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ON THE PARAMETERS OF STRUCTURES OF A DEFORMABLE BED IN A WAVE FLOW

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Numerous works have been devoted to the study of periodic structures of the deformable bed of wave and translational flows; however, until recently there have still not been sufficiently complete and reliable data ([1−6]) for describing the physical aspect of the mechanism of their formation and dynamics. In the present article, results are given from a study of the dependence of the parameters of periodic microstructures (ripples) on the parameters of the wave flow, the constants of the fluid and of the particles, and also of the dependence of the velocity of their displacement and of the magnitude of the particle flux on the parameters of the wave flow.

The investigations were carried out in a 15-meter wave flume with a cross section of 0.5×0.8 m^2, within the limits of variation of the period τ_v and height h_v of the waves 0.9 sec $\leqslant \tau_\mathrm{v} \leqslant$ 5.0 sec, 4 cm $\leqslant h_\mathrm{v} \leqslant$ 20 cm, at a flow depth $H = 0.4$ m, or in dimensionless variables $0.25 \leqslant kH \leqslant 2$ and $0.1 \leqslant h_\mathrm{v}/H \leqslant 0.5$. Allowance for the undesirable action of a standing wave (its amplitude was reduced as much as possible by a combined wave absorber), and also of the wall effect and the transverse periodicity of the ripples ([ξ]), made it possible to reduce the uncontrolled scatter of $h_\mathrm{v}, h_\mathrm{p}, \lambda_\mathrm{p}$ under the most unfavorable regimes to $\sim 5\%$. From control measurements of the maximum values of the bottom velocities of both phases of the wave, made by means of motion-picture filming of the bottom layer, it was established that the relation between h_v and v'_v—the maximum value of the bottom velocity in the I phase of the wave—is given quite accurately by the known relation

$$v'_\mathrm{v} = \frac{2\pi}{\tau_\mathrm{v}} \frac{h_\mathrm{v}}{2} \frac{1}{\operatorname{sh} kH}.$$

1. The results of measuring the dependence of the height h_p and spacing λ_p of wave ripples on the flow parameters, in the variables v'_v and τ_v, are presented in Fig. 1. These dependences in dimensionless quantities have the following form

$$\Pi = 7.24 \cdot 10^{-2} / \operatorname{sh} kH, \qquad \Lambda = 4.02 \cdot 10^{-1} / \operatorname{sh} kH, \qquad (1)$$

where $\Pi = h_\mathrm{p}/(h_\mathrm{v} + h_0)$; $\Lambda = \lambda_\mathrm{p}/(h_\mathrm{v} + h_0)$; k is the wave number; $h_0 = v_0\tau_\mathrm{v}\,\mathrm{sh}\,kH/\pi$, and varies from 8.2 to 3.8 cm as kH varies from 2.7 to 0; v_0 is a constant equal to 9.52 cm/sec. Expression (1) describes the entire variety of ripple parameters in the region of their existence in not very deep and shallow water. From expressions (1) there follows a universal dependence between $h_\mathrm{p}, \eta_\mathrm{p}$, and λ_p,

$$\lambda_\mathrm{p} = 5.55 h_\mathrm{p}, \tag{2}$$

which does not depend on the parameters of the wave flow and is valid for any stage and type of ripples that are in dynamic equilibrium with the flow (i.e., are not being reshaped). This dependence is not affected by flow inhomogeneities, its instability, or standing waves.

