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Abstract

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TRANSIENT OPERATING REGIMES OF CONTINUOUSLY OPERATING HEAT EXCHANGERS WITH THICK WALLS

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In work ([1]) the problem of transient processes in "tube-in-tube" heat exchangers with thin walls was solved (i.e., without taking into account the heat capacity of the walls). In the present communication the solution of the nonstationary problem is extended to a more general case, more frequently encountered in practice (when there is a thick wall separating the moving media); here an effective method of Laplace transformation is used (numerous examples of its application in solving nonstationary problems for immobile media are given in ([2])).

Let us consider heat exchange through a partition between two incompressible liquids under the following restrictions:

1) The flow of the liquid is one-dimensional, which corresponds to motion through tubes in the turbulent regime.
2) The coefficient of thermal diffusivity of the material of the partition is so large that the duration of heat propagation in it may be neglected. In practice this condition is satisfied for plane and cylindrical walls when $F_0 > 0.3$, where F_0 is the Fourier criterion; in this case the constancy of the thermophysical parameters of the liquid and the wall is preserved with an accuracy of up to 2%, which does not lead to noticeable errors.
3) The heat flux along the wall may be neglected, since the driving force of the process—the temperature differences in the wall—is negligibly small in comparison with the temperature differences in the radial directions between the hot and cold heat-transfer media.

In contrast to the conditions adopted in work ([1]), in the transient operating regimes of the apparatus we shall take into account the heat capacity of the wall separating the two media.

To the derivation of the general equations of nonstationary heat transfer ([1]), in this case there is added consideration of the wall as a third intermediate medium with velocity equal to zero.

For the case of counterflow the equations have the form*

$$-\frac{\partial T_1}{\partial t} + \frac{\partial T_1}{\partial x} w_1 = k_1(T - T_1),$$

$$\frac{\partial T}{\partial t} = k_2(T_1 - T) + k_3(T_2 - T), \tag{1}$$

$$\frac{\partial T_2}{\partial t} - \frac{\partial T_2}{\partial x} w_2 = k_4(T - T_2),$$

where $T(x,t)$, $T_1(x,t)$, $T_2(x,t)$ are, respectively, the temperature of the wall, of the hot medium, and of the cold medium in relative units**; x is the coordinate —

* The equations for the case of parallel flow are obtained by replacing w_2 by $-w_2$.

** $T_1(0,0)$ is taken as the unit of measurement, and $T_2(l,0)$ as zero, where l is the length of the apparatus.

length of the apparatus section, measured in the direction of motion of the medium T_1; t is time; $k_1,\ k_2,\ k_3,\ k_4$ are the corresponding constants; $w_1,\ w_2$ are the linear velocities of the moving media.

As the disturbing action, a step change in temperature (Heaviside function) at the inlet to the heat exchanger on the heating heat-transfer medium is assumed.

For example, the transient process is determined at the outlet from the apparatus for this same medium. Owing to the linearity of the system, we may take zero initial conditions; the boundary conditions will then have the form

$$T_1(0,t) = 1, \qquad T_2(l,t) = 0.$$

Application of the Laplace-transform method to the solution of the basic equations (1) leads to a system of two ordinary differential equations

$$\lambda_1 \frac{dT_1}{dx} + [p + k_1(1 - \rho_2)]\, T_1 - k_1 \rho_1 T_2 = 0,$$

$$\lambda_2 \frac{dT_2}{dx} - k_4 \rho_2 T_1 + [p + k_4(1 - \rho_1)]\, T_2 = 0, \tag{2}$$

where

$$\lambda_1 = w_1, \qquad \lambda_2 = -w_2, \qquad T = \int_0^\infty e^{-pt} T_1(x,t)\, dt = T_1(x),$$

$$T_2 = \int_0^\infty e^{-pt} T_2(x,t)\,dt = T_2(x), \qquad \rho_1 = \frac{k_3}{p+k_2+k_3}, \qquad \rho_2 = \frac{k_2}{p+k_2+k_3},$$

with boundary conditions $T_1(0) = \frac{1}{p},\ T_2(l) = 0$.

It is important that for $k_2,\ k_3 \to \infty$, i.e., $\rho_1,\ \rho_2 \to 1/2$, equations (2) transform into the corresponding equations for heat exchangers with thin walls (1), with coefficients $\bar{k}_1 = k_1/2,\ \bar{k}_2 = k_4/2$.

Thus, the solution will be general, valid simultaneously for both cases of thick and thin walls separating the moving media.

The general solution of these equations has the form

$$pT_1 = k_1\rho_1 pC_{(1)} e^{\lambda_{(1)}x} + k_1\rho_1 pC_{(2)} e^{\lambda_{(2)}x},$$

$$pT_2 = [+]\, pC_{(1)} e^{\lambda_{(1)}x} + [-]\, pC_{(2)} e^{\lambda_{(2)}x}, \tag{3}$$

where

$$[+] = \lambda_{(1)}\lambda_1 + p + k_1(1-\rho_2); \qquad [-] = \lambda_{(2)}\lambda_{(1)} + p + k_1(1-\rho_2);$$

$\lambda_{(1)},\ \lambda_{(2)}$ are the roots of the corresponding characteristic equation for system (2); $C_{(1)},\ C_{(2)}$ are constants determined from the boundary conditions.

