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Abstract

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GEOPHYSICS

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ON THE PROBLEM OF FORECASTING SMOOTHED VALUES OF METEOROLOGICAL ELEMENTS AT THE MIDDLE LEVEL OF THE ATMOSPHERE

For predicting the field of the stream function ψ at the middle level of the atmosphere, the equation of transport of absolute vorticity may serve:

$$\frac{\partial \Delta\psi}{\partial t} + \frac{1}{a_0^2 \sin\theta}\left(\frac{\partial\psi}{\partial\theta}\frac{\partial\Delta\psi}{\partial\lambda} - \frac{\partial\psi}{\partial\lambda}\frac{\partial\Delta\psi}{\partial\theta}\right) + 2\omega\frac{\partial\psi}{\partial\lambda} = 0. \quad (1)$$

Here a_0 is the radius of the Earth; ω is the angular velocity of the Earth' s rotation; θ is the complement of latitude (increasing southward); λ is the longitude of the place (increasing eastward); t is time;

$$\Delta\psi = \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right)\frac{1}{\sin^2\theta} + \frac{\partial^2\psi}{\partial\lambda^2}. \quad (2)$$

We shall consider the motion at the middle level of the atmosphere to be turbulent and, starting from equation (1), shall try to find a cumulant-statistical description of this motion. In doing so we shall follow L. V. Keller ([1]), who showed how, for the complete system of equations of hydromechanics, one can construct a closed system of characteristics under the assumption that third-order correlation moments may be neglected.

Equation (1) contains only one function $\psi(\theta, \lambda, t)$. We shall seek smoothed values of this function $\bar{\psi}(\theta, \lambda, t)$. Deviations from the smoothed values will be denoted by $\psi'(\theta, \lambda, t)$:

$$\psi'(\theta, \lambda, t) = \psi(\theta, \lambda, t) - \bar{\psi}(\theta, \lambda, t). \quad (3)$$

Applying smoothing to equation (1), we obtain

$$\frac{\partial \Delta \bar{\psi}}{\partial t} + \frac{1}{a_0^2 \sin\theta}\left(\frac{\partial \bar{\psi}}{\partial \theta}\frac{\partial \Delta \bar{\psi}}{\partial \lambda} - \frac{\partial \bar{\psi}}{\partial \lambda}\frac{\partial \Delta \bar{\psi}}{\partial \theta}\right) + 2\omega \frac{\partial \bar{\psi}}{\partial \lambda} +$$

$$+\frac{1}{a_0^2 \sin\theta}\left[\frac{\partial}{\partial \lambda}\left(\overline{\frac{\partial \psi'}{\partial \theta}\Delta\psi'}\right) - \frac{\partial}{\partial \theta}\left(\overline{\frac{\partial \psi'}{\partial \lambda}\Delta\psi'}\right)\right] = 0. \tag{4}$$

Equation (4) contains, in addition to the sought function $\bar{\psi}$, two more smoothed quantities

$$\overline{\frac{\partial \psi'}{\partial \theta}\Delta\psi'}, \qquad \overline{\frac{\partial \psi'}{\partial \lambda}\Delta\psi'}. \tag{5}$$

These quantities can be expressed in terms of correlation moments. According to L. V. Keller, the correlation moment B_φ^f of two functions φ and f is defined as follows:

$$B_\varphi^f = \overline{\varphi'(\theta-\theta', \lambda-\lambda', t) f'(\theta+\theta', \lambda+\lambda', t)} \tag{6}$$

(φ' and f' are deviations of the functions φ and f from their mean values). This is a function of 5 variables: $\theta, \theta', \lambda, \lambda', t$.

Then

$$\overline{\frac{\partial \psi'}{\partial \theta}\Delta\psi'} = \left(B^{\Delta\psi}_{\partial\psi/\partial\theta}\right)_{\theta'=\lambda'=0}, \qquad \overline{\frac{\partial \psi'}{\partial \lambda}\Delta\psi'} = \left(B^{\Delta\psi}_{\partial\psi/\partial\lambda}\right)_{\theta'=\lambda'=0}.$$

On the other hand, by the very definition (6) we have

$$B_\varphi^{\partial\chi/\partial s} = \frac{1}{2}\left(\frac{\partial B_\varphi^\chi}{\partial s} + \frac{\partial B_\varphi^\chi}{\partial \sigma}\right), \qquad B^\varphi_{\partial\chi/\partial s} = \frac{1}{2}\left(\frac{\partial B_\chi^\varphi}{\partial s} - \frac{\partial B_\chi^\varphi}{\partial \sigma}\right), \tag{7}$$

where χ is an arbitrary function; s is taken in place of θ or λ; σ is taken in place of θ' or λ', respectively; whence

$$B^{\Delta\psi}_{\partial\psi/\partial\theta} = \frac{1}{2}\left(\frac{\partial B_\psi^{\Delta\psi}}{\partial \theta} - \frac{\partial B_\psi^{\Delta\psi}}{\partial \theta'}\right), \qquad B^{\Delta\psi}_{\partial\psi/\partial\lambda} = \frac{1}{2}\left(\frac{\partial B_\psi^{\Delta\psi}}{\partial \lambda} - \frac{\partial B_\psi^{\Delta\psi}}{\partial \lambda'}\right). \tag{8}$$

