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Abstract

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GEOPHYSICS

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ON THE QUESTION OF THE FORMATION OF PLANETARY DEEP FAULTS IN THE EARTH’ S CRUST

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Let us consider the deformation of a spherical layer whose radius of the outer surface is $R = R_1$, and whose radius of the inner surface is $R = R_0$. On the outer surface the action of forces is equal to zero, while on the inner surface displacements arise that are determined by the deformation of the inner core.

Following P. F. Papkovich, we write the displacement vector of a point of the spherical layer **u** in the form

$$\mathbf{u} = (4(m-1)/m)\mathbf{B} - \mathrm{grad}(R\mathbf{B} + B_0), \tag{1}$$

where m is Poisson’ s number, **B** is a harmonic vector, and B_0 is a harmonic scalar.

Taking into account the axial symmetry of the problem, it can be shown that for any point of the spherical layer

$$u_R = \sum_{n=0}^{\infty} H_n(R) P_n(\cos\theta), \qquad u_\theta = \sum_{n=0}^{\infty} G_n(R) P'_n(\cos\theta)\sin\theta, \tag{2}$$

where

$$H_n(R) = A_n(n+1)(n-2+4/m)R^{n+1} + B_n n R^{n-1} + C^{-n}_{nR} n(n+3-4/m) - D^{-n-2}_{nR}(n+1); \tag{3}$$

$$G_n(R) = A_n(n+5-4/m)R^{n+1} + B^{n-1}_{nR} + C^{-n}_{nR}(-n+4-4/m) + D^{-n-2}_{nR}. \tag{4}$$

In this case the stress in the spherical layer is given by the expansion

$$\sigma_R = 2G\sum_{n=0}^{\infty} I_n(R)P_n(\cos\theta),$$

$$\sigma_\theta = 2G\sum_{n=0}^{\infty}\{K_n(R)P_n(\cos\theta) + L_n(R)P_n'(\cos\theta)\cos\theta\},$$

$$\sigma_\psi = 2G\sum_{n=0}^{\infty}\{M_n(R)P_n(\cos\theta) - L_n(R)P_n'(\cos\theta)\cos\theta\},$$

$$\tau_{R\theta} = -2G\sum_{n=0}^{\infty} J_n(R)P_n'(\cos\theta)\sin\theta, \qquad \tau_{R\psi} = 0, \qquad \tau_{\theta\psi} = 0, \tag{5}$$

where G is the shear modulus, and the expansion coefficients are determined by the equalities

$$I_n(R) = A_n(n+1)(n^2-n-2-2/m)R^n+B_nn(n-1)R^{n-2}-C_nn(n^2+3n-2/m)R^{-n-1}+D_n(n+1)(n+2)R^{-n-3}; \tag{6}$$

$$J_n(R) = A_n(n^2+2n-1+2/m)R^n+B_n(n-1)R^{n-2}+C_n(n^2-2+2/m)R^{-n-1}-D_n(n+2)R^{-n-3}; \tag{7}$$

$$K_n(R) = -A_n(n^2+4n+2+2/m)(n+1)R^n - B_nn^2R^{n-2}+ \\ +C_nn(n^2-2n-1+2/m)R^{-n-1} - D_n(n+1)^2R^{-n-3}; \tag{8}$$

$$L_n(R) = A_n(n+5-4/m)R^n + B_{nR}^{n-2} + C_n(-n+4-4/m)R^{-n-1}+ \\ +D_{nR}^{-n-3}; \tag{9}$$

$$M_n(R) = A_n(n+1)(n-2-2/m-4n/m)R^n + B_nnR^{n-2}+ \\ +C_nn(n+3-4n/m-2/m)R^{-n-1} - D_n(n+1)R^{-n-3}. \tag{10}$$

The conditions for the absence of stresses on the outer surface of the layer at $R = R_1$ are expressed by the equalities

$$\sigma_R = 0, \qquad I_n(R_1) = 0; \qquad \tau_{R\theta} = 0; \qquad J_n(R_1) = 0 \quad \text{for } n = 0, 1, 2, \ldots. \tag{11}$$

When the compression changes, the displacements on the inner surface at $R = R_0$ are determined by the expressions

$$u_R = (\partial R/\partial \alpha)\alpha = -{}^2/_3 R_0 P_2(\cos\theta)\alpha; \tag{12}$$

$$u_\theta = 0, \tag{13}$$

which leads to the system of equations

$$H_n(R_0) = 0 \quad \text{for } n \neq 2; \qquad H_n(R_0) = -{}^2/_3 R_0 \alpha \quad \text{for } n = 2; \tag{14}$$

$$G_n(R_0) = 0 \quad \text{for } n = 0, 1, 2, \ldots \infty. \tag{15}$$

Equations (3), (4), (6), and (7), under conditions (12), (14), and (15), make it possible to determine the coefficients A_n, B_n, C_n, and D_n.

