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Fig. 1

Figure 1: Fig. 1

Abstract

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HYDROMECHANICS

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THE PISTON PROBLEM IN MAGNETOHYDRODYNAMICS

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The problem of the motion of a piston in a conducting medium in the presence of a magnetic field has been considered by a number of authors. A. G. Kulikovskii ([1]) considered the motion of a piston in which rarefaction waves and Alfvén discontinuities travel ahead of the piston; Beyser ([2]) considered the motion of a piston perpendicular to the magnetic field. G. S. Golitsyn ([3]) investigated the motion of a piston when the field is parallel to the plane of the piston. I. A. Akhiezer and R. V. Polovin ([4]) and R. V. Polovin ([5]) investigated the motion of a piston in small magnetic fields. G. Ya. Lyubarskii and R. V. Polovin ([6]) solved the problem of the motion of a piston along the normal. V. V. Gogosov ([13]) investigated the motion of a piston when the piston velocity is arbitrary and the field is directed along the normal to the piston.

Fig. 1

In the present work, carried out independently by A. A. Barmin and V. V. Gogosov, the motion of a piston in a conducting medium is considered without restrictions on the piston velocity and the magnetic field.

As in the preceding works, both the piston and the medium are assumed to be ideally conducting. It is assumed that the internal energy of the medium is related to the pressure and density by the relation $e = \gamma p/(\gamma - 1)\rho$. The piston problem is automodel, and therefore, when the piston moves, various combinations of shock waves, rarefaction waves, and Alfvén discontinuities may travel ahead of it. Between the waves there may be regions where all quantities are constant. We shall denote: Y^+, Y^-, respectively, the fast and slow shock wave; P^+, P^-, respectively, the fast and slow rarefaction wave; A, a rotational discontinuity. The speeds of their propagation are such ([7]) that the following combinations are possible: Y^+AY^-, Y^+AP^-, P^+AY^-, P^+AP^-, and any of the waves may have zero intensity.

In what follows a diagram will be constructed in the space of piston velocities, allowing one to determine from the piston velocity which of the combinations occurs. To construct the diagram it is necessary to know the relation between u, v, w behind $Y^{\pm}$-, $P^{\pm}$-, and A-waves. Here u, v, and w are the absolute velocities of the gas, referred to the Alfvén speed in the undisturbed medium. Let us consider this relation.

Shock waves. A. G. Kulikovskii ([1]) and Beyser ([8]) have shown that the conditions on a shock wave can be resolved in terms of the parameters of the state ahead of the wave and the tangential component of the magnetic field

beyond the wave. A. A. Barmin has established that the relation between u and v has the form shown in Fig. 1 by the lines Y^+ and Y^-. The nonevolutionary part of the Y^--curve is not shown in Fig. 1. The maximum change of v in a Y^--wave is equal to H_{τ_0}/H_x and is attained when $H_\tau = 0$.

Rarefaction waves. Rarefaction waves have been studied in ([1,2,8,9]). The relation between u, v in P^+- and P^--waves is shown in Fig. 1 by the curves P^+ and P^-, respectively. The maximum rarefaction in a P^+-wave is attained at the point K, where $H_\tau = 0$. The maximum rarefaction in a P^--wave is attained at the point where $p = 0$.

Rotational discontinuity. At a rotational discontinuity ([11]) only $\vec{H}_\tau$ and $\vec{v}_\tau$, the tangential components of the magnetic field and velocity, undergo a discontinuity, while their magnitudes remain unchanged. The coordinates of the end of the velocity vector satisfy the relation

$$\left[\vec{v}_{\tau_1} - \left(\vec{v}_{\tau_0} + \frac{\mathbf{H}_{\tau_0}}{H_n}\right)\right]^2 = \left(\frac{H_{\tau_1}}{H_n}\right)^2 = \left(\frac{H_{\tau_0}}{H_n}\right)^2 = \text{const},$$

which is the equation of a circle lying in the plane $u = u_0 = \text{const}$, with center at the point

$$v = v_0 - \frac{H_{y_0}}{H_n}, \qquad w = w_0 + \frac{H_{z_0}}{H_n}.$$

Construction of the diagram

We shall assume that the piston problem has a unique solution. The indices $1, 2, 3$ will be used to characterize the states behind the 1st, 2nd, and 3rd waves, respectively; u, v, w are the gas velocities referred to the Alfvén speed in the undisturbed medium. The velocity ahead of the piston is zero; therefore the initial state of the medium ahead of the piston corresponds, in the velocity space u, v, w, to the point O. If one of the waves Y^+, P^+, Y^-, P^- propagates through gas at rest, then the state of the gas behind these waves is represented by points of the corresponding curve.

