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Abstract

Full Text

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ON AZIMUTHAL ASYMMETRY IN THE SCATTERING OF DIRAC PARTICLES

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Physics

1. As Mott showed, the exact formula for the angular distribution of a partially polarized beam of Dirac particles contains a factor of the form ([1])

$$D(\vartheta) = i\left[f(\vartheta)g^*(\vartheta) - f^*(\vartheta)g(\vartheta)\right]. \tag{1}$$

In the case of purely Coulomb scattering by a point center, $D(\vartheta)$ was computed by Mott to first-order accuracy. With the aid of damping theory, A. A. Sokolov succeeded in generalizing Mott' s results to the case of scattering by a force center possessing not only an electric charge but also a magnetic moment, and also in considering the case of an arbitrary potential ([2]).

In the present paper, starting from the values of the phases in the first approximation ([3,4]), $D(\vartheta)$ is calculated for an arbitrary spherically symmetric potential. The differential cross section for an unpolarized beam, corresponding to this approximation, was calculated in ([5]).

2. As is known, in the first Born approximation $f(\vartheta)$ and $g(\vartheta)$ are real and $D(\vartheta) = 0$, i.e., the scattering has no azimuthal asymmetry. In order to obtain complex scattering amplitudes, it is necessary in the exact formulas, when expanding the exponents, to take into account the quadratic terms as well:

$$f(\vartheta) = \frac{1}{k}\sum_{l=0}^{\infty}\left[(l+1)\left(\operatorname{tg}\delta_l^{(1)} + i\operatorname{tg}^2\delta_l^{(1)}\right) + l\left(\operatorname{tg}\delta_l^{(2)} + i\operatorname{tg}^2\delta_l^{(2)}\right)\right]P_l(\cos\vartheta),$$

$$g(\vartheta) = \frac{1}{k}\sum_{l=1}^{\infty}\left[\left(\operatorname{tg}\delta_l^{(1)} + i\operatorname{tg}^2\delta_l^{(1)}\right) - \left(\operatorname{tg}\delta_l^{(2)} + i\operatorname{tg}^2\delta_l^{(2)}\right)\right]P_l^1(\cos\vartheta). \tag{2}$$

It is easy to show that in this case there is no need to take into account the values of the phases in the second approximation; it is sufficient to restrict oneself to the phase values in the first approximation (formulas (19) in ([3])), which, by means of the Laplace transformation

$$rV(r) = \int_0^\infty \overline{V}(t) e^{-rt}\, dt, \qquad \overline{V}(t) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} rV(r) e^{rt}\, dr \tag{3}$$

can be represented in the form

$$\operatorname{tg} \delta_l^{(1)} = -\frac{K}{2c\hbar k} \left\{ \alpha \int_0^\infty \overline{V}(t) Q_l \left(1 + \frac{t^2}{2k^2}\right) dt + \beta \int_0^\infty \overline{V}(t) Q_{l+1} \left(1 + \frac{t^2}{2k^2}\right) dt \right\},$$

$$\operatorname{tg} \delta_l^{(2)} = -\frac{K}{2c\hbar k} \left\{ \alpha \int_0^\infty \overline{V}(t) Q_l \left(1 + \frac{t^2}{2k^2}\right) dt + \beta \int_0^\infty \overline{V}(t) Q_{l-1} \left(1 + \frac{t^2}{2k^2}\right) dt \right\}. \tag{4}$$

where $\overline{V}(t)$ is the transform of the function $rV(r)$; $Q_l(x)$ is a Legendre function of the second kind. The remaining notation here and below coincides with that adopted in ([3]). Substituting these phase values into (2) and carrying out the summation over l, we obtain

$$f(\vartheta) = f_{\mathrm{Re}}(\vartheta) + i f_{\mathrm{Im}}(\vartheta), \qquad g(\vartheta) = g_{\mathrm{Re}}(\vartheta) + i g_{\mathrm{Im}}(\vartheta), \tag{5}$$

where the real parts $f_{\mathrm{Re}}(\vartheta)$ and $g_{\mathrm{Re}}(\vartheta)$ coincide with the scattering amplitudes in the first Born approximation (see, for example, ([5])):

