EQUILIBRIUM AND MOTION OF A SPHERE IN A VISCO-PLASTIC LIQUID

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Abstract

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EQUILIBRIUM AND MOTION OF A SPHERE IN A VISCO-PLASTIC LIQUID

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In contrast to a viscous liquid and, in particular, to a structureless suspension, in which all bodies with a density different from the density of the liquid always fall or rise ([1]), in a visco-plastic medium possessing a limiting shear stress τ_0, for bodies of any density there exists a critical size d_0, below which the gravitational forces do not overcome the structural strength of the medium and the bodies remain motionless in it. For technological processes using visco-plastic liquids (beneficiation of minerals in a heavy suspension, drilling of wells and removal of rock particles by clay solutions), the magnitude of the critical diameter of an equilibrium particle and the laws of motion of particles in a liquid possessing plasticity are of substantial interest.

The resistance force F, acting on a sphere moving in a visco-plastic medium, is determined by the dimensional parameters

$$[v] = LT^{-1}, \quad [d] = L, \quad [\bar{\rho}] = ML^{-3}, \quad [\tau_0] = ML^{-1}T^{-2}, \quad [\eta'] = ML^{-1}T^{-1}, \tag{1}$$

where v is the velocity of motion of the sphere, d is the diameter of the sphere, η' is the plastic viscosity of the medium, and $\bar{\rho}$ is the density of the medium.

From dimensional considerations one may write

$$F = \tau_0 d^2 f(x, y), \tag{2}$$

where $x = \eta' v/\tau_0 d$ and $y = \bar{\rho} v^2/\tau_0$.

For slow motions the arguments of the function will be small, and one may use the Taylor expansion of the latter. Restricting ourselves to terms of the second order in v, we have

$$F = \tau_0 d^2 \left[f(0,0) + f'_x(0,0)\frac{\eta' v}{\tau_0 d} + f'_y(0,0)\frac{\bar{\rho} v^2}{\tau_0} + \frac{1}{2} f''_{xx}(0,0)\frac{\eta' v}{\tau_0 d} + \ldots \right] =$$

$$= \alpha\tau_0 d^2 + \beta\eta' v d + \gamma\bar{\rho} v^2 d^2 + \delta\frac{(\eta')^2}{\tau_0} v^2. \tag{3}$$

To determine the velocity of steady falling, we equate this force to the weight of the sphere after subtraction of the Archimedean force,

$$\frac{\pi}{6} d^3(\rho' - \bar{\rho})g = \alpha\tau_0 d^2 + \beta\eta' v d + \gamma\bar{\rho} v^2 d^2 + \delta\frac{(\eta')^2}{\tau_0} v^2. \tag{4}$$

In particular, for $v_0 = 0$,

$$d_0 = \alpha\left[\frac{6\tau_0}{\pi(\rho' - \bar{\rho})g}\right],$$

where ρ' is the density of the sphere.

For the slowest motions, when the flow of the liquid approaches Bingham flow with the same constant τ_0 as is determined on viscosimeters for relatively rapid motions, the coefficient α can be calculated exactly.

Consider a sphere in equilibrium in a viscoplastic fluid. To determine the force R holding the sphere in a stationary state, we find the sum of the projections of τ_0 on the vertical axis (Fig. 1). On the surface of the sphere, take a strip of length $2\pi r\cos\theta$ and width $r\,d\theta$. The projection of τ_0 acting on an element of the strip is equal to $\tau_0\cos(\pi/2 - \theta) = \tau_0\sin\theta$. Then the force is

$$R = \int_0^{\pi} \tau_0 \sin\theta \cdot 2\pi r \sin\theta \cdot r\,d\theta = 2\pi r^2\tau_0 \int_0^{\pi} \sin^2\theta\,d\theta = \pi^2 r^2\tau_0. \tag{5}$$

From the equality $F_{v=0}$ and R we have

$$\alpha = \frac{\pi^2}{4} \simeq 2.5; \tag{6}$$

hence

$$d_0 = \frac{3}{2}\pi\,\frac{\tau_0}{(\rho' - \rho)g} \simeq \frac{4.5\,\tau_0}{(\rho' - \rho)g}. \tag{7}$$

For flows of dispersed systems that only approximate a Bingham fluid, the value τ_0, measured experimentally, is often considerably higher than the true one, and determination of d_0 for such dispersed systems is associated with experimental determination of the value of the numerical coefficient in (7). Thus, in the experiments of P. M. Khomikovskii and D. I. Shilov ([3]), the value of the numerical coefficient in formula (7) varied from 1.68 to 18, while in the monograph of A.

Fig. 1. Equilibrium of a sphere in a viscoplastic fluid

Figure 1: Fig. 1. Equilibrium of a sphere in a viscoplastic fluid

Kh. Mirzadzhanzade ([4]) an experimental value of the coefficient equal to 18 is given.

