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Fig. 1. System with a nonlinear pulse element

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Abstract

Full Text

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PERIODIC REGIMES IN SYSTEMS WITH NONLINEAR PULSE ELEMENTS

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1. In a pulse system (1) with a nonlinear pulse element (NPE, see Fig. 1), the input of the linear part LP receives a sequence of pulses $z(\bar{t})$, modulated in amplitude, width, or position, and related by a nonlinear dependence to the discrete values of the input signal of the pulse element $u(\bar{t}) = u[n]$. The problem of determining periodic regimes in such a system reduces to the consideration of a nonautonomous system: the pulse element periodically opens the loop of the system, as a result of which the frequency of oscillations in it proves to be rigidly connected with the repetition frequency of the pulse element. This problem has been solved for several special cases (2). In the frequency approach to its solution the following results have been obtained: 1) in a system with a linear part having the properties of a low-pass filter, and 2) with pulse elements: a) amplitude relay elements with a dead zone (3), b) without a dead zone (4), and c) with a width pulse element (5). Periodic motions determined from the first harmonic have been found whose frequencies ω_1 are related to the repetition frequency of the pulse element $\omega_0 = 2\pi/T_0$ by the condition

Fig. 1. System with a nonlinear pulse element

$$N = \frac{\omega_0}{\omega_1} = \frac{2\pi}{\omega_1} \geqslant 2, \quad \text{integer}. \tag{1}$$

Below a frequency approach is set forth for the approximate determination of periodic motions of type (1) in a system with an arbitrary amplitude, width, or time pulse element.

2. Let, in the system of Fig. 1 with a linear part possessing the properties of a low-pass filter, there be established a periodic regime which, at the input of the pulse element, can be approximated by the first harmonic of frequency $\bar{\omega}_1 = 2\pi/N$, $N \geqslant 2$, an integer (it is assumed that the constant component in the periodic motion is absent; $\bar{t} = t/T_0$ is relative time):

$$u(\bar{t}) = C\cos\left(\frac{2\pi}{N}\bar{t} + \psi\right), \quad -\frac{\pi}{N} \leqslant \psi \leqslant \frac{\pi}{N},$$

or, in complex form,

$$U = Ce^{i\psi}. \tag{2}$$

In this case, at the output of the pulse element there will be a periodic sequence of pulses, $z(\bar{t})$, whose frequency coincides with the frequency of signal (2).

The equation of the periodic regime in the first harmonic, according to the harmonic-balance method (6), can be written in the form

$$\left[1 + W\left(j\frac{2\pi}{N}\right)J^*\right]U = 0;$$

here $W(j\bar{\omega})$ is the frequency characteristic of the linear part; $J^* = Z_1/U$ is the equivalent complex gain coefficient of the nonlinear pulse element; Z_1 is the complex amplitude of the first harmonic at the output of the nonlinear pulse element. Hence we find the approximate condition for existence of a periodic regime

$$W\left(j\frac{2\pi}{N}\right) = -\frac{1}{J^*}. \tag{3}$$

3. The complex amplitude of the first harmonic of the response of the pulse element to signal (2) can be represented by a sum of the form

$$Z_1 = \sum_{s=0}^{N-1} A_s. \tag{4}$$

Let us write the periodic sequence of δ-functions with period T_0 in the form

$$\delta_{T_0}(\bar{t}) = \sum_{s=0}^{N-1} \delta_{NT_0}(\bar{t} - s);$$

here

Fig. 2. Amplitude nonlinear pulse element

Figure 2: Fig. 2. Amplitude nonlinear pulse element

$$\delta_{NT_0}(\bar{t}-s) = \frac{1}{NT_0}\left[1 + 2\sum_{l=1}^{\infty}\cos l\frac{2\pi}{N}(\bar{t}-s)\right]. \tag{5}$$

The first harmonic of (5) has the form

$$a_1(s) = \frac{2}{NT_0}e^{-j\frac{2\pi}{N}s},$$

whence for (4) we have

$$Z_1 = \sum_{s=0}^{N-1}\alpha_s K_i \Phi_{\gamma_s}\left(j\frac{2\pi}{N}\right)a_1(s+\beta_s).$$

Here $\alpha_s,\ \beta_s,\ \gamma_s = F_{\alpha,\beta,\gamma}(u[s])$ are the relative amplitude (with allowance for sign), shift, and width of the s-th pulse; $x_n = F_x(u[n])$ is the modulation characteristic of the pulse element; $K_i = \text{const} > 0$ is the nominal pulse amplitude; $\Phi_{\gamma_s}(j\bar{\omega})$ is the complex spectrum of the s-th pulse.

Fig. 2. Amplitude nonlinear pulse element

The complex amplitude of the first harmonic at the output of the amplitude pulse element can also be written in the form of a series

$$Z_1 = \sum_{l=-\infty}^{\infty}B_{1+lN}, \quad l = \lambda\left[1 + N - 2E\left(\frac{N}{2}\right)\right], \tag{6}$$

where

$$B_{1+lN} = \frac{1}{T_0}K_i\Phi_\gamma\left(j\frac{2\pi}{N}\right)B'_{1+lN};$$

B'_{1+lN} is the complex amplitude of the $(1+lN)$-th harmonic of the response to signal (2) of an ordinary nonlinear element of the same type, with respect to the nonlinearity characteristic, as the amplitude-pulse element under consideration.

