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Abstract

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A POINT EXPLOSION IN AN INHOMOGENEOUS ATMOSPHERE

(Presented by Academician Ya. B. Zel'dovich, 19 X 1959)

As is known, the problem of a point explosion in a medium with a constant isentropic exponent γ has an analytic solution ([1]) for that stage of the phenomenon when the energy contained in the medium before the explosion can be neglected in comparison with the energy brought by the blast wave. A comparatively simple solution is obtained because the problem is self-similar: it contains no characteristic parameters of the dimensions of length, velocity, and time. If there is a gradient of the density of the medium in at least one direction, self-similarity is already absent, and an exact solution is not obtained. For small gradients the problem can be treated by the perturbation method ([2,3]).

If the explosion occurs in a very rarefied atmosphere, then a strong shock wave with limiting compression propagates to such distances where the density changes many times in comparison with the density at the place of the explosion. Then linearization is inadmissible. Numerical integration of the problem with three variables is very difficult, even when computed on an electronic machine.

One can propose a semi-quantitative approach based on one essential feature of the exact centrally symmetric solution. Namely, in this solution the energy is distributed almost uniformly over the entire volume of the blast wave, and only near its very front is it 2–3 times greater than the average value over the volume. In this region the entire mass of the substance is also concentrated.

It is natural to suppose that the same property is possessed by the blast wave in an inhomogeneous atmosphere. Indeed, if the pressure inside the wave is constant in space (the pressure is proportional to the energy density), while the mass density is equal to zero, then the equations of hydrodynamics are satisfied in a trivial way in the main part of the volume. Then, in order to describe the propagation of the wave, one must use the conditions at the shock front itself.

If the equation of the wave front is $f(r, z, t) = 0$, then the normal component of the front velocity D_n is determined by the known equality

$$D_n = -\frac{\partial f}{\partial t}\Big/|\nabla f| = \sqrt{\frac{p}{\rho^2\left(\frac{1}{\rho} - \frac{1}{\rho'}\right)}}. \tag{1}$$

Here, as usual in the problem of a strong explosion, the initial pressure has been discarded in comparison with the pressure at the wave front p. In this approximation the density behind the front ρ' is related to the density ahead of the front ρ by the constant ratio

$$\frac{\rho'}{\rho} = \frac{\gamma + 1}{\gamma - 1}. \tag{2}$$

The pressure is expressed in terms of the energy density ε,

$$p = (\gamma - 1)\varepsilon = (\gamma - 1)\lambda \frac{E}{V}, \tag{3}$$

where E is the energy density of the explosion; V is the volume occupied by the blast wave; $\lambda = \lambda(\gamma)$ is a coefficient indicating how many times greater the energy density near the front is than the average density over the volume ([1]). The assumption that λ is constant over the surface underlies the method proposed here.

We shall assume that the equation of the wave front in cylindrical coordinates is solved with respect to the radius: $r = r(z, t)$. Then the total volume of the wave is

$$V(t) = \pi \int_{z_1}^{z_2} r^2(z, t)\, dz, \tag{4}$$

where $r(z_1, t) = r(z_2, t) = 0$. Substituting (2), (3), and (4) into (1) and expressing the density by the barometric formula, we arrive at the partial differential equation for the function r:

$$\left(\frac{\partial r}{\partial y}\right)^2 - e^{-z/z_0} \left[\left(\frac{\partial r}{\partial z}\right)^2 + 1\right] = 0. \tag{5}$$

Here z_0 is the equivalent thickness of the atmosphere; y is an auxiliary variable defined by the equality

$$y = \int_0^t \frac{dt}{\sqrt{V}} \sqrt{\frac{\lambda E(\gamma^2 - 1)}{2\rho_0}}; \tag{6}$$

ρ_0 is the initial density of the air at the point of explosion ($z = 0$).

Equation (5) is solved by the method of separation of variables

$$r = \xi y + \int_0^z dz \sqrt{\xi^2 e^{-z/z_0} - 1}; \tag{7}$$

Fig. 1

Figure 1: Fig. 1

$$\frac{\partial r}{\partial \xi} = y + \int_0^z dz\, \frac{\xi e^{-z/z_0}}{\sqrt{\xi^2 e^{-z/z_0} - 1}} = F(\xi). \tag{8}$$

For small t, or y, the wave must be spherical. For this it is sufficient to set the function $F(\xi)$ equal to zero. Then, eliminating ξ from (8) and substituting into (7), we obtain

$$r = 2z_0 \arccos\left[\frac{1}{2} e^{z/2z_0}\left(1 - x^2 + e^{-z/z_0}\right)\right]; \tag{9}$$

here $x = y/2z_0$.

From this we obtain the positions of the upper and lower points of the wave z_1 and z_2:

$$e^{-z_{1,2}/2z_0} = 1 \mp x, \tag{10}$$

as well as the position and value of its maximum radius

$$e^{-z_m/z_0} = 1 - x^2, \qquad r_m = 2z_0 \arcsin x. \tag{11}$$

Thus, the greatest possible radius of the wave is equal to πz_0. In this case $x = 1$, so that the upper edge of the wave goes to infinity. But this occurs in a finite time τ, which is determined from (6) as

$$\tau = \sqrt{\frac{8\pi z_0^5 \rho_0}{\lambda E(\gamma^2 - 1)}} \int_0^1 \sqrt{\Omega(x)}\, dx, \tag{12}$$

where

$$\Omega(x) = \int_{-2\ln(1+x)}^{-2\ln(1-x)} du\, \arccos^2\left[\frac{1}{2} e^{u/2}\left(1 - x^2 + e^{-u}\right)\right]. \tag{13}$$

The time for the wave to go upward to infinity turns out to be finite, owing to the fact that the wave velocity, according to (1), tends to infinity as $z \to \infty$. Figure 1 shows the curves for converting from x to t and $|\Omega(x)|^{1/2}$. Figure 2 gives, to scale, the calculated sections of the wave by the vertical plane passing through the point of explosion, for several instants of time.

Fig. 2

Figure 2: Fig. 2

Fig. 1

Fig. 2

Of course, the solution loses its meaning before z_1 goes to infinity. Nevertheless, the following conclusion may be drawn: however large the total energy of the explosion may be, a strong shock wave can propagate, according to the law obtained here, downward by no more than $1.38z_0$, or approximately 11 km. With further downward propagation, the shock front will weaken more rapidly at the expense of rarefaction waves leaving it upward into the region, open to vacuum, that has been captured by the wave. The propagation of the wave through undisturbed air will resemble a short impact on matter bordering on a vacuum, considered by Ya. B. Zel' dovich[4].

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CITED LITERATURE

1. L. I. Sedov, *Methods of Similarity and Dimensionality in Mechanics*, Moscow, 1957.
2. V. P. Karlikov, DAN, **101**, 86 (1955).
3. E. I. Andriankin, *Izv. AN SSSR, OTN*, No. 2 (1958).
4. Ya. B. Zel' dovich, *Akust. zhurn.*, **2**, 28 (1956).

Note: Figure translations are in progress. See original paper for figures.