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Abstract

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ON ONE POSSIBLE MECHANISM FOR THE CAPTURE OF CHARGED PARTICLES IN A MAGNETIC FIELD

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As is known, the solution of the problem of the motion of a charged particle in the magnetic field of a magnetic dipole in the "conservative" approximation, i.e., without taking into account the loss of kinetic energy, leads to the conclusion that its capture by the magnetic field is impossible ([1]). However, the "conservative" solution in the present case is not entirely rigorous, since, according to the classical theory of electromagnetic-wave radiation, any curvature of the trajectory of a charged particle entails a certain loss of energy to radiation. Substituting into the well-known formula that determines the flux of radiation energy of an accelerated moving particle,

$$S = {}^2/_3\, e^2\dot{v}^2/c^3 \tag{1}$$

(S is the energy radiated per unit time, e is the charge, v is the velocity, c is the electrodynamic constant), the value of the radius of curvature of the particle in the magnetic field $r = mvc/eH$ (m is the mass of the particle, H is the component of the magnetic field normal to the velocity vector), and the value of the total kinetic energy ($S_0 = mv^2/2$), we obtain the relative losses to radiation on a small arc of the trajectory of the moving particle $d\varphi$:

$$\frac{dS_0}{S_0} = \frac{4}{3}\frac{e^3 H}{c^4 m^2}\, d\varphi. \tag{2}$$

Thus, for example, an electron bending around the Earth' s magnetic field along the equator at a distance of 20,000 km from the center loses, over an arc of 1 radian, radiation energy $\Delta S_0 = 2 \cdot 10^{-18} S_0$, which may serve as an estimate of the order of magnitude of the radiation losses. Despite the extreme smallness of the radiation losses, they prove sufficient for particles whose trajectories and velocities satisfy certain "critical" conditions to be captured by the magnetic field.

Fig. 1

Figure 1: Fig. 1

There are also other causes that create small energy losses of particles: ionization losses, losses due to interaction with plasma inhomogeneities, etc. In previous works on the investigation of particle trajectories in the field of a magnetic dipole, the indicated energy losses were usually neglected ([2]).

An analysis of the character of particle trajectories and of the conditions under which capture is possible can be carried out by the "phase-space" method and the "method of isolines" ([3]). The derivations take an especially simple form for the case of particle motion in the equatorial plane of the dipole; the general case of arbitrary three-dimensional motion can be analyzed qualitatively on the basis of the solutions obtained for the equatorial plane.

The differential equation of motion of a charged particle in the field of a dipole for the case of the equatorial plane is conveniently represented in polar coordinates (ρ, φ). The vector of magnetic-field strength in the equato-

plane is perpendicular to this plane and is, in magnitude,

$$H = M/\rho^3. \tag{3}$$

The curvature of the trajectory of a freely moving charged particle in a magnetic field is equal to [4]

$$K = eH/mvc. \tag{4}$$

On the other hand, as is known, the expression for the curvature of a line in polar coordinates has the form

$$K = \frac{\rho^2 + 2\rho o'^2 - \rho\rho''}{(\rho^2 + \rho o'^2)^{3/2}}. \tag{5}$$

Fig. 1

Using (3), (4), (5), we obtain the differential equation of motion of a charged particle in the field of a magnetic dipole (in the equatorial plane)

$$\frac{\rho^2 + 2\rho o'^2 - \rho\rho''}{(\rho^2 + \rho o'^2)^{3/2}} = \frac{a^2}{\rho^3}, \tag{6}$$

where $a = \sqrt{\dfrac{eM}{mvc}}$ is the "characteristic coefficient," M is the magnetic moment of the dipole.

Fig. 2

Figure 2: Fig. 2

In the "conservative" approximation, the kinetic energy and the velocity of the particle remain unchanged, and consequently the "characteristic" coefficient a also remains unchanged in magnitude. In the case, however, where part of the kinetic energy is expended on radiation or on losses of other types, the particle velocity decreases, and the "characteristic" coefficient increases. For small energy losses the approximate relation is valid

$$\Delta v/v \cong \tfrac{1}{2}\,\Delta S_0/S_0 \cong -2\Delta a/a. \tag{7}$$

Analysis of equation (6) can be carried out by passing to phase coordinates $w = \rho/a;\ u = \dfrac{d\rho}{d\varphi}/a$. The differential equation of the phase curves corresponding to equation (6) has the form

$$\frac{du}{dw} = \frac{w}{u} + 2\frac{u}{w} - \frac{[1 + (u/w)^2]^{3/2}}{uw}, \tag{8}$$

where the minus sign on the right-hand side corresponds to trajectories with "positive" curvature, i.e., to segments of the particle trajectories whose convexity is directed away from the pole of the coordinates. The plus sign corresponds to trajectories with negative curvature. To analyze the character of the phase curves, the "isoclines" of equation (8) were constructed graphically, from which the "direction field" was determined. From the "direction field" the character and the main features of the phase curves were determined. The behavior of the phase curves near singular points, and the asymptotic expressions for the phase curves at very large and very small values of (u, w), were investigated analytically.