For $n \neq 2$

$$A_n = B_n = C_n = D_n = 0. \tag{16}$$

For $n = 2$

$$A_2 = \{4R_0^3\alpha[(7 - 5/m)R_0^5 + (5 - 10/m)R_0^2R_1^3 + 3R_1^5]/E\},$$

$$B_2 = -\{2R_0^3\alpha[(98 - 126/m + 40/m^2)R_0^7 + (42 - 84/m)R_0^2R_1^5 + (35 - 5/m^2)R_1^7]\}/E,$$

$$C_2 = -\{R_0^3R_1^3\alpha[(70 - 40/m)R_0^7 - 42R_0^5R_1^2 + (7 - 5/m)R_1^7]\}/E, \tag{17}$$

$$D_2 = \{2R_0^5R_1^5\alpha[(-42 + 24/m)R_0^5 + (35 - 5/m^2)R_0^3R_1^2 + (7 - 9/m - 10/m^2)R_1^5]\}/E,$$

where

$$E = 3[(196 - 420/m + 200/m^2)R_0^{10} + (350 - 600/m + 400/m^2)R_0^7R_1^3 -$$
$$-252R_0^5R_1^5 + (175 - 25/m^2)R_0^3R_1^7 + (56 - 30/m - 50/m^2)R_1^{10}].$$

Substituting the obtained expressions A_n, B_n, C_n, D_n into equations (5), we obtain the final formulas for the stresses:

$$\sigma_R = 2GI_2(R)P_2(\cos\theta), \qquad \sigma_\theta = 2G\{K_2(R)P_2(\cos\theta) + L_2(R)P_2'(\cos\theta)\cos\theta\},$$

$$\sigma_\psi = 2G\{M_2(R)P_2(\cos\theta) - L_2(R)P_2'(\cos\theta)\cos\theta\}, \tag{18}$$

where

$$\tau_{R\theta} = -2GJ_2(R)P_2'(\cos\theta)\sin\theta;$$

$$I_2(R) = \frac{\alpha}{E}\left\{-\frac{24}{m}\left(1-\frac{t}{k}\right)^2 A_2' - 4B_2' + \frac{(20-4/m)}{(1-t/k)^3}C_2' + \frac{24}{(1-t/k)^5}D_2'\right\}, \tag{19}$$

$$J_2(R) = \frac{\alpha}{E}\left\{\left(28+\frac{8}{m}\right)\left(1-\frac{t}{k}\right)^2 A_2' - 2B_2' - \frac{(2-2/m)}{(1-t/k)^3}C_2' - \frac{8}{(1-t/k)^5}D_2'\right\},$$

$$K_2(R) = \frac{\alpha}{E}\left\{-\left(168+\frac{24}{m}\right)\left(1-\frac{t}{k}\right)^2 A_2' - 8B_2' + \frac{(2-4/m)}{(1-t/k)^3}C_2' - \frac{18}{(1-t/k)^5}D_2'\right\},$$

$$L_2(R) = \frac{\alpha}{E}\left\{\left(28-\frac{16}{m}\right)\left(1-\frac{t}{k}\right)^2 A_2' - 2B_2' - \frac{(2-4/m)}{(1-t/k)^3}C_2' + \frac{2}{(1-t/k)^5}D_2'\right\},$$

$$M_2(R) = \frac{a}{E}\left\{-\frac{120}{m}\left(1-\frac{t}{k}\right)^2 A_2' - 4B_2' - \frac{(10-20/m)}{(1-t/k)^3}C_2' - \frac{6}{(1-t/k)^5}D_2'\right\},$$

$$\Delta_2(R) = -\left(12-\frac{24}{m}\right)\frac{a}{E}\left\{14\left(1-\frac{t}{k}\right)^2 A_2' - \left(1-\frac{t}{k}\right)^{-3} C_2'\right\}P_2(\cos\theta),$$

where

Fig. 1. Distribution of the principal stress σ_1 by latitude and depth. Model D. Isolines are drawn every 25 dyn/cm^2

Figure 1: Fig. 1. Distribution of the principal stress σ_1 by latitude and depth. Model D. Isolines are drawn every 25 dyn/cm^2

Fig. 2. Distribution of the principal stress σ_2 by latitude in the meridian plane (mean value). Model A

