Reports of the Academy of Sciences of the USSR

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1958

SovietRxiv

Abstract

Full Text

Reports of the Academy of Sciences of the USSR
1958. Vol. 120, No. 2

GEOPHYSICS

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A NONLINEAR NONSTATIONARY PROBLEM OF DETERMINING PRESSURE FIELDS OF PLANETARY SCALE AT THE MIDDLE LEVEL OF THE ATMOSPHERE

In 1943 one of the authors of the present paper proposed a method for the long-range forecasting of pressure fields and streamlines at the middle level of the atmosphere by means of hydrodynamics ([1]). Two equations were used to solve the problem: the vorticity equation (A. A. Friedmann' s simplified equation) and one of Euler' s equations. The first of these equations made it possible to determine the stream function $\psi(\theta, \lambda, t)$ (θ is the complement of the latitude, λ is the longitude of the place, t is time); the second gave the relation between the stream function and pressure. The solution of the problem was obtained at the cost of linearizing both equations with respect to the west–east transport.

The advent of high-speed electronic computers made it possible to raise the question of solving the nonlinear problem of long-range weather forecasting by methods of hydrodynamics ([2]). The simplest solution is obtained for the middle level of the atmosphere. Several variants of such a solution were proposed in 1954. The methodology and some results of applying one of these variants are set forth in the present paper.

The initial equation will, as before, be the equation for the stream function (the vorticity-transport equation):

$$\Delta \frac{\partial \psi}{\partial t} + \frac{1}{a_0^2 \sin\theta}(\psi, \Delta\psi) + 2\omega \frac{\partial \psi}{\partial \lambda} = 0. \tag{1}$$

Here a_0 and ω are, respectively, the radius and the angular velocity of rotation of the Earth,

$$\Delta = \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\lambda^2}, \qquad (M, N) = \frac{\partial M}{\partial\theta}\frac{\partial N}{\partial\lambda} - \frac{\partial M}{\partial\lambda}\frac{\partial N}{\partial\theta}.$$

Fig. 1

Figure 1: Fig. 1

Equation (1) contains a first-order differentiation with respect to time; we shall assume that the function ψ is known at the initial instant.

The values of ψ can be forecast by means of time steps. To this end, let us first solve equation (1) with respect to $\partial\psi/\partial t$ (Poisson' s equation on the sphere). If we assume that at the equator $\psi = 0$ (the condition that air masses do not penetrate across the equator), we obtain

$$\frac{\partial\psi}{\partial t} = -\frac{1}{4\pi a_0^2}\int_0^{2\pi}\int_0^{\pi/2} \ln\frac{1-\cos\gamma}{1-\cos\bar{\gamma}}\left[(\psi,\Delta\psi) + 2\omega a_0^2\sin\theta'\,\frac{\partial\psi}{\partial\lambda'}\right]d\theta'\,d\lambda', \quad (2)$$

where

$$\cos\gamma = \cos\theta\cos\theta' + \sin\theta\sin\theta'\cos(\lambda-\lambda'),$$

$$\cos\bar{\gamma} = -\cos\theta\cos\theta' + \sin\theta\sin\theta'\cos(\lambda-\lambda'),$$

and the integration extends over the Northern Hemisphere.

Substituting the initial values of the function ψ into the right-hand side of equation (2), we obtain the value of $\partial\psi/\partial t$ at $t = 0$ and then determine the value of ψ at the mo–

time interval τ, close to the initial one, by the formula

$$\psi_{t=\tau} = \psi_{t=0} + \left(\frac{\partial\psi}{\partial t}\right)_{t=0}\tau. \quad (3)$$

Repeating these calculations the necessary number of times, we find the values of the stream function ψ at the end of the forecast period.

Fig. 1

The subintegral expression of the right-hand side of (2) contains differentiation of third order with respect to the coordinates. One can avoid computing derivatives higher than first order if, along with ψ, one introduces one more unknown function:

$$b = \Delta\psi + m\psi, \quad (4)$$

Fig. 2

Figure 2: Fig. 2

Fig. 3

Figure 3: Fig. 3

where m is a constant, which we shall choose below. Then

$$\frac{\partial \psi(\theta,\lambda,t)}{\partial t} = \frac{1}{2\pi}\int_0^{2\pi}\int_0^{\pi/2} G(\theta,\lambda;\theta',\lambda')\, f(\theta',\lambda',t)\sin\theta'\, d\theta'\, d\lambda', \qquad (5)$$

$$G = \frac{1}{2}\ln\frac{1-\cos\gamma}{1-\cos\gamma'}, \qquad f(\theta',\lambda',t) = -\frac{1}{a_0^2\sin\theta'}(\psi,b) - 2\omega\frac{\partial\psi}{\partial\lambda},$$

and, by (1), (4), and (5),

$$\frac{\partial b(\theta,\lambda,t)}{\partial t} = f(\theta,\lambda,t) + \frac{m}{2\pi}\int_0^{2\pi}\int_0^{\pi/2} G(\theta,\lambda;\theta',\lambda')\, f(\theta',\lambda',t)\sin\theta'\, d\theta'\, d\lambda'. \qquad (6)$$

