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Abstract

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ON PROCESSES OF EXTREMAL CONTROL WITH DYNAMIC TRANSFORMATION AND STORAGE OF THE INPUT SIGNAL IN THE PRESENCE OF DISTURBANCES

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With the expansion of the areas of application of extremal-control systems, one of the important tasks in their further development is to increase the accuracy of the search for the extremum of a function under the action of low-frequency external disturbances. Along with the known methods: complicating the search motions by performing computational operations [2,3] and introducing correction for external disturbances [5,6], in a number of cases the solution of this problem is provided by dynamic transformation of the input signal. It is carried out by a special element, placed between the object and the extremal controller, which passes only sufficiently rapidly varying signals [9–12]. In the simplest case the transfer function of the dynamic transformer has the form

$$W(p) = p\tau/(p\tau + 1) \tag{1}$$

or, taking into account the inertia of its elements,

$$W(p) = p\tau/(p\tau + 1)(p^2\tau_2^2 + p\tau_1 + 1), \tag{2}$$

where $\tau,\ \tau_1,\ \tau_2$ are time constants, and p is the differentiation symbol.

A transformer with transfer function (1) practically does not pass low-frequency signals, while with (2) it has a passband for signals only in the range of certain intermediate frequencies.

The improvement in the operation of the system under external disturbances is explained by the fact that changes of the signal caused by the search motions of the executive element of the controller, lying in the range of intermediate frequencies, are transmitted by the dynamic transformer to the controller without substantial distortions, whereas the effects of low-frequency disturbances are filtered out to a considerable extent or completely. Dynamic transformation of the input signal, moreover, makes it possible to greatly increase the sensitivity

Fig. 1. Coordinate reference systems

Figure 1: Fig. 1. Coordinate reference systems

of the controller and to reduce the influence of noise entering after the controlled object, owing to an increase in the useful-signal/noise ratio.

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The functions of measuring the output of the object and of dynamically transforming the signal may be performed sequentially, by separate devices, or with the aid of one special device—a dynamic sensor of the information signal; this is usually accomplished by simple design means.

Let us consider the operation of an extremal-control system with a signal relay (with storage), having a dead zone z [1,2], in the presence of a dynamic transformer of the input signal, assuming the controlled object to be inertia-free and the speed of the executive element V_x pos—

constant in absolute value. Let us assume that the characteristic of the plant in the region of the maximum is approximated by a parabola (see Fig. 1) $y = -K_x x^2$, where y and x are coordinates with the origin at the point of the maximum of the function.

We shall denote the action of the external disturbance by $f_{\text{v}}(t)$, and the output signal of the converter by z; then, for the case of dynamic transformation of the input with transfer function (1), noting that $z/y = W(p)$, $y = -K_x x^2 + f_{\text{v}}(t)$, we shall have

$$(p\tau + 1)z = -2\tau K V_x x + \tau f'_{\text{v}}(t). \tag{3}$$

In those cases where the action of the external disturbance is a linear function of time, the second term on the right-hand side of (3) is constant, and the quantity z in the steady state due to the action of the disturbance has only a constant component. Since the extremal controller does not respond to the presence of constant quantities at its input, the external disturbance in this case has no effect on the process of extremal control. In this case certain drifts of the system may arise only during the period t_{n} of the onset of the disturbance action, while the process is settling (for $t_{\text{n}} < (2 \div 3)\tau$). However, the indicated drifts are insignificant, since for small τ, t_{n} is small, and they can be limited by the action of a commutator ([2]). This conclusion, important for practice, has been verified by exact graphical constructions of control processes and by a special experiment ([9,10]).

Of particular interest is the consideration of the case when the external disturbance is a quadratic function of time, $K_{\text{v}} t^2$. Then, after the process of searching for the extremum is completed, a limiting cycle is established, to which oscillations of the actuator correspond with amplitude x, equal to x_p and depending on K_{v}. In this case the parameter z can be represented as the sum of a linear and a periodic function of time, i.e. $z = 2\tau K_{\mathrm{v}} t + \xi$, and (3), for the motion of the system from the point $x = -x_p$ at $t = 0$ to the point $x = x_p$ at $t = 2t_p$ (we denote $K = K_x V_x^2$), is written as

$$(p\tau + 1)\xi = 2K\tau(t_p - t) - 2K_{\mathrm{v}}\tau^2. \tag{4}$$

For an exact determination of the parameters of the periodic cycle we shall use the methods developed in papers ([1],[7]).

