A "Lunar" Radio Interferometer

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Abstract

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Astronomy

As is known, two types of radio interferometers are used in radio astronomy: "two-antenna" or "multi-antenna" interferometers, when the interference of radio waves from cosmic discrete sources is produced with the aid of two or many antennas, and the "sea" interferometer, when the interference is produced with the aid of only one antenna installed high near the seashore; the role of the second antenna is played by the surface of the sea.

Alongside this, one can imagine still another type of radio interferometer, making use of the circumstance that the Moon has a very rarefied atmosphere and, consequently, an ionosphere. Radio waves from a discrete source which is on the celestial sphere near the disk of the Moon can reach an observer on the Earth both directly and through the ionosphere of the Moon, being thereby bent by a certain amount, and as a result can interfere at the point of their reception ([1], [2]). Thus the ionosphere of the Moon, or simply the Moon, in this case plays the role of the second antenna or of the sea surface. For convenience we shall call such an interferometer a "lunar" one. As will be seen below, a lunar interferometer has great resolving power and, because of this, may prove to be a very valuable means for determining the angular dimensions of very small discrete sources of cosmic radio emission. In the present note the basic considerations concerning the operation of a lunar interferometer are given, as well as a method for determining the angular dimensions of discrete sources.

The ability of the Moon' s ionosphere to refract radio waves is characterized by the parameter φ_0, called the "total angle of refraction" ([1], [2]). Radio waves from a cosmic source located within this angle near the disk of the Moon will propagate to the Earth both directly and through the ionosphere of the Moon. The magnitude φ_0 depends on the electron concentration of the ionosphere (and consequently on the activity of the Sun) and on the wavelength. For meter waves φ_0 is measured in minutes and, possibly, even in tens of minutes of arc. In Fig. 1 the region φ_0 is marked by a dashed arc.

Let us assume, for simplicity, that the cosmic source has a rectangular shape, of width (in the direction of the relative apparent motion of the Moon and of the discrete source) equal to β_0. If we denote by I_0 the total intensity of the radiation of the source at wavelength λ, then I_0/β_0 will be the intensity per unit width of the source, and $\frac{I_0}{\beta_0}\,d\beta$ the intensity of an element of width $d\beta$. At a certain distance $\varphi - \beta$ of this element from the edge of the Moon' s disk (Fig. 1), a path difference Δs is formed between the direct and the curved rays, whose magnitude is equal to

$$\Delta s = \frac{D}{2}(\varphi - \beta)^2, \tag{1}$$

where D is the distance of the Moon from the Earth. Corresponding to this difference, a receiver on the Earth will receive the following intensity from the element $d\beta$:

$$dI_\lambda = \frac{I_0}{\beta_0}\,d\beta + \frac{I_0}{\beta_0}\,d\beta \cos\frac{\pi}{2}\frac{D}{\lambda}(\varphi - \beta)^2. \tag{2}$$

Integrating, we find the intensity received from the entire surface of the source when it is at a distance φ ($\leqslant \varphi_0$) from the edge of the disk:

$$I_\lambda = I_0[1 + \delta_\lambda(\varphi, \beta_0)], \tag{3}$$

where

$$\delta_\lambda(\varphi, \beta_0) = \frac{1}{\beta_0}\int_0^{\beta_0} \cos a(\varphi - \beta)^2\,d\beta$$

$$(\beta_0 \leqslant \varphi \leqslant \varphi_0). \tag{4}$$

In this expression $a = \pi D/2\lambda$ is the constant of the lunar interferometer; its value for several wavelengths is given in Table 1 ($D = 3.8 \cdot 10^{10}$ cm).

Formula (4) is valid until a partial eclipse of the source by the Moon has occurred. In the latter case we have

$$\delta_\lambda(\varphi, \beta_0) = \frac{1}{\beta_0}\int_0^{\varphi} \cos a(\varphi - \beta)^2\,d\beta \qquad (0 \leqslant \varphi \leqslant \beta_0). \tag{5}$$

Fig. 1.
Labels in the figure: Moon; discrete source; direction opposite to the motion of the Moon; φ_0, β_0, $d\beta$, φ, β.

Thus, formulas (4) and (5) cover the region from $\varphi = 0$ to $\varphi = \varphi_0$. Using them, one can construct the curve of the course of the intensity variation from the moment the interference pattern appears, when the source is at the boundary $\varphi = \varphi_0$, until its complete disappearance.

Table 1

λ, cm	3	10	50	100	150	300	450	600
a	$1.99 \cdot 10^{10}$	$5.96 \cdot 10^{9}$	$1.20 \cdot 10^{9}$	$5.96 \cdot 10^{8}$	$3.98 \cdot 10^{8}$	$1.99 \cdot 10^{8}$	$1.32 \cdot 10^{8}$	10^{8}

The integrations in (4) and (5) can be carried out numerically or graphically. In the special case when $\beta_0^2 \ll \varphi^2$, the result can be obtained in explicit form, and instead of (4) we shall have

$$\delta_\lambda(\varphi, \beta_0) = \frac{\sin a\varphi\beta_0}{a\varphi\beta_0} \cos(a\varphi^2 - a\varphi\beta_0). \tag{6}$$

This, evidently, is a harmonic oscillation with variable amplitude—a beating. The argument of the cosine will vary more rapidly with a change in φ than the argument of the sine. Therefore the first factor in (6) will be the amplitude, and the second factor represents the oscillation itself.

