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Abstract

Full Text

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THEORY OF ELASTICITY

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EXPLOSION IN A COMPACTING UNBOUNDED MEDIUM

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The distinctive character of the development of an explosion in soil is associated with large irreversible deformations of the medium. The present article sets forth the principal results of an experimental and theoretical study of an explosion in a compacting inelastic medium.

Experimental procedure. Explosions of charges weighing 1.0, 6.0, and 24.0 g were produced in sandy soil of density 1.5 g/cm^3 and moisture content 6%. The influence of a free surface on the development of the explosion was excluded in the experiments. Under these conditions the motion of the medium was spherically symmetric.

In the experiments, the velocity of propagation of the wave front and the displacements in time of spherical layers initially located at various distances from the center of the explosion were measured. The symmetry of the motion made it possible to record along any chosen radius. To fix a certain surface, foil 0.1 mm thick was placed in the soil; it moved together with the medium. Registration of the displacements of the foil with time was carried out by means of a multi-contact, immovably fixed probe (Fig. 1). The electric signals were recorded on an OK-24 (IXF) cathode-ray oscillograph.

Fig. 1. Diagram of displacement registration with time.
1—charge, 2—foil, 3—probe

Results of the experiments. Figure 2 gives a typical experimental dependence $r(t)$, obtained for the explosion of a 24 g charge for a layer located at a

Fig. 2. Experimental dependence of layer displacements in time. 1 —motion of the layer, 2 —motion of the wave front

Figure 2: Fig. 2. Experimental dependence of layer displacements in time. 1 —motion of the layer, 2 —motion of the wave front

distance of 10 cm from the center of the charge. Similar curves were obtained for other distances in the range

$$0.3 < \frac{R}{\sqrt[3]{q}} < 1.5.$$

The curves give the displacement field around the charge at various instants of time. By graphical differentiation of the curves $r(t)$, the field of particle-displacement velocities of the medium and its variation with time were found. The principal dependences can be expressed by the formulas

$$D = 40\frac{\sqrt[3]{q}}{R}; \tag{1}$$

$$u = 3.4\left(\frac{\sqrt[3]{q}}{R}\right)^{1.8}; \tag{2}$$

$$v = u\left(\frac{R}{r}\right)^{1.5}, \tag{3}$$

where R, r (m) are the coordinates of the front and the current coordinate; q (kg) is the weight of the charge; D (m/sec) is the velocity of the wave front; u, v (m/sec) are the velocities of displacement of the particles of the medium, respectively at the front and behind the front at a distance r from the center of the charge.

On the basis of the data obtained, the kinetic energy at different instants of time was determined. Its magnitude varies little and is approximately 2–3% of the total energy E.

An estimate of the energy remaining in the products of the explosion showed that the irreversible expenditure of energy on heating the soil during its inelastic deformation amounts to 70-80% of E. With the aid of empirical dependences, by integrating the equations of motion, the envelope of stresses behind the wave front at different instants of time was obtained. It is shown that throughout the investigated region the ratio $\sigma_{\varphi\varphi}/\sigma_{rr} = \sigma_{\theta\theta}/\sigma_{rr} = \alpha$ may be taken as constant and equal to 0.4. This corresponds to the plasticity condition: $\sigma_{rr} - \sigma_{\varphi\varphi} = m(\sigma_{rr} + \sigma_{\varphi\varphi} + \sigma_{\theta\theta})$, where $m = \text{const}$. From formulas (1) and (2) it is seen that, as the wave front propagates, the compaction at the front decreases.

Fig. 2. Experimental dependence of the displacements of a layer in time. 1 — motion of the layer, 2 —motion of the wave front.

Formulation of the problem of an explosion in an unbounded inelastically deformable medium

The equations of motion for centrally symmetric motion have the form:

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial r} = \frac{1}{\rho}\left(\frac{\partial \sigma_{rr}}{\partial r} + \frac{2(1-\alpha)\sigma_{rr}}{r}\right); \tag{4}$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho v)}{\partial r} + \frac{2\rho v}{r} = 0. \tag{5}$$

Experiment showed that the volume deformations have an irreversible character, and the dependence of compactability on stress may be written approximately in the form

$$\left(1 - \frac{\rho_0}{\rho}\right) \equiv \xi = \left(\frac{\sigma}{B}\right)^{1/n}.$$

The plasticity condition for soil is determined by the expression:

$$\sigma_{rr} - \sigma_{\varphi\varphi} = m(\sigma_{rr} + 2\sigma_{\varphi\varphi}).$$

Using the plasticity condition, one may write

$$\sigma_{rr} = B\xi^n.$$

Neglecting wave processes in the expanding products of the explosion, the dependence of the pressure in the cavity on its radius r_0 may be described by the formula

$$\frac{P}{P_0} = \left(\frac{R_0}{r_0}\right)^{3\gamma}, \tag{6}$$

where γ is the isentropic exponent of the products of the explosion.

For $n > 1$, the boundary of the disturbed region will be a shock front, the conditions at which are:

$$\rho_0\dot{R} = \rho(\dot{R} - u), \qquad \rho_0\dot{R}^2 = \rho(\dot{R} - u)^2 - \sigma_{rr}(R), \tag{7}$$

where R is the radius of the front, and u is the velocity of particles at the wave front.