Having obtained from the general expressions (3) the transform for the transient process at the outlet of the apparatus for medium T_1, we write it in the form

$$pT_1 = e^{-\frac{l}{\lambda_1}p} e^{-\frac{k_1 l}{2\lambda_1}(1-\nu)} \frac{1}{1-q} \left\{1 - \frac{[-]}{[+]}\right\} e^{lz},$$

where

$$z = \frac{\nu k_1 p_2}{\lambda_1} - \frac{k_1 k_4 p_1 p_2}{\lambda_1 \lambda_2 \mu} \frac{1}{1 + \sqrt{1 + \frac{k_1 k_4 p_1 p_2}{\lambda_1 \lambda_2 \mu^2}}},$$

$$\mu = \frac{p + k_1(1-p_2)}{2\lambda_1} - \frac{p + k_4(1-p_1)}{2\lambda_2}, \qquad [q = \frac{[-]}{[+]} e^{\lambda_{(2)}l - \lambda_{(1)}l};$$

ν is equal to 1 or 0, respectively, for the cases of thick and thin walls.

In order to simplify the result considerably, one may seek only an approximate solution of the problem.

Expanding e^{lz} in a power series in the small parameter lz and, in the resulting expression pT_1, discarding all terms of fourth order of smallness and higher with respect to $\frac{1}{p}$ as $p \to \infty$, we obtain, for the case of thick walls,

$$pT_1 \simeq e^{-\frac{l}{\lambda_1}p} e^{-\frac{k_1 l}{\lambda_1}} \left(1 + l\frac{k_1 p_2}{\lambda_1} + \frac{1}{2}l^2\frac{k_1^2 p_2^2}{\lambda_1^2} + \frac{1}{6}l^3\frac{k_1^3 p_2^3}{\lambda_1^3} - \frac{1}{2}l\frac{k_1 k_4 p_1 p_2}{\lambda_1 \lambda_2 \mu}\right). \quad (4)$$

For the case of thin walls, retaining terms no higher than first order of smallness, we obtain

$$pT_1 \simeq e^{-\frac{l}{\lambda_1}p} e^{-\frac{\bar{k}_1 l}{\lambda_1}} \left[1 + \frac{\overline{l k}_1 \bar{k}_2}{(\lambda_1 - \lambda_2)p + (\lambda_1 \bar{k}_2 - \lambda_2 \bar{k}_1)}\right], \quad (5)$$

where $\bar{k}_1 = k_1/2,\ \bar{k}_2 = k_4/2.$

In passing to the final expressions of the transient process in the originals, we introduce dimensionless parameter complexes:

a) for the case of thick walls, 5 parameters:

$$\alpha = \frac{lk_1}{2\omega_1}, \qquad \beta = \frac{lk_4}{2\omega_2}, \qquad \alpha_2 = \frac{lk_2}{2\omega_1}, \qquad \beta_2 = \frac{lk_2}{2\omega_2}, \qquad k = \frac{k_3}{k_2},$$

and since $\alpha_2,\ \beta_2$ will enter the final expression only in the form of the sum $\alpha_2 + \beta_2$, the number of parameters can be reduced to 4 by denoting $\alpha_2 + \beta_2 = \gamma$,

b) for the case of thin walls, 2 parameters:

$$\alpha = \frac{l\bar{k}_1}{2\omega_1}, \qquad \beta = \frac{l\bar{k}_2}{2\omega_2}.$$

Thus, we obtain the final solution in the form

$$T_1 \simeq \begin{cases} 0, & \text{for } t < \tau_1, \\ e^{-2\alpha}\psi(t - \tau_1), & \text{for } t \geq \tau_1, \end{cases} \qquad \tau_1 = \frac{2}{1 + \omega_1/\omega_2}.$$

In the case where the heat capacity of the partition is taken into account, we have

$$\psi(t) = 1 + \frac{2\alpha\gamma}{a}(1 - e^{-at}) + \frac{2a^2\gamma^2}{a^2}\left[1 - e^{-at}(at + 1)\right] +$$

$$+\frac{4}{3}\frac{\alpha^3\gamma^3}{a^3}\left[1-e^{-at}\left(\frac{t^2a^2}{2}+at+1\right)\right]+$$

$$+\frac{2\alpha\beta\gamma^2k}{abc}\left[1-\frac{bc}{(a-b)(a-c)}e^{-at}-\frac{ac}{(b-a)(b-c)}e^{-bt}-\frac{ab}{(c-a)(c-b)}e^{-ct}\right],$$

where $a=\gamma(k+1)$; $b,\ c$ are the roots of the quadratic trinomial

$$q^2+[\alpha+\beta+\gamma(k+1)]q+\gamma(\alpha k+\beta),$$

In the case of a thin wall, if its heat capacity is not taken into account, we obtain

$$\psi(t)=1+\frac{\alpha\beta}{2(\alpha+\beta)}[1-e^{-4(\alpha+\beta)t}].$$

Thus, an approximate solution of the nonstationary problem has been obtained in elementary functions under the assumptions of a thick and a thin wall. These solutions make it easy to carry out analytical investigations of the transient-process curve.

Consequently, along with the ease and accessibility of calculating transient processes in given objects, possibilities are opened up for studying the dependence of the course of the transient process on changes in the system parameters for widely used heat-exchange apparatuses.

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CITED LITERATURE

[1] B. N. Devyatov, DAN, 90, No. 5, 771 (1953). [2] A. V. Lykov, *Thermal Conductivity of Nonstationary Processes*, 1948.

Note: Figure translations are in progress. See original paper for figures.