$$f_{\mathrm{Re}}(\vartheta) = -\frac{K}{\hbar c}(\alpha + \beta \cos\vartheta) \int_0^\infty \overline{V}(t)\, \frac{dt}{t^2 + 4k^2 \sin^2(\vartheta/2)},$$

$$g_{\mathrm{Re}}(\vartheta) = -\frac{K}{c\hbar} \beta \sin\vartheta \int_0^\infty \overline{V}(t)\, \frac{dt}{t^2 + 4k^2 \sin^2(\vartheta/2)}; \tag{5a}$$

and for the imaginary parts $f_{\mathrm{Im}}(\vartheta)$ and $g_{\mathrm{Im}}(\vartheta)$, taking formulas (7a)–(7) into account, we obtain the values

$$f_{\mathrm{Im}}(\vartheta) = \frac{K^2}{4c^2\hbar^2 k^3} \int_0^\infty \int_0^\infty \left[(\alpha^2 + 2\alpha\beta x' + \beta^2 \cos\vartheta) \Sigma(x, x', \vartheta) - \alpha\beta \ln \frac{x+1}{x-1} \right] \overline{V}(t) \overline{V}(t')\, dt\, dt', \tag{5 }$$

$$g_{\mathrm{Im}}(\vartheta) = \frac{K^2}{4c^2\hbar^2 k^3} \operatorname{tg}\frac{\vartheta}{2} \int_0^\infty \int_0^\infty \left[\left(2\alpha\beta x' + 2\beta^2 \cos^2\frac{\vartheta}{2} \right) \Sigma(x, x', \vartheta) - \alpha\beta \ln \frac{x+1}{x-1} \right] \overline{V}(t) \overline{V}(t')\, dt\, dt',$$

where

$$x = 1 + \frac{t^2}{2k^2}, \qquad x' = 1 + \frac{{t'}^2}{2k^2},$$

and $\Sigma(x, x', \vartheta)$ is defined by formula (8).

Using (1) and (5), (5), we finally find that in the presence of an arbitrary spherically symmetric scatterer $D(\vartheta)$ is expressed as follows:

$$D(\vartheta) = 2\left[f_{\rm Re}(\vartheta)g_{\rm Im}(\vartheta) - f_{\rm Im}(\vartheta)g_{\rm Re}(\vartheta)\right]. \tag{6}$$

The integrals in (5a), (5) are easily evaluated when $\overline{V}(t)$ is proportional to a δ-function (see below) or to its derivative (exponential potential). In the remaining cases numerical integration is necessary. Let us note that, in order to obtain $D(\vartheta)$ in the next approximation, it is necessary to know the phases in the second approximation (formulas (23) in ([3])). However, the difficulties arising here in the summation have not been overcome.

3. The results obtained can also be applied to the Coulomb field. However, it is first necessary to consider the screened Coulomb field (a Yukawa-type potential)

$$V(r) = -\frac{Ze^2}{r}e^{-\chi_0 r}, \qquad \overline{V}(t) = -Ze^2\delta(t - \chi_0).$$

In this case the δ-functions remove the integration in formulas (5a) and (5). Further, in the expressions for the Yukawa amplitudes one may make the limiting transition to the purely Coulomb field:

$$V(r) = -\frac{Ze^2}{r}, \qquad \overline{V}(t) = -Ze^2\delta(t).$$

Taking into account the limiting value

$$\Sigma\left(1 + \frac{\chi_0^2}{2k^2},\ 1 + \frac{\chi_0^2}{2k^2},\ \vartheta\right) \quad \text{as} \quad \chi_0 \to 0,$$

from (5) we obtain the imaginary parts of the Coulomb scattering amplitudes $f_{\rm Im}(\vartheta)$

and $g^c_{\rm Im}(\vartheta)$ (the real parts are obtained automatically from (5a))

$$f^c_{\rm Im}(\vartheta) = \left(\frac{Ze^2}{2c\hbar}\right)^2 \frac{K^2}{k^3}\left\{(\alpha^2 + 2\alpha\beta + \beta^2\cos\vartheta)\frac{1}{\sin^2(\vartheta/2)}\ln\frac{2k\sin(\vartheta/2)}{\varkappa_0} - 2\alpha\beta\ln\frac{2k}{\varkappa_0}\right\},$$

