of horizontal advection and vertical diffusion of density is unclear.

Indeed, it seemed to us ([6]) that in the process of mass redistribution density diffusion plays a secondary role in comparison with advection. Welander ([4]) practically proceeds from an analogous assumption. Robinson and Stommel ([3]) obtained, in particular, that neglect of vertical thermal diffusion substantially distorts the stationary temperature field. Ichiye ([2]) proposed a model of density distribution in the ocean without advection.

In the present work an attempt is made at a quantitative study of the process of the origin and development of wind-driven currents and mass redistribution, taking into account advection and vertical diffusion of density. The numerical calculations have not yet been completed; here we present only the formulation of the problem and the conclusion concerning the role of density advection in the formation of intense currents near the western coast of the ocean.

We solve the following problem: in the ocean an initial distribution of the density field $\rho = \rho(y, z)$ is prescribed; at the instant $t = 0$ a stationary field of wind (atmospheric pressure) arises; it is required to calculate the process of redistribution of the mass field under the action of the wind.

We neglect the nonlinear terms in the equations of motion, allowing for inaccuracy in the coastal layer. In height the ocean is mentally divided into two layers: an upper homogeneous layer and an underlying main baroclinic layer. In the upper layer we proceed from the following simple scheme:

$$\nu\frac{\partial^2 u}{\partial z^2} + lv = \frac{1}{\rho_0}\frac{\partial P}{\partial x}; \tag{1}$$

$$\nu\frac{\partial^2 v}{\partial z^2} - lu = \frac{1}{\rho_0}\frac{\partial P}{\partial y}; \tag{2}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0. \tag{3}$$

Boundary conditions in height:

$$\text{at } z = 0: \qquad \rho_0\nu\frac{\partial u}{\partial z} = -T_x, \quad \rho_0\nu\frac{\partial v}{\partial z} = -T_y; \tag{4}$$

$$w = -\frac{1}{\rho_0 g}\frac{\partial P}{\partial t}; \tag{5}$$

$$\text{as } z \to \infty \qquad u,\ v \text{ are bounded.} \tag{6}$$

The x-axis is directed eastward, y—northward, z—vertically downward; the pressure anomaly $P = P_1 - \rho_0 g z$ is regarded as independent of z. Outside the equatorial region of the ocean we adopt the condition $T_x = T_y \equiv 0$.

From (1), (2), (4), (6) we easily obtain

$$u + iv = \frac{T_x + iT_y}{\rho_0\nu\alpha(1+i)}e^{-(1+i)\alpha z} + \frac{1}{\rho_0 l}\left(i\frac{\partial P}{\partial x} - \frac{\partial P}{\partial y}\right), \tag{7}$$

where $\alpha = \sqrt{l/2\nu}$.

The formula for w is obtained with the aid of (3), (5), (7). In what follows we shall need only the value of w at the Ekman friction depth h; it has the form (the β-effect of the drift terms is small):

$$w\big|_{z=h} = -\frac{1}{\rho_0 g}\frac{\partial P}{\partial t} + \frac{\beta h}{\rho_0 l^2}\frac{\partial P}{\partial x} - \frac{1}{2\alpha'\rho_0 l}\Delta P_0'. \tag{8}$$

In deriving formula (8), as in all previous works (see, for example, (7)), we expressed the tangential wind stress (T_x, T_y) through the atmospheric pressure at sea level P_0', using Akerblom' s model (8), $\alpha' = \sqrt{l/2\nu'}$, ν' being the coefficient of vertical turbulent viscosity of air.

In (7), (8), P is an unknown function of the horizontal coordinates and time; it must be determined from the solution of the problem for the baroclinic layer. For this layer we start from the system

$$\frac{\partial u}{\partial t} - lv = -\frac{1}{\rho_0}\frac{\partial P}{\partial x}; \tag{9}$$

$$\frac{\partial v}{\partial t} + lu = -\frac{1}{\rho_0}\frac{\partial P}{\partial y}; \tag{10}$$

$$\rho g = \frac{\partial P}{\partial z_1}; \tag{11}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z_1} = 0; \tag{12}$$

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial y} + w\frac{\partial \rho}{\partial z_1} = \varkappa\frac{\partial^2 \rho}{\partial z_1^2}. \tag{13}$$

Here $z_1 = z - h$; the boundary conditions in height are the following: at the boundary between the homogeneous and baroclinic layers the continuity condition for pressure and vertical velocity is prescribed; moreover,

$$\left.\frac{\partial \rho}{\partial z_1}\right|_{z_1=0} = \Gamma(y); \tag{14}$$

$$\text{as } z_1 \to \infty \qquad \rho = P = 0. \tag{15}$$

We shall discuss the question of the horizontal boundary conditions below. Let us first analyze the linearized version of equation (13):

$$\frac{\partial \rho}{\partial t} + \bar{u}\frac{\partial \rho}{\partial x} + v\frac{\partial \bar{\rho}}{\partial y} + w\frac{\partial \bar{\rho}}{\partial z_1} = \varkappa\frac{\partial^2 \rho}{\partial z_1^2}, \tag{16}$$

where $\bar{u} = \bar{u}(y, z_1)$, $\bar{\rho} = \bar{\rho}(y, z_1)$ are respectively the zonal values of velocity and density.