Fig. 1. Equilibrium of a sphere in a viscoplastic fluid

The most convenient method for determining this coefficient is to measure the velocity of motion of specimens in the fluid under investigation. To determine the minimum number of velocity measurements, consider expansion (3). Obviously, in cases where all 4 terms are significant, it is necessary to solve jointly a system of 4 equations. However, in the case of low velocities of motion, for which the ratio of the inertial terms to the viscous terms is small, i.e.

$$\left(\gamma\rho v^2 d^2 + \delta\left(\eta'\right)^2 \tau_0^{-1} v^2\right) / \left(\alpha\tau_0 d^2 + \beta\eta' v d\right) \ll 1, \tag{8}$$

to determine the coefficient α it is possible to confine oneself to the data of 2 velocity measurements.

Equating the weight of the particle in the medium to the sum of the viscous terms and solving the resulting equations with respect to v, we obtain a system of 2 equations, whence the coefficient α is calculated by the formula

$$\alpha = \frac{\pi}{6}\,\frac{d_1^2(\rho_1' - \rho)v_2 - d_2^2(\rho_2' - \rho)v_1}{\tau_0(d_1 v_2 - d_2 v_1)}\,g, \tag{9}$$

where d_1 and d_2 are the diameters of two balls, ρ_1' and ρ_2' are their densities, and v_1 and v_2 are the velocities of their motion in the medium.

To find the laws governing the motion of bodies in a viscoplastic medium, experiments were carried out to measure the falling velocities of balls of different size and density in media with various rheological pa-

parameters (η' and τ_0), preliminarily determined in a rheoviscosimeter with coaxial cylinders [5]. Measurement of the velocity of motion of the balls was carried out on a special apparatus by marking them with radioactive cobalt [2]. The problem of the investigation was reduced to finding the relation between the dimensional parameters (1).

The experimental results were processed in the coordinates $\tilde{d}$, $\tilde{v}$, where

$$\tilde{v} = \left[\left(v d\bar{\rho}/\eta'\right)\left(6\bar{\rho}v^2\right)/\pi d(\rho' - \bar{\rho})\right]^{1/3} = v/v_*, \tag{10}$$

where

Fig. 2

Figure 2: Fig. 2

$$v_* = \left[\pi(\rho' - \bar{\rho})\, g\nu'/6\rho\right]^{1/2}, \tag{11}$$

$$\tilde{d} = \left\{\left[\pi d(\rho' - \bar{\rho})\, g/6\rho v^2\right]\left[\rho^2 v^2 d^2/(\eta')^2\right]\right\}^{1/3} = d/d_*, \tag{12}$$

$$d_* = \nu'/v_*$$

(ν' is the kinematic plastic viscosity).

A parameter simultaneously taking into account the plastic viscosity of the medium and its yield shear stress, and not depending directly on the particle diameter and its velocity, is

$$N_* = \tau_0 d_*^2 \bar{\rho}/(\eta')^2. \tag{13}$$

Substituting the obtained quantities into (2), we find

$$\tilde{d} = N_* \varphi\left(\tilde{v}/N_* d,\ v^2/N_*\right), \tag{14}$$

and finally conclude that

$$\tilde{v} = f(\tilde{d}, N_*). \tag{15}$$

Fig. 2. Dependence of $\tilde{v}$ on $\tilde{d}$ for the motion of a sphere in a viscoplastic fluid. 1 $-N_* = 10.6$; 2 −15.7; 3 −15.9; 4 −18.1; 5 −18.2; 6 −21.8; 7 −24.1; 8 −35.4; 9 −43.4; 10 −48.4; 11 −53.5; 12 −61.2; 13 −66.5; 14 −95.7; 15 −102.3; 16 −108.0; 17 −121.0; 18 −122.0; 19 −140.0; 20 −153.6; 21 −166.6; 22 −170.0; 23 −186.5; 24 −230; 25 −242; 26 −314; 27 −521.

Figure 2 gives the experimental results of measuring the falling velocity of balls in baryte suspensions of different concentrations and dispersity of the solid phase (3.26; 2.19; 1.88 and 1.26 μ) on a logarithmic scale. Through the experimental points lines $N_* =$ const have been drawn. As a result of approximating the graphs, the experimental relation was obtained

$$\lg \tilde{v} = 14 \lg \tilde{d} + 3.9 N_* - 61.289. \tag{16}$$

The formula gives an error of the order of 10% within the investigated range of parameters (v = 0.15–20.0 cm/sec, $\bar{\rho}$ = 1.0–1.65 g/cm^3, η' = 1.0–5.4 cm, τ_0 = 0–40 dyn/cm^2, ρ' = 1.3–1.7 g/cm^3, d = 0.7–1.2 cm).

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Note: Figure translations are in progress. See original paper for figures.