$$g^{c}_{\mathrm{Im}}(\vartheta) = \left(\frac{Ze^2}{2c\hbar}\right)^2 \frac{K^2}{k^3} \operatorname{tg}\frac{\vartheta}{2} \left\{ \left(2\alpha\beta + 2\beta^2\cos^2\frac{\vartheta}{2}\right) \frac{1}{\sin^2(\vartheta/2)} \ln\frac{2k\sin(\vartheta/2)}{\varkappa_0} - 2\alpha\beta\ln\frac{2k}{\varkappa_0} \right\},$$

whence it is easy to obtain the well-known Mott formula

$$D(\vartheta) = \left(\frac{Ze^2}{2c\hbar}\right)^3 \frac{2K^3}{k^5} \frac{\frac{v^2}{c^2}\sqrt{1 - v^2/c^2}}{\sin(\vartheta/2)\cos(\vartheta/2)} \ln\sin(\vartheta/2).$$

4. In the work[5] sums containing products of three Legendre functions of the first kind were calculated. Using the values of these sums and Heine' s integral representation for Legendre functions of the second kind[6], we obtain the values of the following sums:

$$\sum_{l=0}^{\infty}(2l+1)Q_l(x)Q_l(x')P_l(\cos\vartheta) = \Sigma(x, x', \vartheta), \quad (7\ )$$

$$\sum_{l=0}^{\infty} Q_l(x)\left[(l+1)Q_{l+1}(x') + lQ_{l-1}(x')\right]P_l(\cos\vartheta) = x'\Sigma(x, x', \vartheta) - \frac{1}{2}\ln\frac{x+1}{x-1}, \quad (7\ )$$

$$\sum_{l=0}^{\infty}\left[(l+1)Q_{l+1}(x)Q_{l+1}(x') + lQ_{l-1}(x)Q_{l-1}(x')\right]P_l(\cos\vartheta) = \cos\vartheta\,\Sigma(x, x', \vartheta), \quad (7\ )$$

$$\sum_{l=1}^{\infty} Q_l(x)\left[Q_{l+1}(x') - Q_{l-1}(x')\right]P_l^1(\cos\vartheta) = \frac{x - x'\cos\vartheta}{\sin\vartheta}\Sigma(x, x', \vartheta) - \frac{1}{2\sin\vartheta}\ln\frac{x'+1}{x'-1} + \frac{\cos\vartheta}{2\sin\vartheta}\ln\frac{x+1}{x-1}, \quad (7\ )$$

$$\sum_{l=1}^{\infty}\left[Q_{l+1}(x)Q_{l+1}(x') - Q_{l-1}(x)Q_{l-1}(x')\right]P_l^1(\cos\vartheta) = \sin\vartheta\,\Sigma(x, x', \vartheta), \quad (7\ )$$

where

$$\Sigma(x, x', \vartheta) = \frac{1}{2\sqrt{x'^2 - 2xx'\cos\vartheta + x^2 - \sin^2\vartheta}} \times$$

$$\times \ln \frac{(x' - \cos\vartheta)(\sqrt{x'^2 - 2xx'\cos\vartheta + x^2 - \sin^2\vartheta} + x') - x(1 - x'\cos\vartheta) - \sin^2\vartheta}{(x' + \cos\vartheta)(\sqrt{x'^2 - 2xx'\cos\vartheta + x^2 - \sin^2\vartheta} + x') - x(1 + x'\cos\vartheta) - \sin^2\vartheta} \frac{x+1}{x-1}. \tag{8}$$

In passing to the Coulomb field it is necessary to use the limiting value

$$\lim_{\varkappa_0 \to 0} \Sigma\left(1 + \frac{\varkappa_0^2}{2k^2}, 1 + \frac{\varkappa_0^2}{2k^2}, \vartheta\right) = \frac{1}{\sin^2(\vartheta/2)} \frac{2k\sin(\vartheta/2)}{\varkappa_0}.$$

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Note: Figure translations are in progress. See original paper for figures.