Fig. 2

In Fig. 1 the conservative approximation shows the field of phase curves for trajectory segments with positive curvature (with convexity directed away from the pole of coordinates). Fig. 2 shows trajectories in the equatorial plane corresponding to various phase curves in the conservative approximation. A phase curve of type I corresponds to an "enveloping" trajectory; the phase curves II, III, and Va jointly determine a "loop" trajectory; and phase trajectories of type IV and Va determine a "captured" trajectory. Curves of type Vb and the boundary curve VII determine "non-enveloping" trajectories of hyperbolic type. The phase curves VIa and VIb are separatrices separating the various types of curves, and the singular point A corresponds to an unstable circular trajectory with radius $\rho = a =$ const.

Fig. 3

Figure 3: Fig. 3

Let us consider the representation on the phase plane of the process of change in the kinetic energy of the particle. An instantaneous change of the energy by an amount ΔS_0, without a break and without a "kink" in the particle trajectory in space, leads to a change of the coefficient a, while the former values of ρ and $d\rho/d\varphi$ are preserved. In this case the "representative point,"characterizing on the phase plane the instantaneous state of the particle, will change its coordinates by the amounts

$$\begin{aligned}\Delta u &= \frac{\partial u}{\partial a}\Delta a = \frac{1}{2}u\frac{\Delta v}{v} = \frac{1}{4}u\frac{\Delta S_0}{S_0},\\ \Delta w &= \frac{\partial w}{\partial a}\Delta a = \frac{1}{2}w\frac{\Delta v}{v} = \frac{1}{4}w\frac{\Delta S_0}{S_0}.\end{aligned} \tag{9}$$

Fig. 3

With losses of energy $\Delta S_0 < 0$, and consequently, the representative point will be displaced in the direction toward the origin. With continuous losses of energy, the motion of the "representative point" along the phase curve will take place in such a way that it will always be displaced, relative to the "conservative" phase curves, in the direction toward the origin; this is not difficult to verify either by representing a continuous loss of energy as a sum of infinitely small discrete losses and using expressions (9), or by using (8) directly, varying the coefficient a and taking into account the direction of motion of the representative point on the phase plane.

In Figs. 1 and 2 the change in the direction of motion of the representative point in the presence of kinetic-energy losses is shown schematically (by dashed arrows). In the case where the phase curve for a particle moving along the envelope trajectory is, in the conservative approximation, sufficiently close to the separatrix, energy losses will cause the phase curve to cross the separatrix and, consequently, the particle will pass from the envelope trajectory to the "loop" trajectory. Further, as the representative point moves along the left branch of a phase curve of type *III* (Fig. 1), continuing energy losses may lead to the representative point crossing the second separatrix and, consequently, to the transition of the particle onto a captured trajectory determined by the phase curves *IV* and *Va*. In Fig. 3 this process is shown schematically on the phase trajectory; in Fig. 2, by a dashed line, it is shown in the equatorial plane of the dipole.

The transition from the special case of the equatorial plane to arbitrary three-dimensional motion leads to an increase in the dimensionality of the phase space to four dimensions, $u, w, \vartheta, d\vartheta/d\varphi$, where ϑ is the meridional angle; moreover, everywhere except for the special case of the meridional plane, the deformation of the two-dimensional phase subspace u, w occurs continuously, so that the separatrices form four-dimensional hypersurfaces continuous everywhere except in the meridional planes. This proves the validity of the conclusion concerning the possibility of capture in the general case of three-dimensional motion.

The motion of a particle along a trajectory infinitely close to the separatrix is characterized by a critical relation between v and $\rho_{\min}$, which for motion in the equatorial plane has the form:

$$v_{\text{cr}} = eM/mc\rho_{\min}^2. \tag{10}$$

Thus, for any distance $\rho_{\min}$ there is its own critical value of the particle velocity, at which an arbitrarily small loss of energy leads to the capture of the charged particle.

The mechanism of particle capture described is certainly not the only one [(5)].

It is interesting to note that the capture mechanism proposed in the present work leads to a gradual concentration of particles in any inhomogeneous magnetic field with simultaneous emission of electromagnetic waves, which may be of interest from the standpoint of problems of cosmogony. In particular, this mechanism may be of interest for explaining the nature of "radio stars."

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Note: Figure translations are in progress. See original paper for figures.