ON THE CALCULATION OF OBLIQUE SHOCK WAVES IN MAGNETIC GAS DYNAMICS

h_{2y}^{3}

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Abstract

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ON THE CALCULATION OF OBLIQUE SHOCK WAVES IN MAGNETIC GAS DYNAMICS

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The tangential component of the magnetic field behind the front of an oblique shock wave is determined from a cubic equation, which, in the notation of (1), is written in the form

$$h_{2y}^3+h_{1y}\left[1-(2-k)\left(1-\frac{u_{1x}^2}{V_{1x}^2}\right)\right]h_{2y}^2+\rho_1\left(1-\frac{u_{1x}^2}{V_{1x}^2}\right)\left(V_{1x}^2-a_{1Mx}^2\right)(k+1)h_{2y}-$$

$$-(k+1)V_{1x}^2 h_{1y}\rho_1\left(1-\frac{u_{1x}^2}{V_{1x}^2}\right)=0. \tag{1}$$

The intensity of the discontinuity is determined by the Mach number $M_{Mx} = u_x/V_x$ and by the dimensional parameter $V_x^2 - a_{Mx}^2$, which specifies the signal speed and, consequently, the probability of information transfer in the direction perpendicular to the wave front (2); the magnitude of the velocity component u_{1y} parallel to the front does not affect the field jump. The cubic equations for determining the velocities u_{2x}, u_{2y} and the density ρ_2 behind the front are obtained from (1) by linear and fractional-linear substitutions:

$$h_{2y}=\frac{\rho_1 u_{1x}}{h_x}(u_{2y}-u_{1y})+h_{1y}, \tag{2}$$

$$h_{2y}=h_{1y}\frac{\rho_1 u_{1x}^2-h_x^2}{\rho_1 u_{1x}u_{2x}-h_x^2}=\frac{1-u_{1x}^2/V_{1x}^2}{1-\dfrac{u_{1x}}{V_{1x}}\dfrac{u_{2x}}{V_{1x}}}h_{1y}, \tag{3}$$

$$h_{2y}=h_{1y}\rho_2\frac{\rho_1 u_{1x}^2-h_x^2}{\rho_1^2 u_{1x}^2-h_x^2\rho_2}=\frac{1-u_{1x}^2/V_{1x}^2}{1-\dfrac{u_{1x}^2}{V_{1x}^2}\dfrac{\rho_1}{\rho_2}}h_{1y}. \tag{4}$$

That branch of the roots of these cubic equations which, when represented by Cardano’ s formula (3), has a positive imaginary part describes accelerated

Fig. 1

Figure 1: Fig. 1

Fig. 2

Figure 2: Fig. 2

shock waves, which pass, as the field tends to zero, into the oblique shock waves of nonmagnetic gas dynamics. Retarded shock waves, which disappear in perpendicular fields, are described by the branch of roots with negative imaginary part. For $M_{Mx} = 1$, the solution of equation (1) is $h_{2y} = -h_{1y}$—a rotational discontinuity.

In Fig. 1, the solid line gives the dependence of u_{2y} on u_{2x}, representing the shock polar of the strong family for the particular case of an incident flow normal to the front; the energy of the magnetic field is greater than or equal to the internal energy and the kinetic energy of the gas. The upper half of the shock polar corresponds to changes of the field inclination from 0

to $\pi/2$, or from π to $3/2\pi$; the lower one, respectively, from $\pi/2$ to π, or from $3/2\pi$ to 2π.

While the field (Fig. 2) behind the front changes monotonically, the dependence of u_{2y} on u_{2x} has the character of a hump, the top of which corresponds to a certain "resonant" inclination of the magnetic field.

If the incident flow has a velocity component along the front, then the velocity polar is displaced by the magnitude of this component in the corresponding direction along the u_y axis. In Fig. 1 the displaced polar is shown by the dashed line. The vector $\overrightarrow{OA}$ gives the direction of the incident flow, the vector $\overrightarrow{OB}$ the direction of the flow behind the front, and the vector $\overrightarrow{OC}$ the direction of the flow behind the front in the absence of a magnetic field.

