ON REDUCING THE INFLUENCE OF INERTIA IN EXTREMAL CONTROL OF OBJECTS OF THE n-TH ORDER

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Abstract

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ON REDUCING THE INFLUENCE OF INERTIA IN EXTREMAL CONTROL OF OBJECTS OF THE n-TH ORDER

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In the present note, with reference to an object of the n-th order, a method is considered for reducing the influence of the object' s inertia on the process of extremal control. The application of this method to an object of the first order was considered in ([2]).

Let us suppose that the extremal controller operates according to the principle of "remembering the maximum." From this principle itself it follows that reversal of the actuator, i.e. a change in the sign of the velocity $\dot{x}$ of the object input, occurs after passage through the maximum of the dynamic quantity y of the object output measured by the sensing element. But this quantity, in the case of an inertial object, is not, generally speaking, equal to the quantity y^* determined by the static characteristic of the object

$$y^* = f(x). \tag{1}$$

At the same time, the controller must seek the extremum of the static quantity y^*. It is obvious that the oscillatory nature and inaccuracy of the control process will be the smaller, the closer the dynamic and static curves y and y^* are. Since the object is assumed to be stable, an increase in the closeness of the curves y and y^* can be achieved by decreasing the rate of the artificial disturbance (the search rate). If the search rate tends to zero, then the dynamic dependence y tends to the static y^*. However, this path is associated with an excessive increase in the time required to establish the regime. Is there another way of reducing the influence of inertia? An affirmative answer to this question can be given if one takes into account that into the sign relay ([1]), which gives the command for reversal of the input x, it is by no means necessary to introduce the dynamic output quantity y. If it were possible, by some means, knowing the dynamic quantity y, to determine in finite time that value $y_\infty = y^*$ which the output will reach after an infinitely long time, then, by introducing the found y_∞ into the sign relay, it would be possible to ensure a nonoscillatory approach to the value $x = x_0$ corresponding to the maximum, and then the fastest approach to the extremum with small oscillations. Thus, it is necessary to find a method for determining in finite time that value $y_\infty = y^*$ which the real system will reach after an infinitely long interval of time.

Suppose that the controlled object is described by a linear differential equation of the n-th order with constant coefficients

$$y^{(n)} + b_1 y^{(n-1)} + ... + b_n y = 0. \tag{2}$$

Let the static dependence (1) between the output y and the input x be continuous.

If an extremal controller is connected to the plant, then the dynamic output y of the plant will be described by the differential equation

$$y^n + b_1 y^{(n-1)} + ... + b_{ny} = f(x). \tag{3}$$

Let the initial conditions at time $t = 0$ be

$$y = y_0, \quad \dot{y} = \dot{y}_0, \dots, \quad y^{(n-1)} = y_0^{(n-1)}, ... \tag{4}$$

Assuming a stepwise control system, let us apply at $t = 0$ a displacement Δ to the input x; then a transient process will begin according to the law described by the differential equation

$$\bar{y}_1^{(n)} + b_1 \bar{y}_1^{(n-1)} + ... + b_n \bar{y}_1 = a_1, \tag{5}$$

where

$$\bar{y}_1 = y - y_0, \qquad a_1 = f(x_1) - y_0 = \text{const}. \tag{6}$$

The solution of (5) will be

$$\bar{y}_1 = \frac{a_1}{b_n} + \sum_{k=1}^{n} A_{k1} e^{-\lambda_k t}, \tag{7}$$

where λ_k are the roots of the characteristic equation

$$\lambda^n + b_1 \lambda^{n-1} + ... + b_n \lambda = 0; \tag{8}$$

A_{k1} are coefficients depending on λ_k and on the initial conditions. Suppose that the properties of the plant are known and that, consequently, all λ_k are known.

As we saw earlier, the fastest establishment process will take place if the signal relay is fed not with the dynamic output, but with the static one, i.e., the value

y which in the real system will be attained only at $t = \infty$. Consequently, our task is to find the value $y_\infty(x)$ over a finite interval of time.

Consider expression (7).

$$\text{For } t = \infty \qquad \bar{y}_{1\infty} = \frac{a_1}{b_n}. \tag{9}$$

Consequently, in order to find the static value $y_{1\infty}$, it is necessary to know the quantity a_1, which is as yet undefined. If the values $a_1, A_1, \ldots, A_n$ are known, then for any t one can find the corresponding value $\bar{y}_1(t)$.

However, equation (7) can be approached from the standpoint of the well-known principle of duality and $a_1, A_1, \ldots, A_n$ can be regarded as quantities to be determined. These $n+1$ quantities can be found if $n+1$ systems of values $\bar{y}_{1i}, t_i$ are known. From this follows a method for determining, over a finite interval of time, the value $y_{1\infty}$ which the system reaches at $t = \infty$.

