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Abstract

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DEVELOPMENT OF SEA WAVES FROM THEIR ORIGIN TO THE GREATEST STEEPNESS

In previous works ([1],[2]) the development of sea waves was investigated at the most extensive stage: from the moment they attain their greatest steepness to the limiting dimensions attainable in the ocean at the existing wind speed. Proceeding from the theorem on the moment of momentum, it was possible to establish the causes and the law of increase of the wavelength as their height increases. In complete agreement with observations in the ocean, the continuously decreasing ratio of wave height to length, h/λ, for the largest storm waves tends, in accordance with our theory, to the limit 0.04.

However, up to the present time it has not yet been possible in a similar way to trace the development of waves at the initial, very short, stage—from their origin to the moment when they attain the greatest steepness. It was known only that the motion of an air flow over a mirror-smooth water surface creates an instability of the interface; that on this surface waves of small, but quite definite, finite length suddenly arise ([3]); that the height of such initial waves grows faster than their length and that therefore the ratio h/λ, characterizing the steepness of the waves, increases.

Photo and cine filming of waves at this initial stage, carried out in the storm basin of the Marine Hydrophysical Institute of the Academy of Sciences of the USSR, showed that the waves very quickly attain the greatest kinematically possible ([4]) steepness $h/\lambda = 0.142$, and after this the wavelength λ begins to grow faster than the height h, and therefore the wave steepness decreases according to the law described in works ([1],[2]).

In the present note we shall attempt to outline a scheme for the development of waves at the short initial stage, emphasizing that the construction of a rigorous theory is at present still impossible: it is hindered by the extraordinary difficulty of analyzing small waves developing in the presence of an unsteady regime of drift current in the upper layers of the sea. We have learned to investigate the kinematics of large waves in the presence of a drift current which has already encompassed a sufficiently thick layer of water ([5]-[7]); however, the considerations valid for these conditions cannot be applied to the initial stages of wave formation, in view of the obvious difference in drift-current velocities within the thin surface layer.

For lack of exact expressions, let us write, instead of formula (12) from work ([1]), the generalized expression for the moment of the acting forces $\overline{M}$, averaged over the wave period T:

$$\overline{M} = \delta cr\frac{dr}{dt} + 3\delta g\frac{r^2}{c}\frac{dr}{dt} - \delta\frac{g}{c}\frac{r^3}{R}\frac{dR}{dt} + f_1. \tag{1}$$

Here δ is the density of water; c is the phase velocity of the waves; r is the half-height of the waves; R is the so-called radius of the rolling circle ($R = \lambda/2\pi$); f_1 is a function to be determined in the future.

In a similar manner, let us supplement the expression for the derivative dQ/dt of the period-averaged momentum of the water particles, writing, instead of formula (13) of paper ([1]),

$$\frac{d\overline{Q}}{dt} = \delta cr\,\frac{dr}{dt} + \frac{1}{2}\,\delta r^2\,\frac{dc}{dt} + f_2; \tag{2}$$

the form of the function f_2 is also as yet unknown to us.

Since, on the basis of the momentum theorem, the left-hand sides of (1) and (2) are equal to one another, then, equating their right-hand sides, multiplying them by the phase velocity of the waves c, and dividing by δr^2, we obtain

$$3g\,\frac{dr}{dt} - g\,\frac{r}{g}\,\frac{dR}{dt} = \frac{1}{4}\,\frac{d(c^2)}{dt} + (f_2 - f_1)\,\frac{c}{\delta r^2}$$

or

$$3g\,\frac{dr}{dt} - g\,\frac{r}{R}\,\frac{dR}{dt} = \frac{g}{4}\,\frac{dR}{dt} + (f_2 - f_1)\,\frac{c}{\delta r^2}. \tag{3}$$

There is every reason to suppose that the "correction"terms f_1 and f_2 in formulas (1), (2) play a role only at a very short initial stage of wave formation and that their relative magnitude (in comparison with the principal terms) decreases rapidly with time. On the other hand, in the last– "correction" –term of (3), the numerator c of the fraction increases considerably more slowly than the factor r in the denominator. Hence it follows that the quantity $(f_2 - f_1)\frac{c}{\delta r^2}$ very quickly becomes negligibly small in comparison with the first term on the right-hand side of (3). Since the structure of the functions f_1 and f_2 is still unknown, in a schematic analysis of equation (3) we shall make the simplest assumption, which will allow us to estimate the role of the additional–the second–term in (3). Namely, we set

$$\frac{g}{4}\,\frac{dR}{dt} + (f_2 - f_1)\,\frac{c}{\delta r^2} \approx ng\,\frac{dR}{dt}. \tag{4}$$

Substituting expression (4) into (3), we obtain

$$\frac{dr}{dt} = \frac{1}{3}\left(n + \frac{r}{R}\right)\frac{dR}{dt}. \tag{5}$$

This is the differential equation that replaces equation (15) from paper ([1]).

