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E. F. KURGAEV

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Abstract

Full Text

PHYSICAL CHEMISTRY

E. F. KURGAEV

ON THE VISCOSITY OF SUSPENSIONS

(Presented by Academician P. A. Rebinder, January 5, 1960)

A disperse system consisting of a liquid and solid particles suspended in it has a higher viscosity than the viscosity of the liquid of the dispersion medium itself. This phenomenon was first established theoretically by A. Einstein ⁽¹⁾, who, on the basis of the equations of hydrodynamics, derived the following formula for determining the viscosity of suspensions of low concentration:

$$\mu_c = \mu_0(1 + 2.5C), \quad (1)$$

where μ_0 is the viscosity of the pure dispersion medium; C is the ratio of the volume of the particles of the dispersed phase to the total volume of the disperse system.

In deriving the formula, the following assumptions were made: 1) the particles are spherical; 2) the mean distance between neighboring solid particles is very large compared with the dimensions of the solid particles; 3) the kinetic energy of the solid particles in the liquid is negligibly small.

The indicated increase in viscosity, for particles of a given shape, does not depend on the particle size or on the molecular nature of the particles and the medium.

Experimental determinations of the viscosity of suspensions ⁽²⁻⁴⁾ showed that Einstein's formula is valid for $C \leq 0.03-0.05$, which to a certain extent corresponds to the second of the assumptions indicated above. Increasing the coefficient at C to 4.5, as proposed by Guth ⁽⁵⁾, and to 4.75 ⁽⁶⁾, did not give positive results and makes it possible to determine the value of μ_c only in a narrow interval of values of C —from 0.15 to 0.18.

The derivation of the formula for the viscosity of suspensions given below proceeds from the following premises. When solid particles are flowed around, changes occur in the live cross section of the liquid flow; as a result, velocity pulsations arise and local components of the flow acquire a turbulent character. The presence of such pulsations has been confirmed by experimental investigations ^(7, 8). These pulsations cause an increase in the resistance to flow deformation, equivalent to an increase in viscosity, due to molar exchange of momentum in the liquid.

The increased viscosity observed in turbulent flows has been called apparent or virtual viscosity. L. Prandtl, G. Taylor ⁽⁹⁾, and M. A. Velikanov ⁽¹⁰⁾ derived semiempirical formulas for such viscosity. However, it does not appear possible to apply these formulas to a suspension, since they pertain to the case of free turbulence in the absence of restriction of the flow by solid surfaces, or when the latter are at a considerable distance, whereas in a suspension the flow is constrained by solid particles.

We assume the additional viscosity arising in a suspension owing to the presence of solid particles in the liquid to be proportional to the molecular viscosity of the liquid and to the relative change in the momentum of a unit mass of liquid along its path of motion between a section constricted by solid particles and a free section. The proportionality of the viscosity of a suspension to the molecular viscosity of the liquid was adopted in Einstein's formula and confirmed by experiments. The indicated change in the quan-

the quantity of motion referred to the quantity of motion with the mean velocity is equal to

$$\alpha \frac{\bar{v}_{\max} - \bar{v}_{\min}}{\bar{v}} = 2 \frac{\Delta \bar{v}}{\bar{v}} \alpha, \quad (2)$$

where \bar{v}_{\max} is the mean maximum velocity of motion of the liquid in the minimum cross section between solid particles; \bar{v}_{\min} is the mean minimum velocity of motion of the liquid in a cross section free of solid particles; \bar{v} is the mean velocity of motion of the liquid in the suspension; α is the coefficient of the quantity of motion of the flow, characterizing the nonuniformity in the distribution of velocities over the cross section of the flow.

Consequently, according to our hypothesis,

$$\frac{\mu_c}{\mu_0} = 1 + 2 \frac{\Delta \bar{v}}{\bar{v}} \alpha, \quad (3)$$

where μ_c is the viscosity of the suspension; μ_0 is the viscosity of the liquid.

The mean velocity of motion of the liquid in the suspension is equal to

$$\bar{v} = \frac{\bar{v}_{\min}}{1 - C}, \quad (4)$$

where C is the volume concentration of solid particles in the suspension.

Since $\Delta \bar{v} = \bar{v} - \bar{v}_{\min}$, from expression (4) we obtain

$$\Delta \bar{v} / \bar{v} = C. \quad (5)$$

Consequently,

$$\mu_c/\mu_0 = 1 + 2\alpha C. \quad (6)$$

The magnitude of the coefficient α is determined by the expression

$$\alpha = \frac{\int_{\omega} (\bar{v} + \Delta v)^2 d\omega}{\bar{v}^2 \omega}. \quad (7)$$

Evaluation of this integral requires knowledge of the velocity distribution over the cross section of the flow and, for the conditions under consideration, is not possible. An approximate determination of the coefficient α can be carried out on the basis of the following considerations. The quantity α , obviously, is proportional to the dissipation of energy in a turbulent flow, which in turn depends on the square of the velocity of motion of the liquid.

