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B. F. Shorr

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Abstract

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THEORY OF ELASTICITY

B. F. Shorr

THE EFFECT OF NONUNIFORM HEATING UNDER CREEP CONDITIONS ON THE CHANGE IN THE STRESS STATE

(Presented by Academician Yu. N. Rabotnov, 2 VII 1958)

When parts operate under conditions combining considerable stress, a high mean temperature, and a large nonuniformity of the temperature field, creep develops in them, proceeding at sharply different rates at different points of the cross section; this leads to a significant redistribution of stresses with time.

Proceeding from the theory of hardening, one may adopt for a uniaxial stress state ([1])

$$v_p = \operatorname{sign}\sigma \cdot Ae^{-\beta/T}\left(e^{k|\sigma|} - 1\right)\Phi(p), \tag{1}$$

where $v_p = d\varepsilon_p/d\tau$ is the creep rate; T is the absolute temperature; A, β, k are experimental constants; p is a parameter estimating the hardening plastic strain accumulated during creep.

For nonuniformly heated bodies, where the stresses not only may have different signs at a given instant of time but may also change sign in the course of creep, let us set

$$p = \begin{cases} p^+ + \varkappa p^-, & \text{for } \sigma > 0, \\ p^- + \varkappa p^+, & \text{for } \sigma < 0, \end{cases} \tag{2}$$

where

$$p^+ = \int_0^\tau v_p^+\, d\tau_1, \qquad p^- = \int_0^\tau |v_p^-|\, d\tau_1,$$

i.e., p^+ is the plastic strain accumulated during periods of tension, p^- during periods of compression. The coefficient $\varkappa$ characterizes the directionality of the hardening action of plastic strain ([1],[2]). For $\varkappa = 0$, hardening is caused only by strain of the same sign; for $\varkappa = 1$, the strains are summed in absolute value, as was assumed in ([3]) in the analysis of plastic tension of steel. It is assumed that the "rapid" plastic strain ε_{pn}, corresponding to the deformation diagram, does not affect hardening during creep ([1]).

Integrating (1) with allowance for (2), we have

$$\chi(p_i) = \chi(p_{i-1}) + A \int_{\tau_{i-1}}^{\tau_i} e^{-\beta/T(\tau_1)} \left[e^{k|\sigma(\tau_1)|} - 1\right] d\tau_1, \tag{3}$$

where

$$\chi(p) = \int_0^p \Phi^{-1}(p_1)\, dp_1$$

and $\tau_{i-1} \leq \tau_1 \leq \tau_i$.

Approximating the functions σ and β/T on the interval from τ_{i-1} to τ_i by linear expressions, we obtain

$$\chi(p_i) = \chi(p_{i-1}) + Ae^{-\beta/T(\tau_{i-1})} \left[e^{k|\sigma(\tau_{i-1})|} - \frac{1 - e^{-\Delta(\beta/T)_i}}{\Delta(\beta/T)_i}\right] \Delta\tau_i, \tag{4}$$

$$\varepsilon_p(\tau_i) = \varepsilon_p(\tau_{i-1}) + \operatorname{sign}\sigma(\tau_{i-1}) \cdot (p_i - p_{i-1}), \tag{5}$$

which makes it possible to find $\varepsilon_p(\tau_i)$, if $\varepsilon_p(\tau_{i-1})$, p_{i-1}, and $\sigma(\tau_{i-1})$ are known. The quantity $\Delta\tau_i$ must be such that the inequality

$$\frac{1}{2}\left|k\Delta|\sigma|_i - \Delta\left(\frac{\beta}{T}\right)_i\right| \ll 1, \tag{6}$$

is satisfied, where

$$\Delta|\sigma|_i = |\sigma(\tau_i)| - |\sigma(\tau_{i-1})|, \qquad \Delta\left(\frac{\beta}{T}\right)_i = \frac{\beta}{T(\tau_i)} - \frac{\beta}{T(\tau_{i-1})},$$

and the sign of the stress did not change during $\Delta\tau_i$.

