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Fig. 1

Figure 1: Fig. 1

Abstract

Full Text

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THEORY OF ELASTICITY

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ON POSTCRITICAL DEFORMATIONS OF COMPRESSED CYLINDRICAL SHELLS

It has been established experimentally that a cylindrical shell under conditions of axial compression fails long before the stresses determined under the condition that the cylindrical form is preserved reach the strength limit of the material. The reason is that, at a certain value p_k of the compressive force, stability of the cylindrical form is lost; as a result, a redistribution of stresses occurs, and in individual places they reach destructive values.

The process of transition into the postcritical elastic state of a compressed cylindrical shell takes place instantaneously (the shell snaps), and the load perceived by it sharply decreases to a certain value p_k^* , which is usually called the lower critical load, in contrast to the value p_k —the upper critical load.

Determination of the value of the lower critical load has been the subject of many experimental and theoretical investigations*. This problem is rather difficult and, as it seems to us, has not yet received a satisfactory solution. The difficulty lies in the fact that, first, after snapping the shell has a form far removed from the initial one, and a regular approximation of this form is very difficult; second, as the results of experimental investigations show, in practically important cases of the ratios of the thickness and diameter of the shell, the consideration must be carried out with allowance for plastic deformations arising during snapping.

Fig. 1

In the present note we set forth some results of an investigation of the postcritical elastic state of a compressed cylindrical shell (after snapping). We wish to give a qualitative description of the process of transition of the shell into the postcritical elastic state and to determine the value of the lower critical load.

1. We assume that the deformation of the middle surface of a compressed cylindrical shell during the transition into the postcritical elastic state is essentially a geometrical bending. In this connection we approximate the form of the surface under such deformation by a surface Z , isometric to a cylinder (Fig. 1). The surface Z has periodicity of structure in two directions and has special lines (ridges) separating the regular parts in the form of corrugations. The character of the periodicity of the surface Z is determined by two integer parameters m and n , and the form of the regular part is given by a certain periodic function $y(x)$.
2. The bending energy of the shell under postcritical deformation U is conventionally divided into two parts: U_{Δ} and U_{γ} . U_{γ} is the energy due to strong local bending along the ridges γ , while U_{Δ} is the bending energy of the remaining part

* An extensive bibliography on this question is given in the book (1).

surface of the shell. The energy U_A is determined in the usual way, i.e., by the deformation of bending of the initial surface into the surface Z . To determine the energy of local bending U_y , we use the formula obtained in the author's work (2). The function $y(x)$, which determines the shape of the deformed surface of the shell for a given character of waviness (the numbers m and n), is found from the condition of a minimum of the functional $U(y)$ under fixed axial compression, which is expressed in the form of a certain integral relation for the function $y(x)$. Thus, the energy of elastic deformation of the shell is obtained as a function of the axial compression h and two parameters characterizing the general periodicity of the structure.

3. We proceed from the assumption that in the process of transition to a postcritical elastic state, i.e., during snap-through, the character of the periodicity of the structure of the shell shape is preserved and, consequently, remains the same as at the moment of loss of stability. But the parameters characterizing the waviness at the moment of loss of stability are not arbitrary—they are connected by a certain relation. This makes it possible, in the expression U , to eliminate one parameter, and the energy U becomes a known function of the axial compression h and the parameter ξ , which represents, roughly speaking, the ratio of the dimensions of the dents on the shell surface that arise during snap-through. After the expression for the energy of elastic deformation has been obtained, the investigation of the equilibrium state of the shell presents no difficulty.
4. The postcritical elastic equilibrium states of a compressed cylindrical shell under relatively small axial compression are unstable. This, as is known, agrees with experimental data on the nature of the transition of the shell into the postcritical elastic state with snap-through.

If it is assumed that the character of the periodicity of the shape of the shell surface is preserved throughout the entire postcritical deformation, then transition

to a stable postcritical state is possible only when the value of the parameter $\xi < 0.91$. In this case the smallest load perceived by the shell, corresponding to a stable equilibrium state, i.e., the lower critical load, is determined by the formula

$$p_k^* = 0.15E \frac{\delta}{R},$$

where R is the radius of the shell, δ is the thickness, and E is the modulus of elasticity of the material.

If, at the moment of loss of stability, the value of the parameter $\xi > 0.91$, then the transition of the shell to a stable equilibrium state during snap-through is inevitably associated with a change in the character of the periodicity of the structure of the surface shape, i.e., with additional snap-throughs, during which the value of the parameter ξ decreases. This explains the well-known experimentally established fact: the dents on the shell surface after snap-through are, as a rule, compressed along the generator.

5. The results listed above pertain to the case of shells of unlimited elasticity. For real shells, possessing limited elasticity, these results are applicable only to sufficiently thin shells. In particular, the formula given above for p_k^* may be used in the case when stresses of magnitude $\sigma = 3E \frac{\delta}{R}$ do not lead to significant plastic deformations in the shell material.

Investigation of the postcritical equilibrium state of comparatively thick shells is impossible without taking plastic deformations into account.

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CITED LITERATURE

¹ A. S. Vol' mir, *Flexible Plates and Shells*, Moscow, 1956. ² A. V. Pogorelov, *Theory of Convex Elastic Shells in the Postcritical Stage*, Kharkov, 1960.

Note: Figure translations are in progress. See original paper for figures.

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