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ON HYPERSONIC FLOW PAST THIN BODIES AT LARGE ANGLES OF ATTACK

1960

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Abstract

Full Text

HYDROMECHANICS

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ON HYPERSONIC FLOW PAST THIN BODIES AT LARGE ANGLES OF ATTACK

(Presented by Academician A. A. Dorodnitsyn, 6 VIII 1959)

1. The initial assumption of the theory of small perturbations at hypersonic flight speeds is, as is well known, the assumption that the angles of inclination of the surface of the body being flowed around to the direction of the undisturbed flow are small. In accordance with this requirement, both the relative thickness of the body and its angle of attack must be small. When these conditions are satisfied, there is a well-known analogy with unsteady gas motions (the law of plane sections), first substantiated in works ^(1, 2), as well as a similarity law for flows about bodies with the same law of thickness distribution ⁽³⁾. The present paper gives a generalization of the theory of small perturbations to the case of flow past thin bodies at large angles of attack. The results obtained establish a generalized law of plane sections and a similarity law for gas flows about bodies whose transverse dimensions are substantially smaller than their length.
2. Consider a thin or elongated body placed in a uniform supersonic flow at an angle of attack α . Let the greatest transverse dimension of this body be d , and its length l . As the initial assumption we shall take

$$\delta = \frac{d}{l} \ll 1. \tag{1}$$

We shall assume the Mach number M_∞ of the undisturbed flow to be substantially greater than unity, so that one of the conditions holds:

$$M_\infty \delta \sim 1 \quad \text{or} \quad M_\infty \delta \gg 1. \tag{2}$$

Consider the flow in a narrow region adjacent to the body surface, whose transverse dimensions are of the order of the transverse dimensions of the body. At small angles of attack ($\alpha \sim \delta$), evidently, the entire flow field between the shock wave and the body can be included in this region. At large angles of attack, outside this region there will lie only a weakly perturbed part of the flow, which does not influence the remaining flow field by virtue of the hypersonic character of the transverse flow ($M_\infty \sin \alpha \gg 1$). Thus, also in the case of large angles of attack, the problem of flow about the body is reduced to the investigation of the

flow in a small neighborhood of the body. This circumstance makes possible its approximate analytical investigation.

3. Choose a Cartesian coordinate system (x, y, z) , whose z -axis coincides with the axis of the body, while the velocity vector of the undisturbed flow lies in the plane $x = 0$, and introduce the dimensionless independent variables

$$\bar{x} = \frac{x}{d}, \quad \bar{y} = \frac{y}{d}, \quad \bar{z} = \frac{z}{l}. \quad (3)$$

We define the dimensionless unknown functions as follows:

$$\bar{u} = \frac{u}{V_\infty \sin \alpha}, \quad \bar{v} = \frac{v}{V_\infty \sin \alpha}, \quad \bar{w} = \frac{w}{V_\infty \cos \alpha}, \quad \bar{p} = \frac{p}{\rho_\infty V_\infty^2 \sin^2 \alpha}, \quad \bar{\rho} = \frac{\rho}{\rho_\infty}, \quad (4)$$

where u, v, w are the components of the flow-velocity vector; p is the pressure; ρ is the density; the subscript ∞ refers to quantities in the undisturbed flow. The transformation of variables, as is evident, is based on the fact that the transverse velocities must be proportional to the transverse velocity of the undisturbed flow, while the pressure coefficient on the body surface is proportional to its square.

The system of differential equations of gas dynamics in these variables takes the form:

