STUDY OF THE DYNAMIC STABILITY OF PLATES WITH THE AID OF ELECTRONIC DIGITAL MACHINES

Fig. 1

1960

SovietRxiv

Abstract

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THEORY OF ELASTICITY

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STUDY OF THE DYNAMIC STABILITY OF PLATES WITH THE AID OF ELECTRONIC DIGITAL MACHINES

(Presented by Academician N. I. Muskhelishvili, 28 VI 1960)

The stability and post-critical behavior of plates under dynamic loading were investigated in work ([1]) by means of the Bubnov–Galerkin method; the approximating expression for the deflection contained one parameter. In the present article, results are presented for a more accurate solution of this nonlinear problem by the finite-difference method using electronic digital machines. A square plate with an initial deflection is considered, hinged along its edges and subjected to dynamic loading by compressive forces in one direction. It is assumed that, during deformation of the plate, the edges remain rectilinear; the unloaded edges may freely approach each other. The variation of deflections and stresses with time is determined.

Fig. 1

The initial equations in dimensionless parameters have the form ([1])

$$c\nabla^2\nabla^2 w = w_{,xx}(\Phi_{,yy} - p) + w_{,yy}\Phi_{,xx} - 2w_{,xy}\Phi_{,xy} + c\nabla^2\nabla^2 w_0 - w_{,tt}; \quad (1)$$

$$\nabla^2\nabla^2\Phi = w_{,xy}^2 - w_{,xx}w_{,yy} - \left(w_{0,xy}^2 - w_{0,xx}w_{0,yy}\right); \quad (2)$$

here the following notation has been introduced:

$$w = \frac{\bar{w}}{h}, \qquad w_0 = \frac{\bar{w}_0}{h}, \qquad \Phi = \frac{\bar{\Phi}}{Eh^2}, \qquad x = \frac{\bar{x}}{a}, \qquad y = \frac{\bar{y}}{a}, \qquad t = \frac{\bar{t}h}{a^2}V,$$

$$p = \frac{\bar{p}}{E}\left(\frac{a}{h}\right)^2, \qquad c = \frac{1}{12(1-\mu^2)}; \quad (3)$$

$\bar{w}(\bar{x}, \bar{y}, \bar{t})$ and $\bar{w}_0(\bar{x}, \bar{y})$ are, respectively, the total and initial deflection; $\bar{\Phi}(\bar{x}, \bar{y}, \bar{t})$ is the stress function in the middle surface; a and h are, respectively, the side

Fig. 2

Figure 1: Fig. 2

Fig. 3

Figure 2: Fig. 3

and thickness of the plate; $\bar{p}(\bar{t})$ is the intensity of the compressive forces; V is the speed of sound in the material of the plate; $\bar{t}$ is time; the coordinates $\bar{x}, \bar{y}$ are measured along the sides of the plate; indices after a comma denote differentiation with respect to the corresponding variable.

The boundary conditions for the edges $x = 0,\ x = 1$ will be (^{2})

$$w = 0, \qquad w_{,xx} = 0, \qquad \Phi_{,xy} = 0, \qquad \Phi_{,xxx} = 0; \tag{4}$$

analogous conditions hold for the edges $y = 0,\ y = 1$.

The initial conditions are taken in the form

$$w = w_0 = w_0^* \sin \pi x \sin \pi y; \qquad w_{,t} = 0 \quad \text{for } t = 0. \tag{5}$$

Equations (1), (2) and conditions (4), (5) were represented in finite differences using symmetric operators of second-order accuracy. The mesh domain consisted of a series of planes parallel to the plane $t = 0$ and separated by certain intervals in time. The deflection w at each mesh node is determined step by step in time; after computing the deflections at each step, an iteration cycle was carried out, as a result of which the values of the function Φ were calculated.

Fig. 2

The computations were carried out on the M-20 electronic digital computer with a mesh spacing in the x, y directions equal to 1/8. The time step was chosen so as to ensure the stability of the solution of the difference equations. Preliminarily, by the method described in [3], the static equilibrium forms of the plate under supercritical deformation were determined, corresponding to various "generating" deflected forms. The latter corresponded to eigenvalues of the linear problem with the number of half-waves $m = 1$ and $m = 3$ in the x direction and one half-wave along y.

Fig. 3

In Fig. 1, the dashed lines show the dependence of the deflection at the center of the plate w^* on the parameter $t^* = p/p_{cr}$ for $w_0^* = 0.001$, where $p_{cr} = \pi^2/3(1 - \mu^2)$ is understood as the parameter of the static critical stress. The results of solving the dynamic problem are also given here for a load increasing proportionally to time. In the case where the loading rate is relatively small ($p/t = 2.15$), a rapid increase in deflections occurs at a load $p \approx 1.8\,p_{cr}$; thereafter nonlinear oscillations begin about the static equilibrium forms. At a higher loading rate ($p = 5t$), the deflections increase sharply when the load reaches the value $p \approx 2.6\,p_{cr}$; subsequently a change in the sign of the deflection occurs. The motion of the plate is accompanied by a significant change in the shape of the deflected surface.

Figure 2 presents the deflected shapes for the middle section, parallel to x, at successive instants of time; they are marked in Fig. 1 by the corresponding letters.

If the loading rate increases to $p/t = 21.5$, the buckling of the plate from the very beginning occurs not in one half-wave, but in three half-waves.

Figure 3 shows the results of computations for a more complicated loading program. It is assumed that in the first segment the load increases according to the law $p/t = 5$; after reaching the value $p = 1.67p_{cr}$, in one case it remains constant (dashed straight line A), while in the other two it decreases to zero at different rates (straight lines B, C). The solid lines marked by the same letters represent the variation of the deflection arrow with time. Already in case B, the deflection arrow does not exceed the plate thickness throughout the entire buckling process.

To solve the dynamic problem by the method described, it is necessary to carry out about 30 million operations, which takes 15 minutes on the M-20 machine.

Air Force Engineering Academy
named after N. E. Zhukovsky

Received
10 VI 1960

CITED LITERATURE

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Note: Figure translations are in progress. See original paper for figures.