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Abstract

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ON THE MOTION OF A PISTON IN A CONDUCTING MEDIUM

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The problem of a piston moving in a conducting liquid in the presence of a magnetic field has been considered by a number of authors. A. G. Kulikovskii ([1]) considered piston motion in which rarefaction waves and an Alfvén discontinuity proceed ahead of the piston. Beiser ([2]) considered the motion of a piston perpendicular to the magnetic field. I. A. Akhiezer and R. V. Polovin ([3]), and R. V. Polovin ([4]), investigated the motion of a piston in small magnetic fields. G. Ya. Lyubarskii and R. V. Polovin solved the problem of the motion of a piston along the normal ([5]).

In the present work the motion of a piston in a magnetic field of arbitrary intensity is investigated, when the lines of force of the magnetic field are perpendicular to the plane of the piston. As in the previous works, both the piston and the medium are assumed to be ideally conducting. The state in front of the piston is characterized by: $p_0; \rho_0; H_n; H_{\tau_0} = 0; S = c^2/V_{\mathrm{n}}^2$, where $c^2 = \gamma p/\rho; V^2 = H^2/4\pi\rho; q = c_{\pm}^2/c^2$; $c_{\pm}$ are the velocities of the fast and slow magnetohydrodynamic rarefaction waves. $\vec{v}_{\mathrm{n}}(u_{\mathrm{n}}; v_{\mathrm{n}}; w_{\mathrm{n}})$ is the velocity of the piston. The remaining notation is standard.

Ahead of the piston there may propagate:

1) a fast shock wave Y^+, followed either by a slow shock wave Y^- or by a slow rarefaction wave P^-;
2) a fast rarefaction wave P^+, followed either by a slow shock or by a slow rarefaction wave;
3) each of these waves separately. Altogether 8 cases are possible. There can be no Alfvén discontinuity; therefore the problem may be regarded as planar.

If $H_{\tau_0} = 0$, then two cases are possible ([6]). Either behind the wave $H_{\tau_1} = 0$, in which case the field drops out of the relations on the shock wave and the wave will be a purely gas-dynamic Y_{g}; or H_{τ_1} is arbitrary—the wave Y_{v} "switches on" the field and moves relative to the gas remaining behind it with the Alfvén velocity.

Fig. 1

Figure 1: Fig. 1

Fig. 2

Figure 2: Fig. 2

From the relations on the shock wave it follows that, if $S_0 \geq 1$, there can be no Y_{v}-wave. If $S_0 < 1$, then both Y_{g}- and Y_{v}-waves may occur. The Y_{v}-wave is always fast (Y_{v}^+); the Y_{g}-wave may be either fast (Y_{g}^+) or slow (Y_{g}^-).

Rarefaction waves have been investigated in ([1], [2], [8], [9]). In a P^+-wave the absolute value of the tangential component of the field decreases. Therefore, if $H_{\tau_0} = 0$, there can be no P^+-wave that "switches on" the field. We shall show that there can be a purely gas-dynamic fast wave P_{g}^+.

The field in a $P^\pm$-wave is related to the pressure by the relation $H_\tau^2/H_{\mathrm{n}}^2 = (q-1)(S-q^{-1})$. If $H_{\tau_0} = 0$, then either $c_+ = c,\ c_- = V_{\mathrm{n}}$, or $c_+ = V_{\mathrm{n}};\ c_- = c$.

If in the wave $H_\tau = 0$ and $S_0 > 1$, then the rarefaction wave will be the ordinary gas-dynamic one, propagating with velocity $c_+ = c$. The pressure in the wave will decrease until S becomes equal to 1. For $S = 1$, $c_+ = c = V_n = c_-$. For $S < 1$ the P_r^+-wave passes into the gas-dynamic rarefaction wave P_r^-, which is the analogue of the slow magnetohydrodynamic rarefaction wave for $H_\tau = 0$. Its propagation velocity is equal to $c_- = c$. In a P_r^--wave, S can fall to zero.