Formulas (5) and (6) do not contain derivatives of ψ and b higher than first order. The values of b at $t=0$ are found from the initial values of ψ by means of (4) (in doing so it is necessary, only once, to compute the second derivatives of ψ with respect to the coordinates). The values of b at each time step are determined by an extrapolation formula of type (3).

In implementing the solution scheme described here on an electronic computer, we replace the integrals appearing in the right-hand sides of (5) and (6) by a sum of values of f, weighted in a certain way, at the grid points–

of the grid of the Northern Hemisphere of the Earth*. When computing f at the indicated points, the derivatives are replaced by finite differences. For this purpose, in the neighborhood of each point (θ',λ') a local coordinate system is introduced, whose axes

Fig. 2

Fig. 3

X and Y are directed along the tangents to the arcs of two mutually perpendicular great circles passing through the point (θ',λ'). In this case, the values of the functions ψ and b at equidistant points of the local coordinate system are found from the values of these functions at the four nearest

* The computation of the weight multiplying f at the point $\theta'=\theta,\ \lambda'=\lambda$ is carried out taking into account the singularity of the function G at this point.

at the points of the degree grid by interpolation. The use of a local coordinate system makes it possible to ensure constancy of the magnitudes of the finite-difference intervals over the entire region under consideration.* The constant m was chosen so that $(b)_{t=0}$ would be computed as simply as possible.

In carrying out the computations, the stream function ψ was everywhere replaced by its expression in terms of the geopotential of the 700 mb isobaric surface, using the condition of quasigeostrophy.

The problem was solved on the BESM (Large Electronic Computing Machine of the USSR Academy of Sciences). To determine the influence of errors of the finite-difference approximation, as well as the degree of stability of the solution with respect to the growth of random errors, one particular exact solution of equation (1), borrowed from work ([3]), was used:

$$\psi(\theta, \lambda, t) = -a_0^2 \alpha \cos\theta + A P_n^m(\cos\theta) \cos m(\lambda - \sigma t), \tag{7}$$

where A and α are constants; $\sigma = \alpha - 2(\alpha + \omega)/n(n+1)$. Starting from the initial values $\psi(\theta, \lambda)$, specified by (7) for $t = 0$, we computed, by the proposed scheme, future values $\psi(\theta, \lambda, t)$ for $m = 6$, $n = 12$, $\alpha/\omega = 1/24$. The results of the computation were compared with the exact solution (7).

These tests made it possible, in particular, to determine what finite-difference intervals in the coordinates and in time should be used. Then examples were computed of forecasts of the absolute topography chart of the 700 mb surface for various periods (up to 10 days). In these computations the initial data—the values of the heights of the 700 mb surface—were taken at the points of the degree grid of the Earth' s Northern Hemisphere from the pole to 10° N latitude, with intervals of 5° along the meridian and 10° along the circles of latitude. The finite-difference intervals s in the coordinates and the time step τ were chosen as follows: $s = 555$ km, $\tau = 2$ hours.

One example of a forecast of the AT 700 chart 10 days ahead is given in Figs. 1–3. The initial AT 700 chart for 03 hours on 4 V 1957 is given in Fig. 1. Fig. 2 presents the chart for 03 hours on 14 V 1957, and Fig. 3 the chart computed for the same time.

The solution of equation (1), linearized with respect to west–east transport, has repeatedly been tested in forecasts with lead times of several tens of days ([4]). The method proposed here for solving the nonlinear equation (1) may be applied in forecasts with the same lead time. With an increase in the forecast lead time it will be necessary to move away from the mean level and consider a spatial problem, taking into account the accumulating effects of the influx of heat from solar and long-wave radiation, from turbulent heat conduction, and from evaporation.

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of Forecasts

Received
14 II 1958

CITED LITERATURE

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[4] S. A. Mashkovich, Ya. M. Kheifets, *Tr. Tsentraln. inst. prognozov*, issue 60 (1957).

* This cannot be achieved, for example, when using a Cartesian coordinate system in the plane of the stereographic projection of the surface of the Northern Hemisphere.

Note: Figure translations are in progress. See original paper for figures.