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1960

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Abstract

Full Text

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THE LAGRANGE–SYLVESTER FORMULA FOR A TENSOR FUNCTION DEPENDING ON TWO TENSORS

(Presented by Academician L. I. Sedov, 1 IV 1960)

Let H, T_1, T_2 be symmetric tensors of second rank, where H is a tensor function of T_1 and T_2. Suppose that the tensors T_1 and T_2 cannot be simultaneously reduced to canonical form and that the eigenvalues $\lambda_i,\ i = 1, 2, 3$, of the tensor T_1 are distinct. In this case the functional dependence $H = f(T_1, T_2)$ can be represented in the form (1)

$$H = K_1 G + K_2 T_1 + K_3 T_1^2 + K_4 T_2 + K_5 (T_1 T_2 + T_2 T_1) + K_6 (T_1^2 T_2 + T_2 T_1^2), \quad (1)$$

where G is the unit tensor; $K_i,\ i = 1, 2, \ldots, 6$, are scalar functions of the joint invariants of the tensors G, T_1, and T_2.

In a rectangular Cartesian coordinate system coinciding with the principal axes of the tensor T_1, the tensors have the form

$$H = \|H_{ij}\|, \quad G = \|\delta_{ij}\|, \quad T_1 = \|\delta_{ij}\lambda_j\|, \quad T_2 = \|T_{ij}\|.$$

The tensor relation (1) is equivalent to 6 scalar equalities, which in the above-named system are written in the form

$$H_{ij} = K_1\delta_{ij} + K_2\delta_{ij}\lambda_j + K_3\delta_{ij}\lambda_j^2 + K_4 T_{ij} + K_5 T_{ij}(\lambda_i + \lambda_j) + K_6 T_{ij}(\lambda_i^2 + \lambda_j^2), \quad (2)$$

where the indices take the values $ij = 11, 22, 33, 12, 13, 23$. The equalities (2) may be regarded as a system of equations with respect to the unknown quantities $K_1, K_2, \ldots, K_6$. The determinant of this system, by virtue of the assumptions made, is nonzero.

Determining from (2) the quantities $K_i,\ i = 1, 2, \ldots, 6$, and substituting them into (1), we obtain the Lagrange–Sylvester formula for a tensor function depending on two tensors:

$$H = S_{T_1}^{\lambda_i}(\overline{H}_{11}, \overline{H}_{22}, \overline{H}_{33}) + T_2 \cdot S_{T_1}^{\lambda_i}(\overline{H}_{23}, \overline{H}_{13}, \overline{H}_{12}) +$$

$$+S_{T_1}^{\lambda_i}(\overline{H}_{23}, \overline{H}_{13}, \overline{H}_{12}) \cdot T_2 + ST_2; \tag{3}$$

here it is denoted

$$S = -(\overline{H}_{23} + \overline{H}_{13} + \overline{H}_{12}),$$

$$S_T^{\mu_i}(a,b,c) = \frac{(T-\mu_2 G)(T-\mu_3 G)}{(\mu_1-\mu_2)(\mu_1-\mu_3)}\,a + \frac{(T-\mu_3 G)(T-\mu_1 G)}{(\mu_2-\mu_3)(\mu_2-\mu_1)}\,b +$$

$$+\frac{(T-\mu_1 G)(T-\mu_2 G)}{(\mu_3-\mu_1)(\mu_3-\mu_2)}\,C, \tag{4}$$

where $\mu_1, \mu_2, \mu_3, a, b, c$ are arbitrary numbers, and T is a symmetric tensor of second rank,

$$\begin{aligned}
\overline{H}_{11} &= H_{11} - T_{11}\left(\frac{H_{12}}{T_{12}} + \frac{H_{13}}{T_{13}} - \frac{H_{23}}{T_{23}}\right),\\
\overline{H}_{22} &= H_{22} - T_{22}\left(\frac{H_{12}}{T_{12}} - \frac{H_{13}}{T_{13}} + \frac{H_{23}}{T_{23}}\right),\\
\overline{H}_{33} &= H_{33} - T_{33}\left(-\frac{H_{12}}{T_{12}} + \frac{H_{13}}{T_{13}} + \frac{H_{23}}{T_{23}}\right),\\
\overline{H}_{ij} &= -\frac{H_{ij}}{T_{ij}}, \qquad i \neq j, \qquad i,j = 1,2,3.
\end{aligned} \tag{5}$$

In particular, for $T_2 = 0$ the tensor function becomes isotropic: $H = f(T_1)$; consequently, $H_{ij} = 0$ for $i \neq j$, $H_{ii} = H_i = f(\lambda_i)$; moreover, $H_{ii} = H_i$, $i = 1,2,3$, and from (3) there follows the usual Lagrange-Sylvester formula for an isotropic tensor function, which, using the notation introduced above, can be written as

$$H = f(T_1) = S_{T_1}^{\lambda_i}(H_1, H_2, H_3). \tag{6}$$

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Received
30 III 1960

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CITED LITERATURE

1. L. I. Sedov, *Foundations of Nonlinear Mechanics of Continuous Media*, Part 1, 1959, p. 65.

Note: Figure translations are in progress. See original paper for figures.