Many authors have indicated the existence of a weak dependence of λ_p on the viscosity of the fluid ν, the particle diameter d, and their density ρ_t. However, owing to great experimental difficulties, exact data and, thus

moreover, the complete dependence $\lambda_p = f(\rho_t, d, \nu, ...)$ has not been obtained. We know of only one work ([3]), whose author obtained a numerical value of the exponent m for one of the indicated relations $\lambda_p = \lambda_p(d)$, equal to 0.5. Here we shall give, by a semi-empirical method, a derivation of the dependence $h_p = f(\rho_t, g, d, \rho\ , \nu)$. Since the coefficients in (1), through which the desired dependence enters, are dimensionless, one may assume

$$\Pi \sim \left(\frac{d^3 \rho_t g}{\rho\,\nu^2}\right)^n, \tag{3}$$

where ρ and g are the density of the liquid and the acceleration due to gravity (for a liquid flow, instead of ρ_t in (3) one should take $\rho_t - \rho$, to allow for the actual weight of the particles). The exponent n in (3) may be determined experimentally from the dependence of h_p (or λ_p) on any of the constants entering into (3). Since the value $m = 0.5$ ($n = 1/6$) from ([3]) is unreliable, we used (3) to process the data of another work ([1]) on the dependence of the spacing of channel forms (ridges) λ_{pp} on ν and d, from which the author of that work was unable to obtain numerical values for m. For both dependences n proved to be equal to 0.1. Substituting (3) with $n = 0.1$ into (1), we obtain

$$\Pi = 4.14 \cdot 10^{-2} \left(\frac{d^3 \rho_t g}{\rho\,\nu^2}\right)^{0.1} \frac{1}{\mathrm{sh}\,kH}, \tag{4}$$

$$\Lambda = 2.30 \cdot 10^{-1} \left(\frac{d^3 \rho_t g}{\rho\,\nu^2}\right)^{0.1} \frac{1}{\mathrm{sh}\,kH}.$$

Fig. 1. Dependence of the height and spacing of periodic bed structures in a wave flow on the period and near-bottom velocity. With an increase in the period to infinity, the wave structures pass into periodic structures of translational flows (vertical straight line), and their parameters h_{pp} and λ_{pp} become dependent on time (expressions (6)). In practice, however, their dimensions may be limited by the depth of the flow (case of a channel flow). In a wave flow, in which these limitations do not operate, periodic structures reach dimensions exceeding the magnitude of channel forms by several orders (heights reach hundreds of meters, the period spacing—kilometers).

In the variables τ, v' and τ, h (the latter for the case of a flow in shallow water), expressions (4) take the following form:

$$\lambda_p = 5.55 h_p = 0.732 \cdot 10^{-1} \left(\frac{d^3 \rho_t g}{\rho\, \nu^2} \right)^{0.1} (v' + v_0)\tau\,, \tag{5'}$$

$$\lambda_p = 5.55 h_p = 3.65 \cdot 10^{-2} \left(\frac{d^3 \rho_t g}{\rho\, \nu^2} \right)^{0.1} \frac{h + h_0}{H} c_\phi \tau\,, \tag{5''}$$

where c_ϕ is the phase velocity of the wave.

For the case when $S = \infty$ ($v' = 0$), which corresponds to the transition of a wave flow into a translational one, replacing v' by the velocity of the translational flow v and τ by the time of vortex development t, from (4) we obtain expressions for the height h_{pp} and spacing λ_{pp} of channel forms (ridges):

$$h_{pp} = \sigma_{\mathrm{I}} \left(\frac{d^3 \rho_t g}{\rho\, \nu^2} \right)^{0.1} (v + v_0)t\,, \tag{6'}$$

$$\lambda_{pp} = \sigma_{\mathrm{II}} \left(\frac{d^3 \rho_t g}{\rho\, \nu^2} \right)^{0.1} (v + v_0)t\,. \tag{6''}$$

There are several views on the mechanism of formation of periodic structures. Darwin ([2]) already indicated that the formation of ripples is associated with the action of vortices generated in the flow behind inhomogene-

···features of the bottom relief. Later this point of view was abandoned. Recently a different opinion has been expressed ([6]), according to which the formation of ripples (the author studied channel forms) is caused by large-scale pulsations in the flow. Our investigations ([7]) confirmed Darwin' s opinion on the role of vortices in ripple dynamics.