$$\begin{aligned} \bar{u} \frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{u}}{\partial \bar{y}} + \delta \operatorname{ctg} \alpha \bar{w} \frac{\partial \bar{u}}{\partial \bar{z}} &= -\frac{1}{\bar{\rho}} \frac{\partial \bar{p}}{\partial \bar{x}}, \\ \bar{u} \frac{\partial \bar{v}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{v}}{\partial \bar{y}} + \delta \operatorname{ctg} \alpha \bar{w} \frac{\partial \bar{v}}{\partial \bar{z}} &= -\frac{1}{\bar{\rho}} \frac{\partial \bar{p}}{\partial \bar{y}}, \\ \bar{u} \frac{\partial \bar{w}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{w}}{\partial \bar{y}} + \delta \operatorname{ctg} \alpha \bar{w} \frac{\partial \bar{w}}{\partial \bar{z}} &= -\delta \operatorname{tg} \alpha \frac{1}{\bar{\rho}} \frac{\partial \bar{p}}{\partial \bar{z}}, \\ \bar{u} \frac{\partial \bar{\rho}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{\rho}}{\partial \bar{y}} + \delta \operatorname{ctg} \alpha \bar{w} \frac{\partial \bar{\rho}}{\partial \bar{z}} + \bar{\rho} \left(\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} + \delta \operatorname{ctg} \alpha \frac{\partial \bar{w}}{\partial \bar{z}} \right) &= 0, \\ \bar{u} \frac{\partial}{\partial \bar{x}} \left(\frac{\bar{p}}{\bar{\rho}^\gamma} \right) + \bar{v} \frac{\partial}{\partial \bar{y}} \left(\frac{\bar{p}}{\bar{\rho}^\gamma} \right) + \delta \operatorname{ctg} \alpha \bar{w} \frac{\partial}{\partial \bar{z}} \left(\frac{\bar{p}}{\bar{\rho}^\gamma} \right) &= 0, \end{aligned} \quad (5)$$

where γ is the ratio of the specific heats of the gas.

All dimensionless unknown functions, independent variables, and derivatives in this system will be assumed to be of order unity.

Let us consider the boundary conditions. Using the known relations on the surface of a strong discontinuity

$$\bar{z} = \bar{s}(\bar{x}, \bar{y}) \quad (6)$$

and introducing the dimensionless variables (3), (4), we obtain:

$$\begin{aligned}
 \operatorname{ctg} \alpha \bar{w} &= \operatorname{ctg} \alpha + O(\delta), \\
 (1 - \bar{v}) \frac{\partial \bar{s}}{\partial \bar{x}} + \bar{u} \frac{\partial \bar{s}}{\partial \bar{y}} &= 0, \\
 \bar{u} \frac{\partial \bar{s}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{s}}{\partial \bar{y}} &= \delta \operatorname{ctg} \alpha + \frac{\gamma - 1}{\gamma + 1} \left(\frac{\partial \bar{s}}{\partial \bar{y}} - \delta \operatorname{ctg} \alpha \right) \\
 &+ \frac{2}{\gamma + 1} \frac{1}{M_\infty^2 \sin^2 \alpha} \frac{(\partial \bar{s} / \partial \bar{x})^2 + (\partial \bar{s} / \partial \bar{y})^2}{\partial \bar{s} / \partial \bar{y} - \delta \operatorname{ctg} \alpha} + O(\delta)^2,
 \end{aligned} \tag{7}$$

$$\begin{aligned}
 \bar{p} &= - \left[\left(\frac{\partial \bar{s}}{\partial \bar{x}} \right)^2 + \left(\frac{\partial \bar{s}}{\partial \bar{y}} \right)^2 \right] \left(\frac{\partial \bar{s}}{\partial \bar{y}} - \delta \operatorname{ctg} \alpha \right) \left[\bar{u} \frac{\partial \bar{s}}{\partial \bar{x}} - (1 - \bar{v}) \frac{\partial \bar{s}}{\partial \bar{y}} \right] + \frac{1}{\gamma M_\infty^2 \sin^2 \alpha} + O(\delta)^2, \\
 \bar{\rho} &= \left[\frac{\gamma - 1}{\gamma + 1} + \frac{2}{\gamma + 1} \frac{1}{M_\infty^2 \sin^2 \alpha} \frac{1}{(\partial \bar{s} / \partial \bar{y} - \delta \operatorname{ctg} \alpha)^2} \right]^{-1} + O(\delta)^2.
 \end{aligned}$$

The boundary condition on the body surface, whose shape we shall prescribe in the form

$$\bar{z} = \bar{T}(\bar{x}, \bar{y}), \tag{8}$$

will be

$$\bar{u} \frac{\partial \bar{T}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{T}}{\partial \bar{y}} = \delta \operatorname{ctg} \alpha \bar{w}. \tag{9}$$