Let us represent the nonlinear amplitude-pulse element (Fig. 2) as a series connection of an ordinary nonlinear element NE, an ideal pulse element IPE, and a shaping device SD.

with frequency characteristic $K_i\Phi_\gamma(j\bar{\omega})$. The ideal pulse element forms a sequence of instantaneous pulses whose areas are proportional to the discrete values of the input signal $v(\bar{t}) = v[n]$. The response of the nonlinear element to signal (2) has the form

$$v(\bar{t}) = \frac{1}{2} \sum_{k=-\infty}^{\infty} B'_{2k-1} e^{j(2k-1)\frac{2\pi}{N}\bar{t}}.$$

The sequence of instantaneous pulses at the output of the ideal pulse element is

$$w(\bar{t}) = \frac{1}{2T_0} \sum_{k,l=-\infty}^{\infty} B'_{2k-1} e^{j(2k-1+lN)\frac{2\pi}{N}\bar{t}},$$

whence the complex amplitude of the first harmonic is

$$W_1 = \frac{1}{T_0} \sum_{l=-\infty}^{\infty} B'_{1+lN}, \qquad l = \lambda \left[1 + N - 2E\left(\frac{N}{2}\right)\right],$$

and, finally, for (6)

$$Z_1 = K_i \Phi_\gamma \left(j\frac{2\pi}{N}\right) W_1.$$

It follows from (6) that the complex amplitude of the first harmonic at the output of a nonlinear amplitude-pulse element, accurate to within a constant factor for the given element, is the sum of the complex amplitudes of the first harmonic and an infinite set of high-frequency components of the response of an ordinary nonlinear element to (2).

It is essential that the response of a nonlinear pulse element to a sinusoidal signal, in contrast to the response of an ordinary nonlinear element to the same signal, depends not only on the amplitude but also on the frequency and phase of the input signal and, in addition, on the pulse shape, repetition frequency, and duty factor of the pulse element.

4. Relations (4) and (6), applied to (2), determine the equivalent complex gain coefficient of the NPE. Thus, from (4)

$$J^*(C, N, \psi, F, \Phi_\gamma, T_0) = \frac{1}{C} \sum_{s=0}^{N-1} A_s e^{-j\psi}. \tag{7}$$

For each value of the frequency $\bar{\omega}_1 = 2\pi/N,\ N = 2, 3, ...,$ the locus of the right-hand side of equation (3) represents the set of values of the modulus and phase of the frequency characteristic for which there exists a periodic solution of the given frequency. In the general case this is a certain region in the plane of the amplitude-phase characteristic. In the space with coordinates: modulus, phase, frequency, the locus of (3) represents a collection of regions corresponding to different values of the frequency $\bar{\omega}_1 = 2\pi/N,\ N = 2, 3, \ldots$. The graphical solution of (3) consists in finding the intersection points of the frequency characteristic, represented in the coordinates: modulus, phase, frequency, with the indicated regions. The problem is most simply solved by considering the corresponding projections onto the coordinate planes. In this case the ordinary amplitude-phase, amplitude-frequency, and phase-frequency characteristics are used as the calculated ones.

5. The determination of periodic modes in systems with amplitude (and time) modulation is conveniently carried out by referring the shaping device

of the impulse element to the linear part. The calculated frequency characteristic then takes the form

$$W_\Phi(j\bar{\omega}) = K_\mathrm{i}\Phi_\gamma(j\bar{\omega})W(j\bar{\omega}),$$

and from expressions (4), (6), (7) the factor $K_\mathrm{i}\Phi_\gamma\left(j\frac{2\pi}{N}\right)$ is eliminated.

6. Approximation of the periodic signal at the output of the linear part by the first harmonic and the application of the harmonic-balance method are justified only under the condition that the linear part satisfies the filter hypothesis (7). This condition can be fulfilled in impulse reproducing systems, since the linear part of the latter, by necessity, must satisfy the requirement

$$\omega_\mathrm{cp} \ll \frac{\omega_0}{2}; \tag{8}$$

here ω_cp is the cutoff frequency of the linear part. Condition (8) is a consequence of the well-known sampling theorem of Kotelnikov[8].

When (8) is satisfied, the impulse system is close to the continuous one (1); however, time quantization introduces a certain specificity into the nature of the phenomena occurring in the system. The approach described above, within the limits of the restrictions inherent in approximate methods, makes it possible to take into account the influence of time quantization on the character of periodic regimes in systems satisfying condition (8).

7. The results of the work show that the problem of the approximate determination of periodic regimes in systems with nonlinear impulse elements is solved in a unified way for various types of impulse modulation and various types of nonlinearities in impulse elements.

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Note: Figure translations are in progress. See original paper for figures.