If one uses the geostrophic approximation, then $w =$

$$= w(x, y, 0, t) + \frac{\beta}{\rho_0 l^2}\int_0^{z_1}\frac{\partial P}{\partial x}\,dz_1,$$

and, using also (11), we obtain from (16)

$$\frac{1}{g}\frac{\partial^2 P}{\partial t\,\partial z_1} + \frac{\bar{u}}{g}\frac{\partial^2 P}{\partial x\,\partial z_1} + \frac{1}{\rho_0 l}\frac{\partial \bar{\rho}}{\partial y}\frac{\partial P}{\partial x} + \frac{\beta}{\rho_0 l^2}\frac{\partial \bar{\rho}}{\partial z_1}\int_0^{z_1}\frac{\partial P}{\partial x}\,dz_1 =$$

$$= \frac{\varkappa}{g}\frac{\partial^3 P}{\partial z_1^3} - \frac{\partial \bar{\rho}}{\partial z_1} w(x, y, 0, t) = f(x, y, z_1, t). \tag{17}$$

We have obtained one equation for determining $P(x, y, z_1, t)$. It is known that, by solving (17), we must obtain an intensification of the currents toward the western coast, caused by the last term on the left-hand side. The presence of $\partial P/\partial x$ in the second and third terms on the left-hand side indicates that horizontal advection also participates in this process of intensification, strengthening or weakening it. For a visual illustration of this, let us represent the nonzonal pressure field in a very simplified form $P = P_0(x, t)e^{-k(y,t)z_1}$; then, multiplying (17) also by $-ke^{kz_1}$, we obtain

$$\frac{k^2}{g}\frac{\partial P_0}{\partial t} - \left[\frac{k}{\rho_0 l}\frac{\partial \bar{\rho}}{\partial y} - \bar{u}\frac{k^2}{g} + \frac{\beta}{\rho_0 l^2}\frac{\partial \bar{\rho}}{\partial z_1}\left(e^{kz_1} - 1\right)\right]\frac{\partial P_0}{\partial x} = -ke^{kz_1} f. \tag{18}$$

The first two terms in the square brackets are due to the horizontal transport of density; in order of magnitude they are no smaller than the third term, caused by the β-effect; this is easily verified by assigning the following values: $k = 2 \cdot 10^{-5}$ cm^{-1}, $l = 10^{-4}$ sec^{-1}, $g = 10^3$ cm/sec, $\partial \bar{\rho}/\partial y = 10^{-11}$ g/cm^4, $\bar{u} = 10$ cm/sec, $\partial \bar{\rho}/\partial z_1 = 10^{-8}$ g/cm^4, $\beta = 2 \cdot 10^{-13}$.

Further assuming that everywhere $\partial \bar{\rho}/\partial y > 0$, we come to the conclusion that the contribution of density advection is determined mainly by the sign of $\bar{u}$; in southern latitudes (10° S $\leqslant \varphi \leqslant$ 30° S) the zonal transport of masses is directed from east to west, i.e. $\bar{u} < 0$. Consequently, the signs of all three terms in the square brackets of (18) coincide with one another, i.e. advection promotes the "pressing"of currents toward the western shore. In middle latitudes (30° S $\leqslant \varphi \leqslant$ 50° S), on the contrary, $\bar{u} > 0$, and advection "tears" the intense currents away from the western shore.

Thus, in addition to the change of the Coriolis parameter with latitude, whose important role was shown by Stommel ([9]), one should emphasize still another substantial factor contributing to the intensification of currents toward the western coast—the zonal transport of water masses in southern latitudes from east to west. This qualitative conclusion is physically quite plausible.

A more reliable answer to the question of the role of advection can be obtained by solving problem (9)–(15). We solve it numerically by time steps, replacing the derivatives by finite differences, for example $\partial u/\partial t = (u - u_0)/\tau$. The process of the development of wind currents and redistribution of masses is a long one; the time step τ may be taken on the order of days, and for the initial stage even larger, and therefore we neglect terms of order $1/(l\tau)^2$ in comparison with unity. From (9), (10) we obtain

$$u = \frac{v_0}{l\tau} - \frac{1}{\rho_0 l}\frac{\partial P}{\partial y} - \frac{1}{\rho_0 l^2 \tau}\frac{\partial P}{\partial x}; \tag{19}$$

$$v = -\frac{u_0}{l\tau} + \frac{1}{\rho_0 l}\frac{\partial P}{\partial x} - \frac{1}{\rho_0 l^2 \tau}\frac{\partial P}{\partial y}. \tag{20}$$

