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Abstract

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MECHANICS

V. V. LAVROV

THE NATURE OF THE SCALE EFFECT IN ICE AND THE STRENGTH OF THE ICE COVER

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The scale effect in ice, as in other materials, consists in the fact that, when samples of larger dimensions are tested, smaller values of the ultimate strength are obtained, and vice versa. Thus, for example, at one and the same temperature the value of the temporary flexural resistance σ of a beam cut through the entire thickness of an ice cover $h = 34$ cm is 9.0 kg/cm^2, whereas for specimens cut from it (from the broken halves) with a cross section of 4.4×4.5 cm it is, on average, 23.2 kg/cm^2.

(Figure: Fig. 1)

Fig. 1

None of the hypotheses proposed concerning the nature of the scale effect as applied to ice agrees with experiment. The statistical theory of strength, on the other hand, is very difficult to verify in practice because of the absence of information on the average number of inhomogeneities occurring in 1 cm^3 of material.

The fracture of ice in bending or tensile experiments at any temperatures occurs according to the type of fracture of brittle materials and practically instantaneously. It apparently begins in that part of the lower surface of the specimen where the weakest (most defective) place is found. Regardless of the nature of the defect, let us consider the following scheme of operation of a beam and of a specimen cut from it.

Under the action of a force P, the beam bends, as a result of which its length l acquires a certain increment Δl. For a small value of the deflection arrow f (hundredths of a millimeter), the curvature of the elastic line is extremely small, and therefore the increment $\Delta l/2$ may be determined as the difference between the length of the hypotenuse and the length of the leg (Fig. 1)*

$$\frac{\Delta l}{2} = L - \frac{l}{2} = \sqrt{f^2 + \left(\frac{l}{2}\right)^2} - \frac{l}{2}. \qquad (1)$$

(Figure: Fig. 2)

Fig. 2

Since the resistance to tensile stresses of the defective place is less than the strength of the other sections along the length of the specimen, it may be considered, in a first approximation, that the increment Δl is the result of the divergence (rotation) of adjacent planes of the crystal lattice of the defective place through some angle α (Fig. 2)*.

In Fig. 2, $\Delta l/2 = dc$; however, owing to the small curvature of the elastic line, it is permissible to take $\Delta l/2 = ac$, and instead of $f = bd$ to assume approximately that $f = ab$.

With such a scheme of operation of the material, its strength will be determined by the magnitude of the intermolecular bonding forces at point b, i.e., at the base of the defect. Since this quantity is a physical characteristic of the given material, it cannot depend on the size of the specimens, and in that case the fracture of both small and large specimens must occur at a constant value of the angle α, or, more precisely, $\operatorname{tg} \frac{\alpha}{2}$:

$$\operatorname{tg} \frac{\alpha}{2} = \frac{ac}{ab} = \frac{\Delta l}{2ab},$$

where ab is the linear size of the defect.

Since the linear size of the defect for a given kind of material is, on average, a sufficiently constant quantity, and in the same place of fracture of the beam considered by us it is strictly constant, fracture of specimens (beams) occurs at one and the same value of Δl of the outermost stretched layer and at one and the same value of the true stress at the base of the defect σ_b, **independently of the size of the specimens**. However, when calculating by the usual formulas of the strength of materials, we obtain the value not of the true stress σ_b, but of the stress of the outermost stretched fiber, which causes, as was said above, the divergence by the angle α of the lattice planes of the defective place. The constancy of the absolute elongation Δl in specimens of any size requires a difference in them of the value of the relative elongation $\Delta l/l$, and, consequently, also of the value of the normal stresses causing these elongations. The larger the size of the specimen, the smaller the relative elongation at which fracture will occur, and the smaller the calculated value of the strength limit. This value is not difficult to obtain, using the expressions for the deflection and the magnitude of the normal stresses of a freely supported beam:

$$f = \frac{Pl^3}{48EI}; \qquad \sigma = \frac{3Pl}{2bh^2}.$$

* In Fig. 2, as in Fig. 1, half of the beam is considered.

Denoting the quantities relating to the small specimen by the subscript 1, and the quantities relating to the large one by the subscript 2, after simple transformations we obtain

$$\sigma_2 = \sigma_1 \frac{l_1^2 h_2}{l_2^2 h_1} \frac{f_2 E_2}{f_1 E_1}. \tag{2}$$

The magnitude of Young' s modulus or, more precisely, the coefficient of proportionality E, as experiment shows, does not depend on the size of the specimens, i.e. $E_1 = E_2$; then, taking into account that

$$f_2 = \sqrt{L_2^2 - \left(\frac{l_2}{2}\right)^2} = \sqrt{\left(\frac{l_2}{2} + \frac{\Delta l_1}{2}\right)^2 - \left(\frac{l_2}{2}\right)^2} =$$

$$= \sqrt{\left(\sqrt{f_1^2 + \left(\frac{l_1}{2}\right)^2} - \frac{l_1}{2} + \frac{l_2}{2}\right)^2 - \left(\frac{l_2}{2}\right)^2}, \tag{3}$$

we shall have

$$\sigma_2 = \sigma_1 \frac{l_1^2 h_2}{l_2^2 h_1} \frac{\sqrt{\left(\sqrt{f_1^2 + \left(\frac{l_1}{2}\right)^2} - \frac{l_1}{2} + \frac{l_2}{2}\right)^2 - \left(\frac{l_2}{2}\right)^2}}{f_1}. \tag{4}$$

Table 1 gives the results of comparing the values of σ_2 calculated by formula (4) with those actually determined*. The agreement of the experimental values of σ_2 with the theoretical ones should be recognized as quite satisfactory.