It should also be noted that, according to Prandtl's formula, the shear stresses in a turbulent flow are proportional to the square of the velocity gradient, and according to T. Kármán's hypothesis⁽⁹⁾ they depend on the square of the velocity ratio.

All this gives grounds for assuming the quantity α to be proportional to the square of the ratio of the mean velocities in the end cross sections of the portion of the liquid flow adopted by us for consideration, i.e. $\alpha \sim (\bar{v}_{\max}/\bar{v}_{\min})^2$, or

$$\alpha = a \left(\frac{\bar{v}_{\max}}{\bar{v}_{\min}} \right)^2, \quad (8)$$

where a is a certain constant coefficient.

The square of the indicated velocity ratio is equal to the square of the ratio of the maximum and minimum areas of the live cross section of the flow, ω_{\max} and ω_{\min} . In the case under consideration these ratios depend on the volume concentration of solid particles and, along with the velocities in the end cross sections, characterize the specific change of velocities along the length of the flow.

If the value ω_{\max} is taken to be equal to unity, then ω_{\min} is equal to

$$\omega_{\min} = 1 - n^2 \frac{\pi d^2}{4},$$

where d is the diameter of the solid particles; n is the number of particles per unit length of the space occupied by the suspension.

The volume concentration of solid particles in the suspension is equal to

$$C = n^3 \frac{\pi d^3}{6},$$

therefore,

$$\omega_{\min} = 1 - 1.2C^{2/3}, \quad (9)$$

whence

$$\left(\frac{\bar{v}_{\max}}{\bar{v}_{\min}}\right)^2 = \frac{1}{(1 - 1.2C^{2/3})^2}. \quad (10)$$

Comparison with data from experimental determinations of the viscosity of suspensions showed that formula (6), when values of a according to formulas (8) and (10) are substituted into it, gives quite satisfactory calculation results for $a = 1$.

Thus, the final formula for the viscosity of suspensions may be represented in the following form:

$$\mu_c = \mu_0 \left[1 + \frac{2C}{(1 - 1.2C^{2/3})^2} \right]. \quad (11)$$

Table 1 gives the values of μ_c/μ_0 , determined by this formula and experimentally.

Table 1

	$C = 0.05$	$C = 0.10$	$C = 0.15$	$C = 0.2$	$C = 0.25$	$C = 0.3$
μ_c/μ_0 accord- ing to for- mula (11)	1.14	1.36	1.68	2.12	2.85	3.85
μ_c/μ_0 experi- men- tal, accord- ing to Broughton	1.15	1.35	1.62	1.94	—	—

	$C = 0.05$	$C = 0.10$	$C = 0.15$	$C = 0.2$	$C = 0.25$	$C = 0.3$
μ_c/μ_0 experi- men- tal, accord- ing to Traw- inski	1.17	1.35	1.67	2.10	2.8	4.10

It was not possible to verify formula (11) at concentrations higher than those indicated in Table 1. It is known that, when $C \geq 0.25$, intensive mechanical interaction of the solid particles begins^{3,7}, which is not taken into account in our derivation. Therefore formula (11) may be recommended for determining the viscosity of suspensions for C from 0 to 0.25. This range of concentrations is the one most often encountered in the practical operation of chemical-engineering apparatus and is considerably wider than for other existing formulas (Einstein, Hatschek, and Harrison), which thus constitute particular cases of formula (11).

All-Union Scientific Research Institute
of Railway Transport

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References

1. A. Einstein, Ann. Phys., 19, No. 2 (1906).
2. G. Broughton, C. S. Windebank, Ind. Eng. Chem., 4, 407 (1939).
3. H. Trawinski, Chem. Ing. Techn., No. 4 (1953).
4. S. Eirich, M. Bunzl, H. Margaretha, Koll. Zs., 74, 276 (1936).
5. E. Hatschek, Koll. Zs., 7 (301) (1910).
6. Fr. Gan, *Dispersion Analysis*, 1940.
7. E. F. Kurgaev, Izv. AN SSSR, OTN, No. 5, 137 (1958).
8. E. F. Kurgaev, Inzh.-fiz. zhurn., 1, 10, 120 (1958).
9. G. Schlichting, *Boundary-Layer Theory*, IL, 1956.

10. M. A. Velikanov, *Dynamics of Channel Flows*, 1954.

Note: Figure translations are in progress. See original paper for figures.

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