Figure 2: Fig. 2. Distribution of the principal stress σ_2 by latitude in the meridian plane (mean value). Model A

$$
\begin{aligned}
A_2' &= (7-5/m)(1-1/k)^8 + (5-10/m)(1-1/k)^5 + 3(1-1/k)^3,\\
B_2' &= (98-126/m+40/m^2)(1-1/k)^{10} + (42-84/m)(1-1/k)^5\\
&\quad + (35-5/m)(1-1/k)^3,\\
C_2' &= (70-40/m)(1-1/k)^{10} - 42(1-1/k)^8\\
&\quad + (7+5/m)(1-1/k)^3,\\
D_2' &= (-42+24/m)(1-1/k)^{10} + (35-5/m^2)(1-1/k)^8\\
&\quad + (7-9/m-10/m^2)(1-1/k)^5,\\
E &= 3\left[(196-420/m+200/m^2)(1-1/k)^{10}\right.\\
&\quad + (350-600/m+400/m^2)(1-1/k)^7 - 252(1-1/k)^5\\
&\quad \left.+(175-25/m^2)(1-1/k)^3 + (56-30/m-50/m^2)\right].
\end{aligned}
$$

Fig. 1. Distribution of the principal stress σ_1 by latitude and depth. Model D. Isolines are drawn every 25 dyn/cm^2.

Fig. 2. Distribution of the principal stress σ_2 by latitude in the meridian plane (mean value). Model A.

The principal stresses σ_1, σ_2, σ_3 and the greatest shear stress $\tau_{\max}$ are obtained from the solution of the determinant:

$$
\begin{vmatrix}
\sigma_R-\sigma & \tau_{R\theta} & 0\\
\tau_{R\theta} & \sigma_\theta-\sigma & 0\\
0 & 0 & \sigma_\varphi-\sigma
\end{vmatrix} = 0, \tag{20}
$$

whence

$$
\sigma_2 = \sigma_\varphi, \qquad \sigma_{1,3} = \frac{\sigma_R+\sigma_\theta}{2} \pm \tau_{\max}, \qquad \tau_{\max} = \sqrt{\frac{(\sigma_R-\sigma_\theta)^2}{4} + \tau_{R\theta}^2}. \tag{21}
$$

The angle of rotation β between the coordinate systems $(R\theta)$ and $(\sigma_1\sigma_3)$ varies

Fig. 3. Distribution of the principal stress σ_3 by latitude and depth. Model C

Figure 3: Fig. 3. Distribution of the principal stress σ_3 by latitude and depth. Model C

from 0° at the surface and reaches a maximum of 45° in the core in the zone of the 35th parallel.

In application to the figure of the Earth, the distribution of the principal stresses σ_1, σ_2, and σ_3 was considered for four models: A, B, C, and D, with a change in polar compression of 10^{-7}.

Model A. $R_1 - R_0 = 64$ km; $K = 100$; $t = 0.2$; $m = 3.7$; $G = 2 \cdot 10^{11}$ dyn/cm^2.

Model B. $R_1 - R_0 = 127$ km; $K = 50$; $t = 0.2$; $m = 3.7$; $G = 4.5 \cdot 10^{11}$ dyn/cm^2.

Model C. $R_1 - R_0 = 640$ km; $K = 10$; $t = 0.2$; $m = 3.7$; $G = 7 \cdot 10^{11}$ dyn/cm^2.

Model D. $R_1 - R_0 = 2900$ km; $K = 2.2$; $t = 0.2$; $m = 3.7$; $G = 2 \cdot 10^{12}$ dyn/cm^2.

1. The pattern of distribution of the principal stresses by latitude and depth in the layer for the four models A, B, C, and D is the same and is illustrated in Figs. 1, 2, and 3.
2. The principal stresses increase with increasing depth of the layer and are a function of latitude.
3. At the 35th parallel the principal stresses and the volumetric expansion change sign.
4. With an increase in polar compression: a) the principal stress σ_1 is directed along the surface tangentially in the meridional direction; from the equator and the poles to the 35th parallel there are compressive and tensile stresses in the meridional direction; b) the principal stress σ_2 acts perpendicular to the meridional plane, compresses the layer from the poles to the 35th parallel, and stretches it from the 35th parallel to the equator—compressive and tensile stresses of the latitudinal direction; c) the principal stress σ_3 is directed at the surface along the radius vector, compresses the layer from the poles to the 35th parallel and simultaneously stretches it from the 35th parallel to the equator, determining, together with the volumetric expansion, vertical movements. Such is the pattern of coupled deformation of the layer when the compression of the figure changes; with depth the directions of the principal stresses σ_1 and σ_3 change, which is associated with a change in the angle β.

Fig. 3. Distribution of the principal stress σ_3 by latitude and depth. Model C

5. With a decrease in polar compression, the opposite pattern is observed.

6. Attention is drawn to the particularly stressed state of the spherical layer in the zone of the 35th parallel.

If one takes into account that the rotational regime of the Earth changes continuously with time and, consequently, the polar compression changes continuously, then the principal stresses σ_1 and σ_2 arising in this case, by latitude and depth, explain the formation of deep faults and folding in the crustal layer, while the change in the principal stress σ_3 and in the volumetric expansion Δ explains the coupled vertical movements of the crustal layer and the distribution of transgressions and regressions in the past, which should be visualized by the irregular external structure of the lithosphere.

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Note: Figure translations are in progress. See original paper for figures.