Thus, the cause of the scale effect in ice is the combined influence of the fracture mechanism and the geometric factor. As a result that the destruction of the specimens does not occur simultaneously over the entire cross-section and begins, regardless of their dimensions, at the same values of the absolute elongation of the extreme tensile layers; the magnitude of the relative deformation, and consequently also of the normal stresses causing these deformations, is smaller in larger specimens than in small ones.

The proposed explanation of the nature of the scale effect is of practical interest in that it makes it possible, on the basis of laboratory tests of thin ice plates or small specimens, to calculate

Table 1

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* Although the width of the specimens does not enter into formula (4), for ice it is nevertheless better to take specimens of square cross section.

No.	Initial data	l_2, cm	h_2, cm	σ_2, calculated	σ_2, mean, determined			
	l_1, cm	h_1, cm	f_1, cm	σ_1, kg/cm^2				
1	36.0	4.0	0.013	19.0	80.0	11.0	14.8	13.7
2	80.0	11.0	0.020	13.7	20.0	2.3	22.9	23.7
3	35.0	4.5	0.013	23.2	250.0	34.0	8.8	9.0
4	35.0	4.5	0.013	23.2	64.0	8.0	16.0	14.5

the flexural strength limit of a natural ice cover of various thicknesses. For thicknesses of the latter of 1 m and more, this quantity is difficult to determine directly; therefore it has until now remained unknown and in calculations has been erroneously taken as the same for any ice thickness, i.e., without allowance for the scale effect.

To derive a formula analogous to relation (4), we shall use the expressions* for the deflection f of an infinite centrally loaded plate on an elastic foundation and for the maximum tensile stresses on the lower surface of the plate:

$$f = \frac{P}{8k\sqrt{m^2Eh^3/12\,(m^2-1)}}; \qquad \sigma = 0.275(1+\nu)\frac{P}{h^2}\lg\left(\frac{Eh^3}{kb^4}\right).$$

Proceeding in exactly the same way as in deriving formula (4), we finally obtain

$$\sigma_2 = \frac{\sigma_1 \lg\left(E_2h_2^3/k_2b_2^4\right)}{f_1 \lg\left(E_1h_1^3/k_1b_1^4\right)}\sqrt{\frac{h_1\left[\left(\sqrt{f_1^2+r_1^2}-r_1+r_2\right)^2-r_2^2\right]}{h_2}}. \qquad (5)$$

Here σ_2 is the flexural strength limit and h_2 the thickness of the ice cover; σ_1 is the flexural strength limit and h_1 the thickness of the laboratory ice plate; f_1 is the deflection of the laboratory plate at the moment of failure; $E_1 = E_2 \simeq 50\,000$ kg/cm^2; $K_2 = K_1 = 0.001$ kg/cm^3 is the stiffness of the foundation; b_1 and b_2 are the radii of load distribution; r_1 and r_2 are the distances from the point of load application (the center of the plate) to the point where the ordinate of the elastic surface first becomes zero, with

(Figure: Figure 3 graph: Change in the strength limit of the ice cover as its thickness changes.)

Fig. 3. Change in the strength limit of the ice cover as its thickness changes

$$r = 3.92\sqrt{\frac{m^2Eh^3}{12(m^2-1)}},$$

* Here, as for the specimens, we use the expressions of the elastic deflection curve, which, for short-duration loading (several seconds), as experiment shows, is quite permissible also for the case of ice failure. The increase in the deformation of the specimen or plate is directly proportional to the increase in load, although in this case the quantity E has the meaning not of the modulus of elasticity, but of the proportionality coefficient.

where m is a quantity reciprocal to Poisson’ s ratio. The quantity r for a plate is analogous to the quantity $l/2$ for specimens.

Fig. 3 presents the curve $\sigma_2 = \varphi(h_2)$ for freshwater ice at negative temperatures, obtained on the basis of calculations by formula (5), proceeding from the following data: $\sigma_1 = 21$ kgf/cm^2; $h_1 = 0.35$ cm; $b_1 = 0.16$ cm; $f_1 = 0.225$ cm; $E_1 = E_2 = 50\,000$ kgf/cm^2; $m = 3$. The radius of load distribution was increased in proportion to the thickness.

The reality of the values of σ_2 given by the curve is confirmed by direct determinations of the flexural strength of beams cut out through the entire thickness of the ice cover; for example, by the data for a beam of thickness $h = 34$ cm in Table 1 (difference 10%), and also by the results of tests on ice cantilever beams. The average value of σ for beams of this kind at negative temperatures and an ice-cover thickness $h = 65$ cm is approximately 8.4 kgf/cm^2 (allowing for a specially determined correction for stress concentration in the corners), which differs from the value from the curve by 0.2 kgf/cm^2.

(Figure: Fig. 4. Dependence of l_2 on h_2, when the calculation is made using specimens with dimensions $l_1 = 36$ cm, $h_1 = 4$ cm (I) and $l_1 = 86$ cm, $h_1 = 11$ cm (II).)

Fig. 4. Dependence of l_2 on h_2, when the calculation is made using specimens with dimensions $l_1 = 36$ cm, $h_1 = 4$ cm (I) and $l_1 = 86$ cm, $h_1 = 11$ cm (II).

However, determining the strength of the ice cover using thin plates of laboratory ice is less convenient than using small specimens. Therefore, for practical purposes it is recommended to use formula (4), in which the dimension l_2 should be taken from the graph presented in Fig. 4. In it, l_2 increases not directly in proportion to the thickness h_2, but in accordance with the law of variation of the quantity r for ice plates. The values of l_2 were calculated from the known values of $\sigma_1, l_1, h_1, f_1, h_2$ and the assigned values of σ_2 (in accordance with the graph of Fig. 3).

Arctic Scientific Research Institute

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