For a uniaxial state of stress ($\sigma_x = \sigma,\ \sigma_y = \sigma_z = 0$), the strain

$$\varepsilon_x = \frac{\sigma}{E} + \gamma t + \varepsilon_{\text{ph}} + \varepsilon_p, \tag{7}$$

where γ is the coefficient of thermal expansion, for a broad class of problems, can be represented in the form

$$\varepsilon_x = \sum_{k=1}^{n} f_k(x)\psi_k(y,z). \tag{8}$$

With the aid of (7)–(8) and the equilibrium equations, the relation between the stresses $\sigma(\tau_i)$ and $\varepsilon_p(\tau_i)$ is established. Thus, in the case of tension and bending of a straight bar of arbitrary cross section,

$$\sigma(\tau_i) = \sigma_{\mathrm{H}}(\tau_i) + L[E\varepsilon_p(\tau_i)], \tag{9}$$

where

$$\sigma_{\mathrm{H}} = \overline{E}\left(\frac{N}{F} + \frac{M_y}{I_y}z - \frac{M_z}{I_z}y\right) + L(E\gamma t) + L(E\varepsilon_{\mathrm{ph}}),$$

$$L(f) = \overline{E}\left(\frac{1}{F}\int_F f\,dF + \frac{z}{I_y}\int_F fz\,dF + \frac{y}{I_z}\int_F fy\,dF\right) - f, \tag{10}$$

$$\overline{E} = \frac{EF}{\int_F E\,dF}, \qquad I_y = \int_F \overline{E}z^2\,dF, \qquad I_z = \int_F \overline{E}y^2\,dF.$$

Having the initial values σ_0 and $\varepsilon_{p0} = p_0 = 0$ and establishing, from (6), an admissible value $\Delta\tau_1$, we find p_1, then ε_{p1}, σ_1, and so on in the same way.

If the sign of the stresses does not change with time ($p = |\varepsilon_p|$), another path of solution is possible, corresponding to that adopted in work ([1]). Expressing σ in (1) according to (9) and assuming $e^{k|\sigma|} \gg 1$, after separation of variables we obtain

$$\chi_1(p_i) = \chi_1(p_{i-1}) + A\exp\left\{k\left|\sigma_{\mathrm{H}}(\tau_{i-1}) + Q[E\varepsilon_p(\tau_{i-1})]\right| - \frac{\beta}{T(\tau_{i-1})}\right\}\Delta\tau_i, \tag{11}$$

where

$$\chi_1(p) = \int_0^p \Phi^{-1}(p_1)e^{Ekp_1}\,dp_1, \qquad Q(f) = L(f) + f$$

and $\Delta\tau_i$ is such that the inequality

$$\frac{1}{2}\left|k\Delta|\sigma_{\mathrm{H}} + Q(e\varepsilon_p)|_i - \Delta\left(\frac{\beta}{T}\right)_i\right| \ll 1. \tag{12}$$

Knowing p_i, we find $L(E_{spi})$, etc.

In works ([1,4–7]) the following form of the function was recommended:

$$\Phi(p) = p^{-\alpha}, \tag{13}$$

which received experimental confirmation for a moderate duration of tests and $\operatorname{sign}\sigma(\tau) = \text{const}$. Using (11) and (13), certain solutions were obtained for $T = \text{const}$ ([4,8]), and in work ([1])—with allowance for nonuniform heating, but restricted by the condition $\operatorname{sign}\sigma(\tau) = \text{const}$. Solution (11) requires special identification of the zones where $\dot{p}p^{\alpha} < 1$, which complicates the calculation. Solution (4) is free of the indicated restrictions.

Expression (13) is valid only for the unsteady stage of creep, since it follows from it that as $p \to \infty$, $\dot{p} \to 0$. Without complicating the calculation, one may take

$$\Phi(p) = \begin{cases} p^{-\alpha}, & \text{for } p < p_c, \\ p_c^{-\alpha}, & \text{for } p \geq p_c, \end{cases} \tag{14}$$

where in the general case $p_c = f(\sigma, T)$. For $p_c = \text{const}$, expression (1) becomes, as $\tau \to \infty$,

$$v_{p\infty} = \operatorname{sign}\sigma_{\infty} \cdot A_{\infty} e^{-\beta/T_{\infty}} \left(e^{k|\sigma_{\infty}|} - 1\right). \tag{15}$$

For the steady state of creep ($\dot{\sigma} = \dot{T} = \dot{\varepsilon}_{pn} = 0$), from (7) and (8) it follows that

$$v_{p\infty} = \sum_{k=1}^{n} \dot{f}_k(x)\psi_k(y, z). \tag{16}$$

Comparison of expressions (8) and (16) shows that the law of distribution over the cross section of the rod of the steady creep rates coincides with the "kinematic" law of distribution of deformations.