The solution of (4) has the form

$$\xi = 2Kt_p\tau - 2K_{\mathrm{v}}\tau^2 + 2K\tau^2 +$$

$$+(\xi_1 - 2K\tau t_p + 2K_{\mathrm{v}}\tau^2 - 2K\tau^2)\exp(-t/\tau) - 2Kt. \tag{5}$$

We assume that at the instant $t = t_p + t_1$ the function $z(t)$ has a maximum; in this case $\xi = \xi_0$, $\dot{z} = 0$, $\dot{\xi}_0 = -2K_{\mathrm{v}}\tau$.

From the instant $t = t_p + t_1$, z begins to decrease. According to the operating principle of an extremal controller with a sign relay, after a certain decrease of the signal arriving at its input, equal to z_{n}, the system is reversed, and the sign of V_x changes to the opposite. Consequently, denoting $\xi = \xi_2$ at $t = 2t_p$, we have

$$\xi_0 - \xi_2 = z_{\mathrm{n}} + 2K_{\mathrm{v}}\tau(t_p - t_1). \tag{6}$$

After reversal of the system, there occurs a stepwise change of the velocity $\dot{y}$, and consequently of $\dot{z}$ and $\dot{\xi}$, by the amount $4kt_p$, and then the cycle begins to repeat. From the symmetry condition,

$$\dot{\xi}_1 = \dot{\xi}_2 + 4kt_p, \tag{7}$$

where $\dot{\xi}_1 = \dot{\xi}$ at $t = 0$, $\dot{\xi}_2 = \dot{\xi}$ at $t = 2t_p$.

The solution of (5) under the stated conditions gives computational formulas that completely determine the parameters of the limit cycle. We shall write the parameters entering these formulas in dimensionless quantities, for which we introduce the notation (see Fig. 1): $t_c = x_{\max}/V_x$; $\eta = Kt_c^2/y_e$; $\lambda_v = K_v/K$; $\nu_\tau = \tau/t_c$; $\psi = x/x_{\max}$; $\psi_p = x_p/x_{\max} = t_p/t_c$; $\psi_{p\tau} = t_p/\tau$; $\delta_n = z_n/y_e$; $\delta_p = y_p/y_e = \eta\psi_p^2$. As a result we have:

$$\delta_n = 2\eta\nu_\tau^2 \left\{ \left[1 - \lambda_v + \frac{2\exp(-\psi_{p\tau})}{\exp(\psi_{p\tau}) - \exp(-\psi_{p\tau})} \right] \psi_{p\tau} - (1-\lambda_v) \left[\ln \frac{2\psi_{p\tau}}{(1-\lambda_v)[\exp(\psi_{p\tau}) - \exp(-\psi_{p\tau})]} + 1 \right] \right\}. \tag{8}$$

Let us note that, under the action of an external disturbance, the limit cycle is established even when the sign relay has no dead zone.

Consider the regulation process in the presence of a commutator $(1, 2)$, which forcibly reverses the system after a time t_k and has an entry for reversal when the sign relay operates. In this case a limit cycle with period $2t_k$ is also established, but the oscillations of x are asymmetric, since, when moving away from the extremum of the function, the system is reversed by the action of the sign relay, whereas during the reverse motion it is reversed by the commutator. Application of the methods indicated above similarly gives formulas determining the parameters of the limit cycle: for $t_k < 2t_p$ and provided that the inequality

$$2\,\frac{\nu_{k\tau}\exp\nu_{k\tau} - \psi_{p\tau}\left(\exp\nu_{k\tau} - 1\right)}{(1-\lambda_v)[\exp(\nu_{k\tau}) - \exp(-\nu_{k\tau})]} > 1, \tag{9}$$