Relations (6) and (7) are the boundary conditions of the problem. The problem posed differs from the problem of A. S. Kompaneets ([1]) in that, instead of constant compaction under compression, a power law of compactability with pressure is adopted.

We shall seek the solution in the form $\dot{R} = C\left(\frac{R_0}{R}\right)^k$. Taking into account that behind the wave front, during unloading, no change in density occurs, from equation (5) we obtain:

$$v = \xi \dot{R}\left(\frac{R}{r}\right)^2. \tag{8}$$

Using expression (8), let us integrate equation (4), assuming that $\rho \simeq \rho_0$. From the boundary conditions at the wave front we shall find the arbitrary function and compute the stresses at the boundary with the cavity:

$$\begin{aligned}\sigma_{\text{cav}} = & -\rho_0 \frac{C^{\frac{2n}{n-1}}}{\beta^{\frac{2}{n-1}}}\left(1 - \frac{2 - k\frac{n+1}{n-1}}{2\alpha - 1}\right) z^{\frac{2(1-\alpha)}{3}}\left(\frac{R_0}{R}\right)^{k\frac{6n-4+4\alpha}{3(n-1)}} \\ & + \frac{\rho_0}{1+\alpha}\frac{C^{\frac{2(n+1)}{n-1}}}{\beta^{\frac{4}{n-1}}} z^{4/3}\left(\frac{R_0}{R}\right)^{k\frac{6n-2}{3(n-1)}} \\ & - \frac{\rho_0}{2\alpha - 1}\frac{C^{\frac{2n}{n-1}}}{\beta^{\frac{2}{n-1}}}\left(2 - k\frac{n+1}{n-1}\right) z^{1/3}\left(\frac{R_0}{R}\right)^{k\frac{6n-2}{3(n-1)}},\end{aligned} \tag{9}$$

where

$$\beta = \sqrt{\frac{B}{\rho_0}}, \qquad z = \frac{\left(3 - \frac{2k}{n-1}\right)\beta^{\frac{2}{n-1}}}{3C^{\frac{2}{n-1}}}.$$

Equation (5) makes it possible to find the dependence of r_0 on R. Substituting it into the boundary condition (6), we find

$$P = P_0 z^{\gamma}\left(\frac{R_0}{R}\right)^{\gamma\left(3 - \frac{2k}{n-1}\right)}. \tag{10}$$

The meaning of the approximate solution is to choose the coefficients C and k so as to obtain the best agreement between P and σ_{cav} over the entire range under study. Table 1 gives the coefficients C and k, chosen so that in the range $10 < R/R_0 < 100$ the ratio $-P/\sigma_{\text{cav}}$ nowhere differed from 1.0 by more than 15%.

Table 1

Values of C and k for $\alpha = 0.4$ and $\rho_0 = 1.0$ (B in kg/cm^2)

γ	n	k	$B = 10^4$	$B = 10^6$	$B = 10^8$	$B = 10^{10}$	$B = 10^{12}$	$P_0 \cdot 10^{-3}$, kg/cm^2
1.33	3	1.0	1.0	3.3	10	–	–	14.5
1.33	5	1.3	–	2.5	5.2	11	–	14.5
1.33	7	1.5	–	–	4.1	7.2	12.5	14.5
1.25	3	1.05	1.0	3.6	11.7	–	–	12.5
1.25	5	1.25	–	2.2	4.7	9.3	–	12.5
1.25	7	1.4	–	–	3.5	6.0	9.0	12.5

Values of the coefficient $C \cdot 10^{-5}$

For other ρ_0, the quantity k does not change, while the values of C must be taken reduced by a factor of

$$\rho^{\frac{n-1}{2}\left(n+\gamma-\frac{1}{8}\right)}.$$

When α is varied from 0 to 1.0, the coefficient C practically does not change, while the coefficient k decreases. For $B = 10^4$ and $n = 3$, the coefficient k changes from 1.1 ($\alpha = 0$) to 0.9 ($\alpha = 1.0$).

Knowledge of the coefficients C and k makes it possible to compute all the remaining parameters of the motion of the soil. In particular, the displacement-velocity field will be

described by the formula:

$$v = \frac{A}{\sqrt{\rho_0}} \left(\frac{R_0}{R}\right)^{\eta} \left(\frac{R}{r}\right)^2, \qquad \text{where} \quad A = \frac{C^{\frac{n+1}{n-1}} \rho_0^{\frac{1}{n-1}}}{B^{\frac{1}{n-1}}}, \quad \eta = k\frac{n+1}{n-1}.$$

Calculation shows that over the entire investigated range of variation of A and η, these quantities are very small: A varies by less than a factor of 1.5, and the coefficient η by less than 10%. Thus, a very weak dependence of the velocity field on the properties of the medium in inelastically deformable media has been demonstrated.

On the basis of the experimental dependence $\sigma = \sigma(\xi)$ for sandy soil, the following dependences were calculated:

$$D \equiv \dot{R} = 64 \left(\frac{\sqrt[3]{q}}{R}\right)^{1.1}, \qquad u = 3.15 \left(\frac{\sqrt[3]{q}}{R}\right)^{1.9}.$$

Satisfactory agreement with the experimental formulas (1) and (2) permits one to suppose that the regularities found will be valid for a broad class of soils.

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CITED LITERATURE

1. A. S. Kompaneets, DAN, **109**, No. 1, 49 (1956).

Note: Figure translations are in progress. See original paper for figures.