It follows from the frozen-in conditions ([6]) that the velocity of the gas at the piston is equal to the velocity of the piston. Therefore one of these waves will travel ahead of the piston if the piston velocity lies on one of the curves Y^+, P^+, Y^-, P^-. From any point of the curves Y^+ and P^+ one may move along circles describing a rotational discontinuity. The totality of these circles forms a surface σ. On this surface begin the curves corresponding to the Y^-- and P^--waves.

The line of the P^--wave issuing from the point $u = 0,\ v = 0$ lies outside the surface σ. From the continuous dependence of u, v, w on the initial data, the uniqueness and the existence of the solution, one may conclude that all the remaining P^- lines also lie outside the surface σ. Similarly, all the Y^- lines lie inside the surface σ.

The Y^- lines issuing from the circle that is the intersection of σ and the plane $u = u_1 = \text{const}$ end at a single point, which corresponds to the tangential component of the magnetic field behind the Y^- wave being zero. Indeed, in the u, v plane the lines $Y^-_{w=0}$ end at the point E, where

$$v_E - v_1 = \frac{H_{\tau_1}}{H_n}\left[\frac{\rho_0}{\rho_1}\right]^{1/2}.$$

The Y^- lines whose origin lies on the Alfvén circle can be obtained by rotating the line $Y^-_{w=0}$ about the straight line passing through the points

$$u_1, \quad v_1 + \frac{H_{\tau_1}}{H_n}\left[\frac{\rho_0}{\rho_1}\right]^{1/2}, \quad 0 \quad \text{(the center of the circle)}$$

and

$$u_E, v_E, w_E$$

(the point E). Indeed, the planes in which the curves Y^- and P^- lie are the same, and are obtained by rotating the u, v plane about the straight line passing through the center of the circle and the point E.

We shall call the set of points E a line of separation. This line is not evolutionary ([12]). Similarly, all lines P^- issuing from the circle can be obtained by rotating the line P^- lying in the plane $w = 0$. Let us denote by δ the surface formed by rotation of the lines Y^-, P^- issuing from the point $u = 0, v = 0$.

The point K corresponds to the maximum rarefaction in the P^+-wave. At it $H_\tau = 0,\ 4\pi p/H_n^2 < 1$. The straight line issuing from this point into the surface σ, parallel to the u-axis, corresponds to the gas-dynamic shock wave corresponding to the slow magnetohydrodynamic shock wave. This straight line ends at the point L, which is the beginning of the line of separation. The continuation of this straight line outside the surface σ corresponds to the line P_τ^-—the gas-dynamic wave corresponding to the P^--wave for $H_\tau = 0$.

The lines P^- end at the points where p_3—the pressure behind the wave—is equal to zero. These points give the vacuum surface B.

If the density of the medium at the piston is zero, then the boundary condition at the piston will be ([2])

$$p_3 = 0;$$

$$(v_\text{p} - v_3)H_x - (u_\text{p} - u_3)H_y = 0, \qquad (w_\text{p} - w_3)H_x - (u_\text{p} - u_3)H_z = 0. \qquad (1)$$

Equation (1) follows from the equality $E_\tau = 0$ in the vacuum zone. The straight lines (1), issuing from the circle which is the intersection of the plane $u = \text{const}$ and the surface σ, can be obtained, analogously to the lines Y^-, P^-, by rotating the corresponding straight line lying in the u, v plane. If the piston velocity lies beyond the vacuum line, then, moving from the vacuum surface, we must arrive at this point along the straight lines (1).

As a result we have the following combinations:
1) P^+AY^-, if the piston velocity lies inside σ to the left of δ.
2) Y^+AY^-, if the piston velocity lies inside σ to the right of δ.
3) P^+AP^-, if the piston velocity lies between the surfaces σ and B to the left of δ.
4) Y^+AP^-, if the piston velocity lies between the surfaces σ and B to the right of δ.
5) If the piston velocity lies outside the surface B, then there will be a vacuum zone in front of the piston.

On the surface σ the intensity of the waves P^-, Y^- is equal to zero. On the surface δ the intensity of the waves P^+, Y^+ is equal to zero. In the plane u, v below the straight line ML and the line of separation there is no rotational discontinuity.

Let us note that if the piston velocity lies on the circle that is the intersection of the surface σ and the plane $u = 0$, then one rotational discontinuity will travel in front of the piston.

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Note: Figure translations are in progress. See original paper for figures.