Fig. 1

Fig. 2

Fig. 3

It is evident that, in solving the problem of oblique collision of inclined shock waves and their reflection from a wall, the most interesting regimes are those in the neighborhood of the "resonant" inclination of the field. In particular, the formation of certain types of protuberances near sunspots can possibly be explained as a "resonant" or nearly "resonant" ejection of a mass of gas behind

Fig. 3

Figure 3: Fig. 3

Fig. 4

Figure 4: Fig. 4

the shock-wave front.

The results obtained above can also be applied to the construction of a nomogram for calculating the flow past a wedge by an ideally conducting supersonic flow (^{4}).

Let the flow velocity u_1 and the field H, inclined at an angle α to the flow, as well as the angle of deflection of the flow behind the front θ (Fig. 3), be prescribed, and determine the angle of inclination of the front φ and the velocity behind the front. To do this, varying u_{1x} with $u_{1y} = 0$ and for the prescribed field modulus, we construct a family of polars. We obtain the second family of curves of the nomogram by joining the points of the polars corresponding to the same—

Fig. 4

to the same inclination of the field to the flow (Fig. 4). Varying the angle β at fixed α, we choose such a γ that the line of equal inclination corresponding to this value of the angle intersects the straight line drawn from the point $A(0, -u_1 \sin\beta)$ at the angle $\delta = \beta + \theta$ at the point through which passes the polar corresponding to $u_{1x} = u_1 \cos\beta$. The resulting vector $\overrightarrow{AB}$ gives the velocity behind the front of the shock wave, whose inclination φ is then equal to β. The remaining parameters behind the front are then easily determined.

For a perpendicular wave, the jump in temperature and entropy is readily obtained in explicit form, using the results of (1),

$$T_2 - T_1 = \frac{(k-1)}{R}\mu\left\{\frac{u_1^2}{2} + V_1^2\frac{2k-1}{k+1} + \frac{a_{1M}^2}{2}\left[\frac{k+1}{2-k} - \frac{a_{1M}^2}{u_1^2} - \sqrt{1 + 4\frac{2-k}{k+1}\frac{u_1^2 V^2}{a_{1M}^4}\left(\frac{k+1}{2-k} + 4\frac{a_{1M}^2}{u_1^2}\right)}\right]\right\}; \qquad (5)$$

$$S_2 - S_1 = c_v\left\{\ln\left[1 - \frac{h_2^2 - h_1^2}{2p_1} + \frac{\rho_1 u_1(u_1 - u_2)}{p_1}\right] - k\ln\frac{u_1}{u_2}\right\} \qquad (6)$$

(u_2 and p_2, see (1)).

For $k = 2$ the expressions are considerably simplified:

$$T_2 - T_1 = \frac{\mu}{R}\left(\frac{u_1^2}{2} + V_1^2 - \frac{5}{2}\frac{a_{1M}^4}{u_1^2}\right); \qquad (7)$$

$$S_2 - S_1 = c_v \left\{ \ln \left[1 - \frac{h_1^2}{2p_1} \left(2\frac{u_1^4}{a_{1M}^4} - 1 \right) + \frac{\rho_1 u_1^2}{p_1} \left(1 - \frac{a_{1M}^2}{u_1^2} \right) \right] - 2\ln \frac{a_{1M}^2}{u_1^2} \right\},$$

where

$$a_M^2 = \frac{2c^2 + u^2 + 2V^2}{3}.$$

The magnitude of the jump decreases as the field increases, with the other parameters unchanged, since in this case the gas-magnetic speed of sound ahead of the front increases, $c_M = \sqrt{c^2 + V^2}$, and the Mach number $M = u/c_M$ tends to unity; this also explains the stretching of the velocity polar along the u_x axis. At the field value $h = \sqrt{\rho(u^2 - c^2)}$ the discontinuity becomes weak. This is illustrated by the geometrical interpretation given for the perpendicular shock wave in (1).

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Note: Figure translations are in progress. See original paper for figures.