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Abstract

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Hydromechanics

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Some Self-Similar Motions of a Gas with Plane Waves in Magnetohydrodynamics

(Presented by Academician L. I. Sedov, 20 III 1960)

The equations of one-dimensional unsteady motion of an ideal gas in the presence of a transverse magnetic field $h_z = H_z^2/8\pi$, for infinitely large conductivity and negligibly small viscosity and thermal conductivity, have the form ([2])

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} + \frac{1}{\rho}\frac{\partial}{\partial x}(P + h_z) = 0,$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho v}{\partial x} = 0, \qquad \frac{\partial}{\partial t}\frac{P}{\rho^k} + v\frac{\partial}{\partial x}\frac{P}{\rho^k} = 0, \tag{1}$$

$$\frac{\partial h_z}{\partial t} + v\frac{\partial h_z}{\partial x} + 2h_z\frac{\partial v}{\partial x} = 0.$$

The system of equations (1) has particular solutions of the form ([3])

$$v = \frac{V(x)}{1+mt}, \qquad \rho = \frac{R(x)}{(1+mt)^s}, \qquad p = \frac{P(x)}{(1+mt)^{s+2}}, \qquad h_z = \frac{h(x)}{(1+mt)^{s+2}}, \tag{2}$$

where $m = \text{const}$, $s = \text{const}$; $V(x)$, $R(x)$, $P(x)$, and $h(x)$ are functions determined from the equations:

$$V\frac{dV}{dx} + \frac{1}{R}\left(\frac{dP}{dx} + \frac{dh}{dx}\right) = mV,$$

$$R\frac{dV}{dx} + V\frac{dR}{dx} = smR,$$

$$kP\frac{dV}{dx} + V\frac{dP}{dx} = (s+2)mP, \tag{3}$$

$$2h\frac{dV}{dx} + V\frac{dh}{dx} = (s+2)mh.$$

Integrating (3) for $h(x) = 0$, we obtain

$$V = c_1 z^{-\frac{1}{k-1}} \{[(k+1)s+2]z - k + 1\}^{\frac{\frac{2}{k-1}(s+1)-2}{(k+1)s+2}},$$

$$R = c_2 z^{\frac{1-s}{k-1}} \{[(k+1)s+2]z - k + 1\}^{\frac{s-1}{k-1} + \frac{2-(k-1)s}{2+(k+1)s}}, \tag{4}$$

$$x = x_* + \frac{c_1}{m}\int (1-kz) z^{-\frac{k}{k-1}} \{[(k+1)s+2]z - k + 1\}^{\frac{\frac{2}{k-1}(s+1)-2}{(k+1)s+2} - 1},$$

where $z = P/RV^2$, and c_1, c_2, and x_* are arbitrary constants.

For $s = \dfrac{2}{k-1}$, the solutions (4) are given in ([3]). In that work it is shown that, with the aid of (4), one can construct flows from plane nonstationary sources whose intensity varies according to the law

$$Q = (1+mt)^{-s-1} R(x_0) V(x_0) \exp\left[sm\int_{x_0}^{x} \frac{dx}{V}\right]. \tag{5}$$

For $h_z \neq 0$, with the aid of (2) one can also construct flows due to plane nonstationary sources whose intensity varies according to the law (5), in which $V(x)$ is determined from system (3).

There are two first integrals of adiabaticity and frozen-in field for system (3):

$$P = c_1 R^{1+\frac{2}{s}} V^{1+\frac{2}{s}-k},$$

$$h = c_2 R^{1+\frac{2}{s}} V^{\frac{2}{s}-1}. \tag{6}$$

For $s = -1$ ($Q = Q(x)$) there is the momentum integral of L. I. Sedov ([1])

$$P + h + RV^2 = c_3. \tag{7}$$

Here c_1, c_2, and c_3 are arbitrary constants.

In this case the functions $P(V), R(V), h(V)$, and $x(V)$ can be expressed explicitly. For $s \neq -1$, the two remaining integrals of the system of equations (3) are determined from the equations

$$\frac{dJ}{dY} = \frac{1}{2-k}\frac{J}{Y}\frac{J-(3s+6-2k)c_1Y-(2+3s)c_2}{J-(s+2)c_1Y-(s+2)c_2}, \tag{8}$$

$$Y^{\frac{k-1}{2-k}}\frac{dY}{dx} = \frac{2(2-k)mJ}{3J-(2-k)Y\dfrac{dJ}{dY}}, \tag{9}$$

where $Y = V^{2-k}$, $J = R^{-2/s}V^{3-2/s}$.

Equation (8) can be integrated in finite form for certain values of the constant s. For $s = -2 + \frac{2}{3}k$ and $s = -2$, equation (8) is a Bernoulli equation. For

$$s = -2\frac{k-1}{k+1}$$

equation (8) is a Darboux equation, which, by the change of variables $J = \varphi Y$, is reduced to a Bernoulli equation and, consequently, is integrated in finite form.

With the aid of the exact solutions (2) one can investigate the process of amplification of the magnetic field in a hydromagnetic dynamo ([4]). Figure 1 presents a schematic diagram of the installation, which consists of the following main components: T—a tube made of insulating material, M—a magnetic core, K—excitation coils, Π and Π'—contact plates, A—an ammeter. The installation may be regarded, in simplified form, as a direct-current generator with self-excitation. Its operation consists in the following: when a conducting medium moves in the tube with velocity V, and when there is some residual magnetization of the poles of the magnetic core, an initial electromotive force arises between the contact plates, proportional to the velocity of motion of the medium and to the magnetic flux. If, in this case, the contact plates are closed through the excitation winding, then, as current passes through it, the magnetic flux in the magnetic core increases. If losses in the electric circuit are not taken into account, then the current in the excitation coils and, consequently, the magnetic-field intensity H_z between the poles of the magnetic core will increase with time as the velocity v ($m < 0$) of the conducting medium in the tube increases. This condition is satisfied by values $s > -2$. From formulas (2) it follows that for $s > -2$, $v \to \infty$ and $h_z \to \infty$ as $t \to -1/m$. However, when losses in the electric circuit, which are proportional to the square of the current, are taken into account, the actual value of the current and, consequently, also of the magnetic-field intensity will have a finite value as $v \to \infty$.

With the aid of L. I. Sedov' s momentum integral one can investigate the process of acceleration of a conducting medium in induction pumps, indicated in the survey ([5]). According to Lenz' s rule, the current induced in the conducting

Fig. 1

Figure 1: Fig. 1

medium will have such a direction that the ponderomotive forces arising thereby will impede the motion of the medium. However, if through contact plates a continuously increasing current of the opposite direction is passed into the conducting medium, then the ponderomotive forces that arise will continuously accelerate the conducting medium. The physical picture of the motion of the conducting medium described by formulas (2) for $s = -1$ is as follows. At the initial instant $t = 0$, motion from a stationary source takes place in the tube, the intensity of which varies according to law (5) ($s = -1$). At this instant the initial distribution of density, particle velocity, pressure, and intensity of the transverse magnetic field is prescribed, as determined by the exact solutions (2). From the instant $t = 0$, a continuously increasing current is supplied to the conducting medium through contact plates; with this current the ponderomotive forces continuously accelerate the conducting medium ($m < 0$). At the same time the intensity of the magnetic field between the poles of the magnetic core will also increase with time. This follows directly from formulas (2). Thus, an unsteady motion of the conducting medium from a stationary source takes place in the tube.

Fig. 1

In conclusion, we note that flows of type (2) can also be considered in the case of unsteady axisymmetric motion of a gas in the presence of magnetic forces.

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Note: Figure translations are in progress. See original paper for figures.