In the mechanism of interaction between the flow and the deformable bottom, two mutually opposite processes can be distinguished. One of them is associated with the action of the vortical part of the flow (in expression (5′) it corresponds to the product $v'\tau$), which determines the supply of material to the crest of the ripples. With increasing v' and τ, the circulation velocity Γ of the vortex and its geometric dimensions increase, which in turn increases both the amount of material transported and the distances (in the vertical and horizontal directions) over which it is displaced. This process, therefore, leads to an increase in h_p and λ_p. The other process is due to the action of the potential part of the flow, which determines the removal of material from the ripple crests (in (5) it corresponds to the factor $\left(\frac{d^3\rho\, g}{\rho\, \nu^2}\right)^{0.1}$). With decreasing viscosity and density of the flow and increasing particle weight $d^3\rho\, g$, the stability of the latter on the ripple crest increases; with increasing velocity it decreases; therefore, in contrast to the first process, the second tends to reduce the height of the ripples. The magnitude of the ripple parameters h_p and λ_p is determined by the dynamic equilibrium of these two processes, represented by expressions (5).

2. In a translational flow, the velocity of ripple migration and the magnitude of the flux of sediment particles moving in the near-bottom layer are in a simple dependence on the current velocity. In a wave flow these dependences prove to be more complicated. In this case, the integral flux of particles over the wave period Q consists of 8 terms: Q_1^+, Q_1^-—fluxes moving in suspension in the near-bottom layer, and q_1^+, q_1^-—fluxes moving by traction in the first phase of the wave, and the analogous four for the second phase of the wave. Let us introduce normalized coefficients of local near-bottom asymmetry of the wave flow, reflecting the disturbance of near-bottom velocities by local irregularities of the bottom contour:

$$s_l' = \frac{1 + v''/v'}{1 + v'/v''}, \qquad s_l'' = \frac{1 + v'/v''}{1 + v'/v''}, \tag{7}$$

in which v' and v'' are the undisturbed, and v' and v'' the disturbed, values of the near-bottom velocities of the wave flow in the 1st and 2nd phases of the wave, and the probability integrals

$$f_{t'\tau'} = \frac{1}{\sigma\sqrt{\pi}} \int_{t'}^{\tau'} \exp\left\{-\left[\frac{t-(t_f+t')}{\sigma}\right]^2\right\} dt,$$

which take into account the distribution, over the wave phases, of particles settling out of suspension with mean settling time t_f, depending on the moments of suspension in the 1st phase t' and in the 2nd phase t'', and the durations of these phases τ' and τ''. Then the experimentally obtained dependence of the particle flux on the period and near-bottom velocities (or on the height and depth of the flow) per unit length of the ripple front will have the form

$$Q = \sum_{i=1,2} (Q_i^\pm + q_i^\pm) =$$

$$= \left[(\alpha f_{t'\tau'} + \gamma_1')(v' s_l' - v^0)^4 + (\alpha f_{0\tau'} + \gamma_2')(v'' s_l'' - v^0)^4\right] \tau\, \mathbf{k}_1 +$$

$$+ \left[(\alpha f_{\tau'\tau} + \gamma_1'')(v' s_l' - v^0)^4 + (\alpha f_{t''\tau} + \gamma_2'')(v'' s_l'' - v^0)^4\right] \tau\, \mathbf{k}_2, \tag{8}$$

$v' = \pi h\,/\tau\,\mathrm{sh}\,kH$; $v'' = \pi h\,/S\,\tau\,\mathrm{sh}\,kH$; $S = v'/v''$ is a characteristic of the asymmetry of the undisturbed wave flow; v^0 is the critical velocity,

at which the interaction of the wave flow with an already deformed bed begins; α and γ_i^j are proportionality coefficients (for $\tau_\mathrm{w} = 2.3$ sec, $H = 0.4$ m and $d = 0.25$ mm, $\alpha \simeq 1 \cdot 10^{-7}$ cm$^{-2} \cdot$ sec^2 and $v_\mathrm{cr}^0 = 10$ cm/sec); $\mathbf{k}_1$ and $\mathbf{k}_2$ are unit vectors of the directions of propagation of the particle fluxes.

