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Abstract

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ON THE CALCULATION OF EVAPORATION FROM BOUNDED WATER BODIES WITH ALLOWANCE FOR HORIZONTAL MIXING

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In paper ([1]) a solution was obtained for the problem of the distribution of humidity over an evaporating surface, taking into account, along with the influence of wind and vertical turbulent exchange, also horizontal turbulent exchange both in the direction of the wind and in the transverse direction. The presence in this solution of a singularity on the evaporating surface prevented the direct calculation of evaporation by differentiation.

In paper ([2]) a special device was described which makes it possible, using, along with the solution of the problem, also its original differential equation, to obtain an expression for evaporation. With the aid of this device an expression for evaporation was found in a somewhat simplified problem—when horizontal mixing in the direction perpendicular to the wind is neglected. Analysis of the corresponding formulas showed that the influence of horizontal mixing (even when it is not fully taken into account) can be very substantial, especially in the case of small dimensions of the evaporating region.

In this connection it is of interest to compute the evaporation also for the general case considered in paper ([1]), i.e., with complete allowance for the influence of horizontal turbulent mixing*. The corresponding expression for evaporation can be obtained with the aid of the above-mentioned special device. In particular, let us consider an evaporating surface having the form of a rectangle with sides $2l$ along the wind and $2h$ across the wind, and let us put the concentration of water vapor equal to the constant value q_0 on it and to zero outside it. Then the expression for the evaporation E has the form

$$
\begin{aligned}
E = {} & \frac{\mu^{1-2\nu}}{2^{3\nu}\pi\Gamma(\nu)}\left(\frac{u_1^2}{k_{x1}k_{z1}}\right)^{\nu}\rho k_{z1}q_0 \times \\
& \times \left\{ \int_{-\bar{h}-\bar{y}}^{\bar{h}-\bar{y}} \left[M_\nu(\bar{x}+\bar{l},\eta) - M_\nu(\bar{x}-\bar{l},\eta)\right] d\eta + \right. \\
& + \int_{-\bar{h}-\bar{y}}^{\bar{h}-\bar{y}} \left[(\bar{x}+\bar{l})M_{1+\nu}(\bar{x}+\bar{l},\eta) - (\bar{x}-\bar{l})M_{1+\nu}(\bar{x}-\bar{l},\eta)\right] d\eta + \\
& \left. + \int_{-\bar{l}-\bar{x}}^{\bar{l}-\bar{x}} \left[(\bar{y}+\bar{h})M_{1+\nu}(-\xi,\bar{y}+\bar{h}) - (\bar{y}-\bar{h})M_{1+\nu}(-\xi,\bar{y}-\bar{h})\right] d\eta \right\},
\end{aligned} \tag{1}
$$

- We note that in formula (4) of paper ([1]), through an oversight of the authors, the factor $\frac{1}{2\pi}$ was omitted.

where the following notation has been introduced: x is the horizontal coordinate along the wind; y is the horizontal coordinate across the wind; ρ is the air density; $k_x = k_{x1}z^m$ is the coefficient of horizontal exchange along the wind; $k_y = k_{y1}z^m$ is the coefficient of horizontal exchange across the wind; $k_z = k_{z1}z^n$ is the coefficient of vertical exchange; $u = u_1 z^n$ is the wind speed; z is height

$$\mu = \frac{2+m-n}{2}; \qquad \nu = \frac{1-n}{2+m-n};$$

$$\bar{x} = \frac{u_1}{2k_{x1}}x; \qquad \bar{l} = \frac{u_1}{2k_{x1}}l; \qquad \bar{y} = \frac{u_1}{2\sqrt{k_{x1}k_{y1}}}y; \qquad \bar{h} = \frac{u_1}{2\sqrt{k_{x1}k_{y1}}}h;$$

$$M_p(\alpha,\beta) = e^{\alpha}\frac{K_p\left(\sqrt{\alpha^2+\beta^2}\right)}{(\alpha^2+\beta^2)^{p/2}}.$$

The power-law dependence of the exchange coefficients and wind speed on height has been adopted in accordance with ([1,2]).

With the aid of expression (1), evaporation can be calculated at any point of the region under consideration. For analysis, however, it is more convenient to use the specific evaporation, i.e., the evaporation from the entire region referred to its area. The expression for the specific evaporation ε_0 has the form