where $\nu_{k\tau} = t_k/\tau$,

$$\delta_n = 2\eta\nu_\tau^2 \left\{ (1-\lambda_v)(\nu_{k\tau} - 1) + 2\,\frac{\nu_{k\tau} - \psi_{p\tau}[1 - \exp(-\nu_{k\tau})]}{\exp(\nu_{k\tau}) - \exp(-\nu_{k\tau})} - \right.$$

$$\left. -(1-\lambda_v)\ln 2\,\frac{\nu_{k\tau}\exp(\nu_{k\tau}) - \psi_{p\tau}[\exp(\nu_{k\tau}) - 1]}{(1-\lambda_v)[\exp(\nu_{k\tau}) - \exp(-\nu_{k\tau})]} \right\}. \tag{10}$$

If (9) is not satisfied (for small $\nu_{k\tau}$), formula (10) loses its meaning, the periodic cycle has another form, and

$$\rho_\tau = \frac{[1 + \exp(-\nu_{k\tau})]\,[\delta_n/2\eta\nu_\tau^2 - \nu_{k\tau}(1-\lambda_v) + 2\nu_{k\tau}]}{2[1 - \exp(-\nu_{k\tau})]}. \tag{11}$$

For $t_k = t_p$, and also for $t_k = 2t_p$, as is to be expected, (10) turns into (8).

Figure 2 presents the results of calculating the parameters of the limit cycles for $\lambda_v = 0.5$, carried out by formulas (8), (10), (11). The change in the error δ_p, arising at the moments of the greatest displacement of the system from the extremum of the function, is shown as a function of the ratio τ/t_c for various values of the regulator dead zone δ_n. The dashed curves correspond to operation of the system without a commutator, and the solid curves to operation with a commutator at $t_k/t_c = 0.15$. As is seen, the introduction of a commutator in the presence of a dead zone δ_n greatly increases the accuracy of regulation.

Calculation shows that there are optimal values of the time constant of the dynamic transformation τ.

It should be noted that, in the absence of a commutator, as is seen from (8), and as λ_v approaches 1, the error δ_p grows without bound, and for $\lambda_v \geqslant 1$ the system loses its operability. In the presence of a commutator such growth of the error does not occur, and the system remains operable for $\lambda_v \geqslant 1$. Thus, for example, at $t_k/t_c = 0.15$, $\lambda_v = 2$, $\delta_n = 0.5\%$, and $\nu_\tau = 0.1$, we have $\psi_p = 0.35$, which at $\eta = 0.5$ gives $\delta_p = 6\%$.

The indicated limit cycles are stable, which is established by representing them on the multi-sheeted phase surface [4,8] in the coordinates z, x and applying the method of point transformations with the construction of a Lamerey diagram.

Examples of Lamerey diagrams for a system with parameters $\lambda_v = 0.5$; $\eta = 0.5$, $\bar{v}_T = 0.1$ at $\delta_n = 0$ (1) and $\delta_n = 1\%$ (2) are shown in Fig. 3, where $\bar{\psi}_i$ and $\bar{\psi}_{i+1}$ are the moduli of the values of ψ at the reversal points of the system. The points $\bar{\psi}_1, \bar{\psi}_2, \bar{\psi}_3$, etc., determine the moduli of ψ at the moments of system reversals after the first, second, third, etc., passage through the extremum of the function.

Fig. 2. Dependence of the regulation accuracy on the time constant of the dynamic converter for various insensitivity zones of the regulator.
1 $-\delta_n = 0$; 2 -0.2%; 3 -0.5%; 4 -1%; 5 -1.5%

In those cases where, under regulation by the maximum of the function, the action of the external disturbance is a quadratic function of time and lowers the level of the extremum, as the construction of the phase trajectories shows, the system retains the ability to search for the maximum of the function, and limit cycles are established with definite amplitudes, which decrease as λ_v increases.

In the case where the low-frequency external disturbance is not a linear or quadratic function of time, limit cycles are not established; however, an approximate estimate of the greatest regulation error can likewise be made by using the formulas obtained. More accurately and comparatively simply, these problems are solved by the method of graphical construction of phase trajectories.

Fig. 3. Lamerey diagrams

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Note: Figure translations are in progress. See original paper for figures.