For the general form of the dependence of the velocity of displacement of ripples on the parameters of the wave flow, we have obtained the expression

$$\mathbf{v}_\mathrm{p} = \frac{(S_\mathrm{w}+1)\varepsilon}{S_\mathrm{w}\beta\sqrt{\Omega_\mathrm{p}}} \Big\{[\alpha\eta_1(f_{t'\tau'} + f_{\tau'\tau_\mathrm{w}}) + \gamma_1' + \gamma_1''](v_\mathrm{w}' s_i' - v_\mathrm{cr}^0)^4 \mathbf{k}_1 +$$
$$+ [\alpha\eta_2(f_{0\tau'} + f_{\tau''\tau_\mathrm{w}}) + \gamma_2' + \gamma_2''](v_\mathrm{w}'' s_i'' - v_\mathrm{cr}^0)^4 \mathbf{k}_2\Big\}, \tag{9}$$

where $\beta = \mathrm{tg}\left(\dfrac{S_\mathrm{w}+1}{S_\mathrm{w}}\dfrac{h_\mathrm{p}}{\lambda_\mathrm{p}}\right)$; ε is a coefficient; $\eta_1 = \dfrac{l^-}{\lambda_\mathrm{p}}$, $\eta_2 = \dfrac{l^+}{\lambda_\mathrm{p}}$; l^+ and l^- are the magnitudes of the projections of the gentle and steep slopes of the ripple; Ω_p is the volume of material displaced during the time of ripple displacement over the distance λ_p. Since $\sqrt{\Omega_\mathrm{p}}$ is proportional to the roots of $Q_i^\pm$ and $q_i^\pm$, the exponents of the factors $(v_\mathrm{w}^j s_i^j - v_\mathrm{cr}^0)$ in the terms of expression (9) are equal to 2.

For a symmetric wave flow on homogeneous sections of periodic structures, (8) and (9) become zero. At the edges of structures or in the neighborhoods of other inhomogeneities, even in a symmetric flow Q_w and $\mathbf{v}_\mathrm{p}^0$ differ from zero. In the limiting case when the wave flow passes into a translational flow ($\tau_\mathrm{w} = \infty$, $v_\mathrm{w}'' = 0$, $S_\mathrm{w} = \infty$), only two terms remain in (8) and (9). Replacing the notation of the wave-flow parameters by the parameters of a translational flow and passing to the particle flux referred to unit time, we obtain for Q_p and $\mathbf{v}_\mathrm{pp}$ —the particle flux and velocity of displacement of channel ripples (dunes) in a translational flow—

$$\mathbf{Q}_\mathrm{p} = (\bar\alpha' + \bar\gamma_1')(v_\mathrm{p} - v_\mathrm{cr}')^4 \mathbf{k}_1, \tag{8'}$$

$$\mathbf{v}_\mathrm{pp} = \frac{\bar\varepsilon(\bar\alpha'\bar\eta_1 + \bar\gamma_1')^*}{\bar\beta}(v_\mathrm{p} - v_\mathrm{cr}')^2 \mathbf{k}_1. \tag{9'}$$

The results obtained make it possible to approach the consideration of the entire variety of periodic forms of wave and translational flows (channel and aeolian) from a unified point of view. All forms are divided into two groups. The first comprises structures for which in (3) $n > 0$. The principal determining parameter for them is the height of the forms h_p. The observed differences among forms of this group are due only to the stages and conditions in which they develop. The second group should include forms for which in (3) $n < 0$. The principal determining parameter for them is the periodicity step λ_p. Examples of the latter are aeolian microforms covering the surface of larger structures.

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Note: Figure translations are in progress. See original paper for figures.