Using the representation for $\Delta\psi$ (2), we can continue this transformation and express our correlation moments exclusively through the correlation moment B_ψ^ψ. Along the way we introduce new variables:

$$\theta_1 = \theta - \theta', \qquad \theta_2 = \theta + \theta', \qquad \lambda_1 = \lambda - \lambda', \qquad \lambda_2 = \lambda + \lambda'.$$

After simple calculations we obtain

$$B^{\Delta\psi}_{\partial\psi/\partial\theta} = \frac{\partial}{\partial\theta_1}\left(\Delta_2 B^{\psi}_{\psi}\right), \qquad B^{\Delta\psi}_{\partial\psi/\partial\lambda} = \frac{\partial}{\partial\lambda_1}\left(\Delta_2 B^{\psi}_{\psi}\right), \tag{9}$$

where

$$\Delta_2 = \frac{1}{\sin\theta_2}\frac{\partial}{\partial\theta_2}\left(\sin\theta_2\frac{\partial}{\partial\theta_2}\right) + \frac{1}{\sin^2\theta_2}\frac{\partial^2}{\partial\lambda_2^2}. \tag{10}$$

Finally, (4) takes the form

$$\frac{\partial\Delta\overline{\psi}}{\partial t} + \frac{1}{a_0^2\sin\theta}\left(\frac{\partial\overline{\psi}}{\partial\theta}\frac{\partial\Delta\overline{\psi}}{\partial\lambda} - \frac{\partial\overline{\psi}}{\partial\lambda}\frac{\partial\Delta\overline{\psi}}{\partial\theta}\right) + 2\omega\frac{\partial\overline{\psi}}{\partial\lambda} +$$

$$+\frac{1}{a_0^2\sin\theta}\left[\left(\frac{\partial^2}{\partial\lambda_2\partial\theta_1} - \frac{\partial^2}{\partial\lambda_1\partial\theta_2}\right)\Delta_2 B^{\psi}_{\psi}\right] = 0$$

$$\text{for}\quad \theta_1 = \theta_2 = \theta, \quad \lambda_1 = \lambda_2 = \lambda. \tag{11}$$

To close the problem, we must construct a second equation relating $\overline{\psi}$ and B^{ψ}_{ψ}. For this purpose let us form the expression $\partial B^{\Delta\psi}_{\Delta\psi}/\partial t$. Since

$$\frac{\partial B^{\Delta\psi}_{\Delta\psi}}{\partial t} = B^{\partial\Delta\psi/\partial t}_{\Delta\psi} + B^{\partial\Delta\psi/\partial t}_{\Delta\psi},$$

then, using (1) and (6), we find

$$a_0^2\frac{\partial B^{\Delta\psi}_{\Delta\psi}}{\partial t} = \frac{1}{\sin\theta_1}\left[B^{\Delta\psi}_{\partial\psi/\partial\lambda\cdot\partial\Delta\psi/\partial\theta} - B^{\Delta\psi}_{\partial\psi/\partial\theta\cdot\partial\Delta\psi/\partial\lambda}\right] +$$

$$+\frac{1}{\sin\theta_2}\left(B^{\partial\psi/\partial\lambda\cdot\partial\Delta\psi/\partial\theta}_{\Delta\psi} - B^{\partial\psi/\partial\theta\cdot\partial\Delta\psi/\partial\lambda}_{\Delta\psi}\right) - 2\omega a_0^2\left(B^{\Delta\psi}_{\partial\psi/\partial\lambda} + B^{\partial\psi/\partial\lambda}_{\Delta\psi}\right). \tag{12}$$

We now use the basic postulate of L. V. Keller on the possibility of neglecting third moments. In accordance with this postulate

$$B^{f\,\partial\chi/\partial s}_{\varphi} = \frac{\partial\overline{\chi}_2}{\partial s_2}B^{f}_{\varphi} + \overline{f}_2\frac{\partial B^{\chi}_{\varphi}}{\partial s_2},$$

$$B^{\varphi}_{f\,\partial\chi/\partial s} = \frac{\partial \overline{\chi}_1}{\partial s_1} B^{\varphi}_{f} + \overline{f}_1 \frac{\partial B^{\varphi}_{\chi}}{\partial s_1}. \tag{13}$$