$$\varepsilon_0 = \frac{\mu^{1-2\nu}}{2^{3\nu}\pi\Gamma(\nu)} \left(\frac{u_1^{1/\nu}}{k_{x1}k_{z1}}\right)^{\nu} \rho k_{z1} q_0 \times$$
$$\times \left\{ \int_0^{2\bar{l}} \int_0^{2\bar{h}} \left(1 - \frac{\bar{y}}{2\bar{h}}\right) [M_{1+\nu}(\bar{x},\bar{y}) + M_{1+\nu}(-\bar{x},\bar{y})] \frac{\bar{x}}{\bar{l}}\, d\bar{y}\, d\bar{x} \right.$$
$$+ \int_0^{2\bar{l}} \int_0^{2\bar{h}} \left(1 - \frac{\bar{y}}{2\bar{h}}\right) [M_{\nu}(\bar{x},\bar{y}) - M_{\nu}(-\bar{x},\bar{y})] \frac{1}{\bar{l}}\, d\bar{y}\, d\bar{x} \tag{2}$$
$$\left. + \int_0^{2\bar{l}} \int_0^{2\bar{h}} \left(1 - \frac{\bar{x}}{2\bar{l}}\right) [M_{1+\nu}(\bar{x},\bar{y}) + M_{1+\nu}(-\bar{x},\bar{y})] \frac{\bar{y}}{\bar{l}}\, d\bar{y}\, d\bar{x} \right\}.$$

Calculations by formula (2) can be carried out only by numerical methods. However, for two limiting cases—very small and very large dimensions of the evaporating region—the calculations can be performed exactly. These limiting cases are also of independent interest. In considering them, for simplicity we shall take $l = h$ and $k_{x1} = k_{y1} = k_0$.

For the case of large dimensions of the evaporating region ($\bar{l} = \bar{h} \gg 1$), we obtain from (2)

$$\varepsilon_0 = \frac{2^{1-3\nu}\mu^{1-2\nu}}{(1-\nu)\Gamma(\nu)} \left(\frac{u_1}{k_{z1}}\right)^{\nu} \rho k_{z1} q_0 \frac{1}{l^{\nu}}, \tag{3}$$

and for small dimensions of the evaporating region

$$\varepsilon_0 = \frac{2^{2(1-\nu)}\mu^{1-2\nu}}{\pi} \int_0^1 (1-t) \left[\frac{1}{t^{2\nu}} - \frac{1}{(1+t^2)^{\nu}}\right] dt \cdot \left(\frac{k_0}{k_{z1}}\right)^{\nu} \rho k_{z1} q_0 \frac{1}{l^{2\nu}}. \tag{4}$$

According to (3), the specific evaporation decreases with increasing linear dimensions of the evaporating region as $l^{-\nu}$ and does not depend on horizontal mixing. Hence it follows that horizontal mixing has no effect on the specific evaporation from large homogeneous reservoirs.

According to (4), the specific evaporation increases with decreasing linear dimensions of the evaporating region as $l^{-2\nu}$ and does not depend on the wind speed. In this case the influence of horizontal mixing on evaporation is very substantial. Complete neglect of horizontal mixing leads, in the case of small regions, to results that are incorrect not only quantitatively but even qualitatively.

It is also possible to take approximate account of the influence of horizontal mixing on evaporation, namely by considering in the original equation of the problem either only mixing in the direction of the wind ($k_x \neq 0,\ k_y = 0$), or only mixing in the direction transverse to the wind ($k_x = 0,\ k_y \neq 0$). The first variant was considered by us in paper ([2]), and the second by G. Kh. Tseytin in papers ([3],[4]). As applied to evaporating regions of small size, both of these approximate approaches are equivalent, which is natural, since the exact expression (4) does not contain the wind speed and, consequently, all horizontal directions prove to be equipollent. These approximate approaches lead to the following expression for the specific evaporation from small regions:

$$\varepsilon_0' = \frac{2^{1-2\nu}\mu^{1-2\nu}\Gamma(1/2+\nu)}{(1-2\nu)\sqrt{\pi}\,\Gamma(\nu)} \left(\frac{k_0}{k_{z1}}\right)^{\nu} \rho k_{z1} q_0 \frac{1}{l^{2\nu}}. \tag{5}$$

Expression (5) differs from the exact value (4) only by a numerical factor, namely

$$\frac{\varepsilon_0'}{\varepsilon_0} = \frac{\sqrt{\pi}\,\Gamma(1/2+\nu)}{2(1-2\nu)\Gamma(\nu)\displaystyle\int_0^1 (1-t)\left[\frac{1}{t^{2\nu}} - \frac{1}{(1+t^2)^{\nu}}\right] dt}. \tag{6}$$

The possible values of ν in our problem are contained in the range from 0 to 1/2, and the characteristic ν for various conditions of thermal stratification of the near-surface layer of the atmosphere are: $\nu = 1/12$ for convective conditions; $\nu = 1/9$ for equilibrium conditions; $\nu = 1/6$ for inversion conditions. The ratio $\varepsilon_0'/\varepsilon_0$ varies over the indicated range of values of ν from 1 to 1/2, respectively, and for the indicated characteristic values of ν is $\varepsilon_0'/\varepsilon_0 = 0.89$ for convective conditions; $\varepsilon_0'/\varepsilon_0 = 0.84$ for equilibrium conditions; $\varepsilon_0'/\varepsilon_0 = 0.77$ for inversion conditions.

Naturally, incomplete accounting for horizontal mixing leads to underestimated values of evaporation. This underestimation is the more significant, the greater the degree of thermal stability of the near-surface layer of the atmosphere.

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