The presence of φ in the denominator of (6) shows that the amplitude is not only a harmonically varying function of φ, but also a monotonically decreasing function of the same φ. As a result, the interference pattern in the case of a lunar interferometer must be a curve of damped beating. The intervals between the maxima of the beating, $\Delta\varphi$, do not depend on φ and satisfy the condition $\Delta\varphi\, a\beta_0 = \pi$. Hence we have

$$\beta_0 = \frac{\pi}{a\Delta\varphi}. \tag{7}$$

This simple formula determines the apparent size of the cosmic radio source β_0; for this it is sufficient to take from the interference record

quantity $\Delta\varphi$ in angular measure, and from Table 1 the quantity a for the given wavelength.

As for the interval between the maxima of the oscillation itself, it already depends on φ. If we denote the magnitude of this interval on the time scale by t_s, then it is not difficult to derive, for large φ ($> \beta_0$):

$$t_s = \frac{2\pi}{ca(2\varphi - \beta_0)}, \tag{8}$$

where c is the path, in angular measure, traversed by the Moon per unit time ($c \simeq 0.55$ min^{-1}). For large values of φ we may write

$$t_s = \frac{\pi}{ca\varphi}. \tag{9}$$

Calculations carried out by this formula give, for $\varphi = 1'$: $t_s \sim 1$ sec for decimeter waves and $t_s \sim 10$–20 sec for meter waves. The duration of one beat on the time scale is somewhat longer—of the order of 1 min. (For example, at $\lambda = 3$ m and $\beta_0 = 0'.2$, the duration of one beat is 1.7 min and does not depend on φ.) Generally speaking: a) when working with a lunar interferometer one should have measuring and recording equipment with a time constant of the order of a second or of several seconds; b) the time constant t_s decreases in going from long waves to short waves. Apparently, in the case of a lunar interferometer it is more expedient to work in the meter wavelength range; c) the time constant is relatively larger for small values of φ. However, φ has a lower limit determined by diffraction of radio waves at the edge of the Moon' s disk. The minimum value $\varphi_{\min}$ is determined from the condition $2a\varphi_{\min}^2 = 1$. For meter waves $\varphi_{\min}$ is of the order of $0'.1$–$0'.2$.

The obtained value of the oscillation period imposes a certain limitation on the passband width of the receiver $2\Delta f$ (in frequency units), since otherwise the interference pattern may be smeared. Therefore it is also necessary to estimate the maximum value $(\Delta f)_{\max}$.

The oscillation of the interference pattern itself is caused by changes in the argument of the cosine in (6). Obviously, in the case of complete smearing of the interference pattern we shall have the following condition (for the case $\beta_0 < \varphi$):

$$\Delta a \varphi^2 = 1. \tag{10}$$

For Δa we have

$$\Delta a = -\frac{a}{\lambda}\Delta\lambda = \frac{a}{f}\Delta f. \tag{11}$$

Therefore

$$(\Delta f)_{\max} = \frac{1}{a\varphi^2} f. \tag{12}$$

This relation may be generalized for some specified degree of smearing ε ($0 \le \varepsilon \ll 1$):

$$(\Delta f)_{\max} = \frac{\varepsilon}{a\varphi^2} f, \tag{13}$$

where $\varepsilon = 1$ for complete smearing. In the case, for example, of meter waves, for $\varphi \sim 1'$, $\varepsilon \sim 0.1$, one obtains $(\Delta f)_{\max} \sim 1$ MHz.

Figure 2 gives the theoretical curve of the interference pattern for the case $\lambda = 3$ m and $\beta_0 = 0'.2$. The calculations from $\varphi = 0$ to $\varphi = 0'.5$ were performed

Fig. 2

Figure 1: Fig. 2

by numerical integration of expressions (4) and (5), and further—using (6). The value $\varphi = 0$ corresponds to complete occultation of the source by the Moon. The interference maxima and minima (i.e. maxi-

small values of $\delta_\lambda(\varphi, \beta_0)$ reach 20% and more in comparison with the normal intensity of the source. In general, the maximum value of $\delta_\lambda(\varphi, \beta_0)$ is the greater, the smaller the angular dimensions of the source β_0 are and the closer the source is to the visible edge of the disk of the Moon, i.e., the smaller φ is.

It should be pointed out that the dimensions—or, more precisely, the order of magnitude of the dimensions—of very small discrete sources can also be determined with the aid of ordinary two-antenna or sea interferometers. However,

Fig. 2

in this case great difficulties arise, connected above all with the fact that in the first case one has to work with a variable baseline, and in the second case with a variable height of the antenna above sea level. These difficulties are eliminated when working with a lunar interferometer. The principal shortcoming of the lunar interferometer should be considered to be that its operation is limited to the region of the ecliptic.

In conclusion, let us note that the moment of appearance of the interference pattern (damped beats), as follows from the arguments given above, gives a lower value for the magnitude of the total angle of refraction φ_0 at a given wavelength and for the given state of the Moon' s ionosphere. We thereby obtain an indirect possibility of carrying out a kind of daily monitoring of the state of the Moon's ionosphere. However, this question is better left until, first of all, experimental confirmation has been obtained of the possibility of producing interference of radio waves from cosmic sources with the aid of antennas and the Moon.

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