Here $\overline{\chi}_1 = \overline{\chi}(\theta_1, \lambda_1, t)$, $\overline{\chi}_2 = \overline{\chi}(\theta_2, \lambda_2, t)$, $\overline{f}_1 = \overline{f}(\theta_1, \lambda_1, t)$, $\overline{f}_2 = \overline{f}(\theta_2, \lambda_2, t)$.
Applying (13) to (12), we obtain

$$a_0^2 \frac{\partial B^{\Delta\psi}_{\Delta\psi}}{dt} = \frac{1}{\sin\theta_1}\left[\frac{\partial\bar{\psi}_1}{\partial\lambda_1}\frac{\partial B^{\Delta\psi}_{\Delta\psi}}{\partial\theta_1} - \frac{\partial\bar{\psi}_1}{\partial\theta_1}\frac{\partial B^{\Delta\psi}_{\Delta\psi}}{\partial\lambda_1} - \frac{\partial\Delta_1\bar{\psi}_1}{\partial\lambda_1} B^{\Delta\psi|\theta}_{\Delta\psi} - \left(2\omega a_0^2\sin\theta_1 - \frac{\partial\Delta_1\bar{\psi}_1}{\partial\theta_1}\right) B^{\Delta\psi|\partial\lambda}_{\Delta\psi}\right] +$$

$$+\frac{1}{\sin\theta_2}\left[\frac{\partial\bar{\psi}_2}{\partial\lambda_2}\frac{\partial B^{\Delta\psi}_{\Delta\psi}}{\partial\theta_2} - \frac{\partial\bar{\psi}_2}{\partial\theta_2}\frac{\partial B^{\Delta\psi}_{\Delta\psi}}{\partial\lambda_2} - \frac{\partial\Delta_2\bar{\psi}_2}{\partial\lambda_2} B^{\Delta\psi|\theta}_{\Delta\psi} - \left(2\omega a_0^2\sin\theta_2 - \frac{\partial\Delta_2\bar{\psi}_2}{\partial\theta_2}\right) B^{\Delta\psi|\partial\lambda}_{\Delta\psi}\right].$$

Then applying a transformation of type (9), we finally obtain:

$$\begin{aligned} a_0^2\Delta_1\Delta_2\frac{\partial B^{\psi}_{\psi}}{\partial t} &= \frac{1}{\sin\theta_1}\left[\frac{\partial\bar{\psi}_1}{\partial\lambda_1}\frac{\partial}{\partial\theta_1}(\Delta_1\Delta_2 B^{\psi}_{\psi}) - \frac{\partial\bar{\psi}_1}{\partial\theta_1}\Delta_1\Delta_2\frac{\partial B^{\psi}_{\psi}}{\partial\lambda_1} - \right. \\ &\left. -\frac{\partial\Delta_1\bar{\psi}_1}{\partial\lambda_1}\Delta_2\frac{\partial B^{\psi}_{\psi}}{\partial\theta_1} - \left(2\omega a_0^2\sin\theta_1 - \frac{\partial\Delta_1\bar{\psi}_1}{\partial\theta_1}\right)\Delta_2\frac{\partial B^{\psi}_{\psi}}{\partial\lambda_1}\right] + \\ &+\frac{1}{\sin\theta_2}\left[\frac{\partial\bar{\psi}_2}{\partial\lambda_2}\frac{\partial}{\partial\theta_2}(\Delta_1\Delta_2 B^{\psi}_{\psi}) - \frac{\partial\bar{\psi}_2}{\partial\theta_2}\Delta_1\Delta_2\frac{\partial B^{\psi}_{\psi}}{\partial\lambda_2} - \right. \\ &\left. -\frac{\partial\Delta_2\bar{\psi}_2}{\partial\lambda_2}\Delta_1\frac{\partial B^{\psi}_{\psi}}{\partial\theta_2} - \left(2\omega a_0^2\sin\theta_2 - \frac{\partial\Delta_2\bar{\psi}_2}{\partial\theta_2}\right)\Delta_1\frac{\partial B^{\psi}_{\psi}}{\partial\lambda_2}\right]. \end{aligned} \tag{14}$$

The two functional-differential equations (11) and (14) make it possible to determine the two functions $\bar{\psi}$ and B^{ψ}_{ψ} from their prescribed initial values. The solution may be carried out in time steps, first determining from equation (11) (Poisson' s equation) $\partial\bar{\psi}/\partial t$, and from equation (14) $\partial B^{\psi}_{\psi}/\partial t$ (a double solution of Poisson' s equation).

Smoothing may be carried out in various ways. In particular, if smoothing is understood as averaging over a circle of latitude, then we obtain a system of equations by solving which one can give a direct forecast of the circulation index (bypassing the forecast of the zonal circulation).

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CITED LITERATURE

1. L. V. Keller, *Zhurn. geofiz. i meteorol.*, **2**, No. 3–4 (1925).

Note: Figure translations are in progress. See original paper for figures.