4. Let us simplify the relations obtained, using the initial assumptions (2), (1). From the third equation of system (5) and the first of the boundary conditions (7) it follows that this relation is valid for

of the entire flow field. Then, neglecting quantities of the second order of smallness in equations (5), we obtain an approximate system of the form*

$$\begin{aligned}
 \bar{u} \frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{u}}{\partial \bar{y}} + \delta \operatorname{ctg} \alpha \frac{\partial \bar{u}}{\partial \bar{z}} &= -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial \bar{x}}, \\
 \bar{u} \frac{\partial \bar{v}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{v}}{\partial \bar{y}} + \delta \operatorname{ctg} \alpha \frac{\partial \bar{v}}{\partial \bar{z}} &= -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial \bar{y}}, \\
 \bar{u} \frac{\partial \bar{p}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{p}}{\partial \bar{y}} + \delta \operatorname{ctg} \alpha \frac{\partial \bar{p}}{\partial \bar{z}} + \rho \left(\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} \right) &= 0, \\
 \bar{u} \frac{\partial}{\partial \bar{x}} \left(\frac{\bar{p}}{\rho^\gamma} \right) + \bar{v} \frac{\partial}{\partial \bar{y}} \left(\frac{\bar{p}}{\rho^\gamma} \right) + \delta \operatorname{ctg} \alpha \frac{\partial}{\partial \bar{z}} \left(\frac{\bar{p}}{\rho^\gamma} \right) &= 0.
 \end{aligned} \tag{10}$$

The boundary condition on the surface of the body takes the form

$$\bar{u} \frac{\partial \bar{T}}{\partial x} + \bar{v} \frac{\partial \bar{T}}{\partial y} = \delta \operatorname{ctg} \alpha. \quad (11)$$

For small angles of attack ($\alpha \sim \delta$), relations (10), (11), and (7) pass into the equations and boundary conditions of the usual theory of small perturbations (4). For large angles of attack ($\alpha \sim 1$), in the boundary conditions (7), along with terms of order δ^2 , terms of order $1/M_\infty^2 \sin^2 \alpha$ must also be neglected; by virtue of conditions (2), these are of the same or of an even higher order of smallness.

Integration of (10) with the boundary conditions (11) and (7) gives an approximate solution of the problem posed above. We note that the ratio of the discarded terms to the retained ones in all relations is of the second order of smallness, which ensures a sufficiently high degree of approximation to the exact solution (just as occurs in the usual theory of small perturbations of a hypersonic flow).

5. By formally replacing the independent variable z by the time variable

$$t = \frac{z}{V_\infty \cos \alpha} \quad (12)$$

we obtain a system of differential equations and boundary conditions whose solution corresponds to the unsteady motion of a gas in the plane $z = \text{const}$, caused by the action of an expanding and translating piston. In this case the shape of the piston is determined by the shape of the cross section of the body, the law of its expansion by the law of distribution of the areas of the cross sections along the length of the body, and the velocity of translation by the angle of attack of the body. This analogy establishes a generalized law of plane sections, according to which the disturbances introduced by a thin body moving at hypersonic speed are essentially reduced to the displacement of gas particles in planes perpendicular to the axis of the body.

6. Relations (6), (8), (10), (11) contain two parameters

$$k_\delta = \delta \operatorname{ctg} \alpha, \quad k_M = M_\infty \sin \alpha. \quad (13)$$

This proves the validity of the similarity law according to which flows about bodies with the same law of distribution of areas and shapes of transverse sections will be similar, i.e., all dimensionless functions ($\bar{u}, \bar{v}, \bar{p}, \rho$) at the corresponding points of the flows ($\bar{x}, \bar{y}, \bar{z}$) are equal, if the similarity parameters k_δ and k_M have the same value in all cases.

* The remaining velocity \bar{w} , if necessary, may be determined with the aid of Bernoulli's equation.

7. The system (10), for large α , contains the small parameter $k\delta$, and its approximate integration in this case can be carried out by the method of successive approximations. The initial (zero) approximation in this case reduces to the exact solution of the problem of the flow past the plane contours of the transverse sections of the body by a hypersonic stream with Mach number $M_{n\infty} = M_\infty \sin \alpha$. The subsequent iterations will, of course, lead to linear equations.

In conclusion, we note that the solution of (10) under the boundary conditions (7) and (11) does not depend on the number M_∞ , if $\alpha \sim 1$. This indicates that the aerodynamic characteristics of slender bodies at large angles of attack reach their hypersonic limit, corresponding to the number $M_\infty \rightarrow \infty$, much earlier than at small α .

Received
31 VII 1959

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Note: Figure translations are in progress. See original paper for figures.

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