Substituting the values of u, v from (19), (20) into (12) and taking into account condition (8) for $z = h$ ($z_1 = 0$), we find the expression for the vertical velocity

$$w = -\frac{1}{\rho_0 g}\frac{\partial P}{\partial t} + \frac{\beta h}{\rho_0 l^2}\frac{\partial P}{\partial x} - \frac{1}{2\alpha' \rho_0 l}\Delta P_0' - \frac{1}{l\tau}\int_0^{z_1}\left(\frac{\partial v_0}{\partial x} - \frac{\partial u_0}{\partial y}\right) dz_1 +$$

$$+\frac{1}{\rho_0 l^2 \tau}\int_0^{z_1}\Delta P\, dz_1 - \frac{\beta}{l^2\tau}\int_0^{z_1} u_0\, dz_1 + \frac{\beta}{\rho_0 l^2}\int_0^{z_1}\frac{\partial P}{\partial x}\, dz_1. \tag{21}$$

Substituting the values of u, v, w from (19), (20), (21) into (13) and using (11), we arrive at the equation $\left(\frac{1}{\rho_0 g}\frac{\partial P}{\partial t}\frac{\partial \rho}{\partial z_1} \ll \frac{\partial \rho}{\partial t}\right)$:

$$\begin{aligned}\rho - \chi\tau\frac{\partial^2\rho}{\partial z_1^2} + \frac{v_0}{l}\frac{\partial\rho}{\partial x} - \frac{g\tau}{\rho_0 l}\frac{\partial\rho}{\partial x}\int_0^{z_1}\frac{\partial\rho}{\partial y}\, dz_1 - \frac{g}{\rho_0 l^2}\frac{\partial\rho}{\partial x}\int_0^{z_1}\frac{\partial\rho}{\partial x}\, dz_1 \\ -\frac{u_0}{l}\frac{\partial\rho}{\partial y} + \frac{g\tau}{\rho_0 l}\frac{\partial\rho}{\partial y}\int_\infty^{z_1}\frac{\partial\rho}{\partial x}\, dz_1 - \frac{g}{\rho_0 l^2}\frac{\partial\rho}{\partial y}\int_0^{z_1}\frac{\partial\rho}{\partial y}\, dz_1 \\ -\frac{1}{l}\frac{\partial\rho}{\partial z_1}\int_0^{z_1}\left(\frac{\partial v_0}{\partial x} - \frac{\partial u_0}{\partial y}\right) dz_1 + \frac{\beta}{\rho_0 l^2}\frac{\partial\rho}{\partial z_1}\int_0^{z_1}\int_\infty^{z_1}\Delta\rho\, dz_1\, dz_1 \\ -\frac{\beta}{l^2}\frac{\partial\rho}{\partial z_1}\int_0^{z_1} u_0\, dz_1 + \frac{g\beta\tau}{\rho_0 l^2}\frac{\partial\rho}{\partial z_1}\int_0^{z_1}\int_\infty^{z_1}\frac{\partial\rho}{\partial x}\, dz_1 + \frac{\beta\tau h g}{\rho_0 l^2}\frac{\partial\rho}{\partial z_1}\int_\infty^{z_1}\frac{\partial\rho}{\partial x}\, dz_1 = \rho_0' + \frac{\tau}{2\alpha'\rho_0 l}\frac{\partial\rho}{\partial z_1}\Delta\rho_0'.\end{aligned} \tag{22}$$

Thus, the problem of theoretically constructing the process of the emergence, development, and establishment of wind currents in a baroclinic ocean is reduced to solving the complicated nonlinear integro-differential equation (22).

The horizontal boundary conditions are constructed as follows. On the "liquid" contours bounding the given ocean region from the north and from the south, we prescribe the condition that the density field be independent of x, t, i.e., we assume that there $\rho = \rho(y, z_1)$. For example, when carrying out computations for the part of the North Atlantic under the influence of the Azores maximum (10° N. lat. $\leqslant \varphi \leqslant 60^\circ$ N. lat.), we assume that for $\varphi \leqslant 10^\circ$ N. lat. and $\varphi \geqslant 60^\circ$ N. lat. the tangential friction of the wind is absent, and that the density field is subject only to the influence of stationary thermodynamic factors having a zonal character. The solid boundaries of the region under consideration must be approximated by a computational grid with the given horizontal step L. Then any coastal segment of length $\leqslant L$ will be parallel either to the x-axis or to the y-axis. On such segments we prescribe the condition that the normal component of velocity be equal to zero. If, in addition, the velocities of the currents are replaced by their geostrophic approximation, then it is easy to show that this condition is equivalent to the condition of constant pressure on the coastline. Thus, the boundary condition for each fixed level $z_1 = z_1^0$ will be the condition

$$\int_{\infty}^{z_1^0} \rho \, dz_1 = 0$$

on the contour of the grid coastline.

It should be noted that in an idealized, homogeneous ocean the zonal mass transport as a factor in the intensification of currents toward the western coast manifests itself not through accounting for nonlinear terms in the equations of motion, but through a more careful accounting of the boundary conditions for w, i.e., from the condition

$$w\big|_{z=0} = -\left(\frac{\partial \zeta}{\partial t} + u\frac{\partial \zeta}{\partial x} + v\frac{\partial \zeta}{\partial y}\right).$$

In the case of a baroclinic ocean, however, it can be shown that the effect of such a boundary condition is small in comparison with density advection.

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Note: Figure translations are in progress. See original paper for figures.