Thus, if in $P^\pm$-waves $H_{\tau_0} = 0$, the waves degenerate into an ordinary gas-dynamic rarefaction wave P_r.

Fig. 1

Fig. 2

If ahead of a P^--wave $H_{\tau_0} = 0$ and $q_0 \neq 1$ (excluding the gas-dynamic wave), then $S_0 = q_0^{-1}$. In a P^--wave $q < 1$, and therefore $S_0 > 1$. If ahead of a P^--wave $H_{\tau_0} = 0$ and $S_0 \leqslant 1$, then the rarefaction wave will be the ordinary gas-dynamic one.

The gas velocity ahead of the piston is equal to the piston velocity ([5]).

From what has been said above it follows:

1. If $S_0 > 1$, then the following waves may precede the piston: $\mathcal{Y}_r^+$; P_r; P^-; $\mathcal{Y}_r^+ P^-$; $P_r^+ P^-$.
2. If $S_0 < 1$, then the following waves may precede the piston: $\mathcal{Y}_r^-$; $\mathcal{Y}_r^+$; P_r; $\mathcal{Y}_v^+$; $\mathcal{Y}_r^+ P^-$; $\mathcal{Y}_v^+ P^-$; $\mathcal{Y}_v^+ \mathcal{Y}^-$.
3. If $S_0 = 1$, then the following waves may precede the piston: P_r; $\mathcal{Y}_r^+$; $\mathcal{Y}_r^+ P^-$.

The choice of one or another combination of waves depends on the piston velocity. This dependence is shown in Fig. 1 ($S_0 > 1$) and Fig. 2 ($S_0 < 1$), where along the X-axis $u_p/V_0 = u^*$ is plotted, and along the Y-axis $v_p/V_0 = v^*$. The line of the P^--wave separates the regions $\mathcal{Y}_r^+ P^-$ and $P_r^+ P^-$. The line $\mathcal{Y}_v^+$ separates the regions $\mathcal{Y}_v^+ \mathcal{Y}^-$ and $\mathcal{Y}_v^+ P^-$. The line of the P_*^--wave separates the regions $\mathcal{Y}_v^+ P^-$ and $\mathcal{Y}_r^+ P^-$.

A single shock gas-dynamic wave is formed ahead of the piston for $v^* = 0,\ u^* > 0$ when $S_0 \geqslant 1$, and for $v^* = 0,\ 0 < u^* < \dfrac{2}{\gamma+1}(1 - S_0) \equiv u_1^*$ ($\mathcal{Y}_r^-$-wave),

$$u^* > \frac{2}{\gamma-1}(1-S_0)\left[\frac{\gamma+1}{\gamma-1} - \frac{2}{\gamma-1}S_0\right]^{-1/2} \equiv u_2^* \quad (\mathcal{Y}_r^+\text{-wave})$$

for $S_0 < 1$.

The piston velocity lying on the segment $u_1^* u_2^*$ is equal to the velocity behind a $\mathcal{Y}_r$-wave for which the $\mathcal{Y}_r$-wave is non-evolutionary. On the segment $u_1^* u_2^*$ terminate the lines of $\mathcal{Y}^-$-waves whose beginning lies on the line of the $\mathcal{Y}_v$-wave. The $\mathcal{Y}_v$-wave is also non-evolutionary ([10]). However, from the assumption of existence and uniqueness of the solution of the given self-similar problem it follows that the combinations of waves $\mathcal{Y}_v^+ P^-$ and $\mathcal{Y}_v^+ \mathcal{Y}^-$ are the unique solution of the problem for the corresponding piston velocities.

A vacuum may form ahead of the piston if the piston velocity vector lies beyond the vacuum line. The vacuum line in Figs. 1 and 2 is marked—

is indicated by hatching and passes through the points: $v^* = 0;\ u^* = -\dfrac{2}{\gamma-1}\dfrac{c_0}{V_0}$ (vacuum is reached in the gas-dynamic rarefaction wave) and $u^* = 0;\ v^* = +3.67 c_0/V_0$ (the case considered in work [2]).

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Note: Figure translations are in progress. See original paper for figures.