# ASTRONOMY

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## Abstract

## Full Text

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# A GENERAL CASE OF STATIONARY DISTRIBUTION OF THE APPARENT DENSITY IN THE HEAD OF A COMET UNDER CONDITIONS OF CONSTANT ISOTROPIC EMISSION

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It has been established ($^1$) that, under constant isotropic emission of molecules, the isophotes in the head of a comet are concentric circles, and the apparent density in a stationary distribution is inversely proportional to the distance from the common center of the isophotes. The problem was solved under two simplifying conditions: 1) the center of emission is motionless relative to the Sun; 2) the line of sight is perpendicular to the direction from the center of emission to the Sun.

Retaining the first condition, we shall show that the law of distribution of the apparent density does not depend on the position of the observer, i.e., on the angle between the line of sight and the direction from the center of emission to the Sun.

Let us denote the coordinates of a point in the old rectangular cometocentric system by $x'$, $y'$, $z'$. The axis $Ox'$ is directed toward the Sun, and the axis $Oz'$ —toward the observer. The equations of motion of the molecules in this system are

$$\begin{aligned} x' &= v_0 t \sin\alpha - \frac{1}{2} g t^2, \\ y' &= v_0 t \cos\alpha \cos\varphi, \\ z' &= v_0 t \cos\alpha \sin\varphi. \end{aligned} \tag{1}$$

We retain here the notation adopted earlier: $v_0$ is the initial velocity, assumed constant; $\alpha$ is the angle of departure; $\varphi$ is the azimuth of the molecule; $g$ is a constant acceleration ($g \gg 0$); $t$ is time (with $x' = y' = z' = 0,\ t = 0$).

It was shown in detail ($^2$) how the apparent density is calculated for the equations of motion (1). If the emission coefficient $n$ does not depend on $\alpha$ and $t$, then the apparent density is computed by the formula

$$N(x',y') = \frac{n\pi}{v_0\sqrt{{x'}^2 + {y'}^2}}. \tag{2}$$

Let now the line of sight make some angle $\psi$ with the direction from the center of emission to the Sun. We pass to a new rectangular cometocentric coordinate system, in which the axis $Oz$ is also directed toward the Earth, but the axis $Ox$, remaining in the comet-Earth-Sun plane, makes an angle $\pi/2 - \psi$ with the direction toward the Sun.

In this new system the equations of motion will have the form

$$\begin{aligned} x &= v_0 t \sin\alpha \sin\psi - \frac{1}{2} g t^2 \sin\psi + v_0 t \cos\alpha \sin\varphi \cos\psi, \\ y &= v_0 t \cos\alpha \cos\varphi, \\ z &= v_0 t \cos\alpha \sin\varphi \sin\psi - v_0 t \sin\alpha \cos\psi + \frac{1}{2} g t^2 \cos\psi. \end{aligned} \tag{3}$$

After eliminating $\alpha$ and $\varphi$ from equations (3), we obtain

$$\frac{1}{4} g^2 t^4 - (A + gz\cos\varphi) t^2 + x^2 + y^2 + z^2 = 0, \tag{4}$$

where $A = v_0^2 - gx\sin\psi$.

The equation of the envelope surface in the new system is

$$A^2 - g^2(x^2 + y^2) + 2Agz\cos\psi - g^2 z^2 \sin^2\psi = 0. \tag{5}$$

When computing the spatial density by the formula $\rho = n\cos\alpha/J$, where $J = D(x,y,z)/D(\alpha,\varphi,t)$, we must take into account the invariance of $J$ under a linear transformation of coordinates, since a rotation does not change the volume element. Therefore:

$$\rho = \frac{n}{v_0^2 t^2 (v_0 - gt\sin\alpha)},$$

but in eliminating $t$ and $\alpha$ we must use equations (3), obtained from equations (1) by means of the coordinate transformation. In this way, we can avoid cumbersome computations by taking the already known expression for $\rho$ in the old system, and then passing to the new system using the transformation formulas for a rotation about the axis $Oy'$ through the angle $\pi/2 - \psi$.