Solving (15) with respect to σ_{∞}, one can, with the aid of (16), establish the law of distribution of stresses in the steady stage of creep. Thus, for a straight rod of arbitrary cross section,

$$\sigma_{\infty} = \frac{\operatorname{sign}(a + by + cz)}{k} \ln\left[\,|a + by + cz|\, e^{\beta/T_{\infty}} + 1\right], \tag{17}$$

where the constants a, b, and c are found from the equilibrium equations.

In Fig. 1 the stresses are shown in a tensile rod for a parabolic temperature distribution across its width, and the process of rearrangement over time of the curves corresponding to two calculated cases is visible: in the absence of

Fig. 1. Tension of a rod made of alloy EI 437A at $\Delta t = 120^{\circ}$

Figure 1: Fig. 1. Tension of a rod made of alloy EI 437A at $\Delta t = 120^{\circ}$

Fig. 2. Change of the coefficients $n_b^{\tau_0}$ (curves 0), $n_{b0}^{\tau_0}$ ( ) and $n_{b\infty}^{\tau_0}$ (∞) in a stretched rod. A—in the coldest point, —in the hottest.

Figure 2: Fig. 2. Change of the coefficients $n_b^{\tau_0}$ (curves 0), $n_{b0}^{\tau_0}$ ( ) and $n_{b\infty}^{\tau_0}$ (∞) in a stretched rod. A—in the coldest point, —in the hottest.

temperature stresses ($\gamma = 0$) and with allowance for temperature stresses corresponding to the actual value of γ. In both cases, with the passage of time, the stresses approach one and the same curve σ_∞, which differs sharply from the initial stress distribution σ_0.

Fig. 1. Tension of a rod made of alloy EI 437A at $\Delta t = 120^{\circ}$

For varying stresses, the safety factor at the time τ_0 may be estimated as $n_b^{\tau_0} = \sigma_b^{\tau_0}/\sigma_e$, where $\sigma_b^{\tau_0}$ is the long-time strength limit of the material over the time τ_0, and σ_e is the equivalent stress. Admitting the possibility of summing the degrees of damage and adopting the relation of stress to time

before fracture in the form $\sigma^m \tau = \text{const}$, where $m = m(T)$, we obtain

$$\sigma_{\text{e}} = \left[\frac{1}{\tau_0} \int_0^{\tau_0} \sigma^m(\tau)\, d\tau \right]^{1/m}. \tag{18}$$

If the stresses do not change, then $n_b^{\tau_0} - n_{b0}^{\tau_0} = \sigma_b^{\tau_0}/\sigma_0$; if the time of redistribution of stresses is substantially less than the service life of the part, then $n_b^{\tau_0} \approx n_{b\infty}^{\tau_0} = \sigma_b^{\tau_0}/\sigma_\infty$. Usually, for dangerous points $n_{b\infty}^{\tau_0} > n_{b0}^{\tau_0}$, i.e., creep promotes the convergence of the "stress field" with the "resistance field" of the material ([9]).

Figure 2 shows the change in the quantities $n_b^{\tau_0}$, $n_{b0}^{\tau_0}$, and $n_{b\infty}^{\tau_0}$ for the dangerous points of the rod considered above.

Fig. 2. Change of the coefficients $n_b^{\tau_0}$ (curves 0), $n_{b0}^{\tau_0}$ ( ) and $n_{b\infty}^{\tau_0}$ (∞) in a stretched rod. A—in the coldest point, —in the hottest.

Thus, in calculating parts nonuniformly heated to high temperatures, it is insufficient to confine oneself to the study only of the initial stressed state (even taking temperature stresses into account); it is necessary to take into account the change of stresses with time.

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Note: Figure translations are in progress. See original paper for figures.