As was done in ([1]), let us introduce into the analysis a variable that directly characterizes the steepness of the waves: $y = r/R$. After the change of variables, instead of (5) we obtain

$$R\frac{dy}{dt} + y\frac{dR}{dt} = \frac{n}{3}\frac{dR}{dt} + \frac{y}{3}\frac{dR}{dt}$$

or

$$\frac{dy}{\frac{n}{2} - y} = \frac{2}{3}\frac{dR}{R}. \tag{6}$$

The equation obtained does not differ in form from equation (18) of work ([1]). It is, however, extremely significant that in work ([1]) it was always $y > 1/8$, whereas at the stage of wave development under investigation it is always $y < n/2$.

Accordingly, let us write a new initial condition: at the moment of wave inception, for $y = 0$, one must have $R = R_1$ and $\lambda = \lambda_1$. Here λ_1 is the wavelength that corresponds to the theory of P. N. Uspenskii ([3]), investi-

···that has added a disturbance to the mirror-smooth surface of the water under the action of the wind.

Integration of equation (8) under this initial condition gives

$$\frac{r}{R} = y = \frac{n}{2}\left[1 - \left(\frac{R_1}{R}\right)^{2/3}\right],$$

or, in other words:

$$\frac{h}{\lambda} = \frac{n}{2\pi}\left[1 - \left(\frac{\lambda_1}{\lambda}\right)^{2/3}\right]. \tag{7}$$

There is reason to suppose that $n/2\pi > 0.142$.* But since the particular value $h/\lambda = 0.142$ corresponds to Michell’ s limiting steep wave ([4]), the increase of wave steepness according to a law close to (7) will evidently cease when the wavelength reaches some value λ_2, determined by the condition

$$\frac{n}{2\pi}\left[1-\left(\frac{\lambda_1}{\lambda_2}\right)^{2/3}\right]=0.142. \quad (8)$$

Let us assume that after this the dimensions of the waves will increase somewhat to some value $\lambda = \lambda_0$, when the "correction" term in (3) becomes negligibly small in comparison with the first term of (3). Then it may be asserted that, upon reaching the value $\lambda = \lambda_0$, equation (5) of the present paper will automatically turn into the old equation (15) from paper ([1]), and the steepness of the waves will begin to decrease according to law (21), found in ([1]).

As a result, the change in steepness of developing wind waves may be represented by the diagram in Fig. 1, in which the very first stage—from $\lambda = \lambda_1$ to $\lambda = \lambda_2$ —obeys the approximate relation (7); the second stage—a very short stage from $\lambda = \lambda_2$ to $\lambda = \lambda_0$—is characterized by the temporary constancy of wave steepness ($h/\lambda = 0.142$) as their dimensions increase; finally, the longest stage—from $\lambda = \lambda_0$ up to the greatest dimensions of storm waves in the ocean—is described by the old relation (21) from paper ([1]).

It is of interest that observations in the storm basin of the Marine Hydrophysical Institute made it possible to detect a slightly noticeable partial breaking of the wave crests precisely at the just-described second (intermediate) stage of wave formation; here the steepness of the waves tends to increase according to a law close to (7), but the wave profile is unstable for $r/R > 0.45$ and $h/\lambda > 0.142$, and the wave crests must under these conditions break partially ([6],[7]).

In the schematic Fig. 1, the segment of the first curve is drawn under the assumption that the coefficient n in formula (9) remains constant. In reality it decreases continuously because of the continuous decrease of the "correction" term in (3). The role of this term becomes negligibly small at the end of the second stage, by the moment of transition from the segment parallel to the abscissa axis to the curve constructed by formula (21) of paper ([1]).

Thus, the actual dependence of wave steepness on their length (or, in other words, the dependence of wave steepness on time, during which the waves develop) must be expressed by a curve with smoothed breaks.

For comparison, Fig. 2 shows the actual change in the steepness of wind waves with time, based on photographic recording of waves in the storm basin (for the method see paper ([7])).

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* For $n/2\pi = 0.142$, in the first stage the steepness of the waves would only asymptotically approach this value, never attaining it; neither the second nor the third stage would exist, in contrast to the actual picture of the phenomenon. For $n/2\pi < 0.142$, the steepness of the waves would asymptotically change from zero to $n/2\pi < 0.142$, which is still more contrary to reality.

Fig. 1 and Fig. 2

Figure 1: Fig. 1 and Fig. 2

In view of the difficulty of making accurate measurements for very small waves, it is, of course, impossible to vouch for the actual length of the intermediate segment of the curve, parallel to the abscissa axis, or for the sharpness of the break in passing from this segment to the third, descending segment. However, the general character of the curve in Fig. 2 agrees well with the theoretical scheme of Fig. 1. It must be assumed

Fig. 1 **Fig. 2**

that in the future the determination of the form of the functions f_1 and f_2 will lead only to a change in the form of the first stage of the curve (from f_1 to f_2) and in the length of the second stage (up to λ_0), but will not alter the conception of the essence of the phenomena described in the present article.

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Note: Figure translations are in progress. See original paper for figures.