Let us take for $\rho$ formula (17) on p. 276 of work $^{(2)}$ and write it in the form

$$\rho = \frac{n}{v_0} \frac{v_0^2 + gx'}{({x'}^2 + {y'}^2 + {z'}^2)\sqrt{v_0^4 - 2v_0^2 gx' - g^2{y'}^2 - g^2{z'}^2}}.$$

After transformation we find

$$\rho = \frac{n}{v_0} \frac{A + gz\cos\psi}{(x^2 + y^2 + z^2)\sqrt{A^2 - g^2(x^2 + y^2) + 2Agz\cos\psi - g^2 z^2 \sin^2\psi}}. \quad (6)$$

We obtain the expression for the apparent density by computing the integral

$$N(x, y) = \frac{n}{v_0} \int_{z_1}^{z_2} \frac{A + gz\cos\psi}{(a^2 + z^2)\sqrt{m + rz - pz^2}}\, dz, \quad (7)$$

where $A = v_0^2 - gx\sin\psi$; $a^2 = x^2 + y^2$; $m = A^2 - g^2(x^2 + y^2)$; $r = 2Ag\cos\psi$;

$$p = g^2 \sin^2\psi.$$

The limits of integration $z_1$ and $z_2$ are determined by the values of $z$ at the points of intersection of the line of sight with the envelope surface. From equation (5) we find

$$z_1 = \frac{A\cos\psi - C}{g\sin^2\psi}; \qquad z_2 = \frac{A\cos\psi + C}{g\sin^2\psi}; \quad \text{here } C^2 = A^2 - g^2(x^2 + y^2)\sin^2\psi.$$

$z_1$ and $z_2$ are real roots of the radicand trinomial in integral (7); therefore it can be transformed as follows:

$$N(x, y) = \frac{n}{v_0\sqrt{p}} \int_{z_1}^{z_2} \frac{A + gz\cos\psi}{(a^2 + z^2)\sqrt{(z_2 - z)(z - z_1)}}\, dz. \quad (8)$$

This integral is rationalized by the substitution

$$\sqrt{(z_2 - z)(z - z_1)} = t(z - z_1),$$

and we obtain

$$N(x, y) = \frac{2n}{v_0\sqrt{p}(a^2 + z_1^2)} \int_0^\infty \frac{Pt^2 + U}{t^4 + bt^2 + c}\, dt, \quad (9)$$

where

$$P = A + gz_1 \cos\psi; \quad U = A + gz_2 \cos\psi; \quad b = 2\frac{a^2 + z_1 z_2}{a^2 + z_1^2}; \quad c = \frac{a^2 + z_2^2}{a^2 + z_1^2}.$$

Let us compute the integral in formula (10), putting $K = \sqrt{c}$, $M = \sqrt{2K - b}$.

$$\int\limits_0^\infty \frac{Pt^2 + U}{t^4 + bt^2 + c}\, dt = \frac{(KP + U)\pi}{2K\sqrt{4K - M^2}},$$

where $4K - M^2 > 0$, since the roots of the denominator of the integrand are complex. Thus we find

$$N(x, y) = \frac{\pi n(KP + U)}{v_0\sqrt{p(a^2 + z_1^2)}K\sqrt{4K - M^2}}. \tag{10}$$

After substituting the conventional notations, we finally obtain

$$N(x, y) = \frac{\pi n}{v_0\sqrt{x^2 + y^2}}. \tag{11}$$

Formula (11) shows that, once a stationary distribution is attained under constant isotropic emission, the apparent density will be inversely proportional to the distance from the center of emission, independently of the position of the observer relative to the comet.

One may also draw the converse conclusion. If from observations we obtain the law $N \sim r^{-1}$, then over a considerable interval of time the emission coefficient $n$ was constant and did not depend on $\alpha$.

In view of the fact that isotropic emission from a solid nucleus is difficult to imagine, the distribution law expressed by formula (11) may be regarded as independent proof of the correctness of the hypothesis of the existence of parent molecules, since photodissociation of parent molecules in a cloud noticeably larger than the solid nucleus can create the effect of isotropic emission.

Formula (11) has been derived under the assumption of a stationary distribution, which requires that, along the line of sight within the interval of integration, molecules that have arrived both along flat and along lofted trajectories be observed at every point. Since the spatial density in the stream of molecules moving along lofted trajectories is small in comparison with the density along flat trajectories, the shape of the isophotes and the law $N(x, y) \sim r^{-1}$ may be satisfied within the errors of modern observations even for a nonstationary distribution.

Such violations of the conditions adopted in solving our problem as, for example, a sharp discontinuous change in the acceleration $g$, or a discontinuous change in the emissive capacity of the molecules, must lead to deformation of the circular isophotes; in this case one may observe the so-called anomalous and ordinary gas tails. Similar phenomena may be caused by photoionization of the molecules composing the head of the comet.

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## REFERENCES CITED

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2. D. O. Mokhnach, *Bulletin of the Institute of Theoretical Astronomy, Academy of Sciences of the USSR*, **6**, No. 5 (78) (1956).

*Note: Figure translations are in progress. See original paper for figures.*

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