After shifting the input by the amount Δ, one must introduce $n+1$ time delays and measure the values of the variable $\bar{y}_1$ at the ends of these intervals (delays). Let the intervals be identical and equal to τ. As a result, a system of $n+1$ equations with $n+1$ unknowns is obtained:

$$\bar{y}_{11} = \frac{a_1}{b_n} + \sum_{k=1}^{n} A_{k1} e^{-\lambda_k \tau},$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots \tag{10}$$

$$\bar{y}_{1n} = \frac{a_1}{b_n} + \sum_{k=1}^{n} A_{k1} e^{-\lambda_k (n+1)\tau}.$$

Computing the quantity a_1/b_n with the aid of a computing device, one can find the static value of the output y_1^*

$$y_1^* = y_0 + \overline{y}_{1\infty} = y_0 + \frac{a_1}{b_n}.$$

Then a new input displacement by Δ is applied and, again, at the ends of $n+1$ time intervals the value of the dynamic output y is measured; in the second stage the motion will be described by the differential equation

$$\overline{y}_2^{(n)} + b_1 \overline{y}_2^{(n-1)} + \ldots + b_n \overline{y}_2 = a_2, \tag{11}$$

where

$$\overline{y}_2 = y - \left(y_0 + \overline{y}_{1,n+1}\right), \qquad a_2 = f(x_2) - \left[f(x_0) + \overline{y}_{1,n+1}\right], \tag{12}$$

with the derivatives taken with respect to time $t' = t - (n+1)\tau$, and, consequently, $dt' = dt$. In what follows we omit the prime symbol. The solution of this equation will have the form

$$\overline{y}_2 = \frac{a_2}{b_n} + \sum_{k=1}^{n} A_{k2} e^{-\lambda_k t}, \tag{13}$$

where A_{k2} are coefficients depending on λ_k and on the initial conditions at $t' = 0$, i.e., at $t = (n+1)\tau$. These initial conditions are easily determined by differentiating expression (7) $n-1$ times and substituting the value $t = (n+1)\tau$. As a result, the static value y_2^* is determined for $x = x_0 + 2\Delta$

$$y_2^* = y_0 + \overline{y}_{1,n+1} + \frac{a_2}{b_n}.$$

By introducing y_2^* and y_1^* into the signum relay, the magnitude of the output increment is found:

$$\delta_1 = y_2^* - y_1^* = \left(y_0 + \overline{y}_{1,n+1} + \frac{a_2}{b_n}\right) - \left(y_0 + \frac{a_1}{b_n}\right),$$

or

$$\delta_1 = \overline{y}_{1,n+1} + \frac{a_2 - a_1}{b_n}.$$

The sign of δ_1 determines the direction of the subsequent displacement of x; after a new input displacement by Δ, the measurement process continues in an analogous manner. Thus, with one measuring device, $n+1$ time intervals are required.

Case of two measuring devices. Suppose that two measuring devices are used in the controller: a measuring device for the output y and a measuring device for some one of the $n-1$ derivatives; it is not difficult to show that in this case the number of time delays can be reduced by half. Suppose, for example, that a first-derivative sensor is used; it will measure the quantity $\dot{y}_1$, obtained by differentiating expression (7):

$$\dot{y}_1 = -\sum_{k=1}^{n} \lambda_k A_{k1} e^{-\lambda_k t}. \tag{14}$$

Then, taking into account that with a derivative measuring device one can perform $k+1$ measurements over k intervals, it is easy to establish that, for even n, $n/2$ time delays are sufficient, while for odd n, $(n+1)/2$ time delays are sufficient. As the number of measuring devices is increased, the number of time delays decreases correspondingly. If measuring devices are used for the output and for the first $n-1$ derivatives, then a single time delay is sufficient.

Let us now suppose that the controlled object is aperiodic, but the values of the roots of the characteristic equation are unknown. In this case—

the number of unknowns increases to $2n+1$. If the controller has a single output meter, then $2n+1$ equations of the form (10) are required, which requires $2n+1$ time delays. If meters of the output y and of some derivative are used, then the number of time delays for even n will be equal to n, and for odd n, to $(2n+1)/2$. As the number of derivative meters increases, the number of delays decreases correspondingly. If meters of the output and of the first $n-1$ derivatives are available, 2 time delays are sufficient.

If the process is nonperiodic and, moreover, the character of the transient process is known, then the number of time delays is the same as in the case of an aperiodic process. If monotone external disturbances act in the system, they can be accounted for in an analogous manner, adding, if necessary, one time delay.

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CITED LITERATURE

1. V. V. Kazakevich, *On extremal control*, Dissertation, MVTU, 1944.

2. V. V. Kazakevich, DAN, 126, No. 3 (1959).

Note: Figure translations are in progress. See original paper for figures.