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Abstract

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A METHOD FOR THE SYNTHESIS OF MULTITERMINAL CONTACT CIRCUITS

(Presented by Academician A. N. Kolmogorov on 26 X 1957)

The algebraic method for the synthesis of series-parallel contact circuits was first developed by V. I. Shestakov ([1]). The first attempt at the synthesis of mult-terminal contact circuits of general form belongs to A. G. Lunts ([2,3]), who proposed a method based on the use of so-called characteristic functions.

In the present note a simple method is proposed for the synthesis of mult-terminal contact circuits of general form and, in particular, a method for the synthesis of multi-terminal nonrepeated circuits, which is especially simple*.

A. Method for the synthesis of mult-terminal circuits of general form. Let there be given a set of Boolean functions of the variables $a, b, \dots, c$:

$$A_{\alpha\beta}(a, b, \dots, c), \qquad \alpha, \beta = 1, 2, \dots, n;\ \alpha < \beta. \tag{1}$$

We shall synthesize the circuit of a contact n-terminal network in which between each pair of terminals α and β there exists the total contact conductivity $A_{\alpha\beta}(a, b, \dots, c)$ (two-way conductivity is meant). It is easy to show that such a synthesis is possible if and only if the conditions

$$A_{\alpha\beta} \geq A_{\alpha\gamma} A_{\gamma\beta}, \qquad \alpha, \beta, \gamma = 1, 2, \dots, n;\ \alpha < \beta \tag{2}$$

are satisfied.

The proposed synthesis method consists in the successive extraction of external (i.e., attached by at least one end to a terminal) contacts of the multi-terminal network according to a special procedure.

There may be indicated three possible ways of extracting an external contact:

I. Extraction of a longitudinal contact according to the circuit of Fig. 1a, where X is a two-terminal network containing contact x, and Y is a multi-terminal

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Fig. 1

Figure 1: Fig. 1

network containing the part of the multi-terminal network A remaining after extraction of contact x. We shall denote their total conductivities by X_{kh} and $Y_{\alpha\beta}$ $(\alpha, \beta = 1, \dots, k', \dots, n;\ \alpha, \beta \neq k;\ \alpha < \beta)$.

II. Extraction of a transverse contact according to the circuit of Fig. 1b (the notation is analogous).

III. Extraction of a longitudinal-transverse contact according to the circuit of Fig. 1c (the notation is analogous).

It is easy to prove the following theorem:

Theorem 1. *Any external contact of an arbitrary multi-terminal network can be extracted by means of one of the three methods I, II, or III.*

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* The work was reported on 10 V 1957 at the seminar on technical applications of mathematical logic (headed by V. I. Shestakov) and on 5 X 1957 at the All-Union Conference on the theory of relay-action devices in Moscow.

In connection with this, for purposes of synthesis one may restrict oneself to the three indicated methods of separating external contacts.

After the separation of some contact x from the multipole A, a multipole A' remains. The Boolean conductivities between the poles of the multipole A' can be determined by solving certain systems of equations of Boolean algebra.

Fig. 1

1. Let us consider the case of separating a longitudinal contact. From Fig. 1a it is seen that between the complete conductivities of the multipoles A, X, and Y there exists the relation

$$\begin{aligned} X_{kk'}Y_{k'\beta} &= A_{k\beta}, \qquad \beta = 1, \dots, n;\ \beta \neq k, k'; \\ Y_{\alpha\beta} &= A_{\alpha\beta}, \qquad \alpha, \beta = 1, \dots, n;\ \alpha, \beta \neq k, k';\ \alpha < \beta \end{aligned} \tag{I}$$

(assuming that k' is included in the list $1, 2, \dots, n$). We shall call this system of equations system No. k. Altogether there exist n such systems of equations of Boolean algebra.

The method for determining the quantities $X_{kk'}$ and $Y_{\alpha\beta}$ $(\alpha, \beta = 1, \dots, n;\ \alpha, \beta \neq k;\ \alpha < \beta)$ is as follows. We examine the functions $A_{\alpha\beta}$ $(\alpha, \beta = 1, \dots, n;\ \alpha, \beta \neq k';\ \alpha < \beta)$. If in some group $A_{k\beta}$ $(\beta = 1, \dots, n;\ \beta \neq k, k')$ there is a common variable x in all functions of the group, and if it enters each function of the

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group as a factor, then we solve system (I) No. k, substituting in it the variable $X_{kk'} = x$, and determine the functions $Y_{\alpha\beta}$ $(\alpha, \beta = 1, \ldots, n;\ \alpha, \beta \neq k;\ \alpha < \beta)$.

2. Let us consider the case of separating a transverse contact. From Fig. 1b it is seen that between the complete conductivities of the multipoles A, X, and Y there exists the relation

$$\begin{aligned} X_{kl} + Y_{k'l'} &= A_{kl}, && k < l; \\ Y_{k'\beta} + X_{kl} Y_{l'\beta} &= A_{k\beta}, && \beta = 1, \ldots, n;\ \beta \neq k, l, k', l'; \\ X_{kl} Y_{k'\beta} + Y_{l'\beta} &= A_{l\beta}, && \beta = 1, \ldots, n;\ \beta \neq k, l, k', l'; \\ Y_{\alpha\beta} + X_{kl} Y_{\alpha k'} Y_{l'\beta} + X_{kl} Y_{\alpha l'} Y_{k'\beta} &= A_{\alpha\beta}, && \alpha, \beta = 1, \ldots, n;\ \alpha, \beta \neq k, l, k', l';\ \alpha < \beta. \end{aligned} \tag{II}$$

We shall call this system of equations system No. $k-l$. There are as many such systems as there are pairs of numbers k, l.

The method for determining the quantities X_{kl} and $Y_{\alpha\beta}$ $(\alpha, \beta = 1, \ldots, n;\ \alpha, \beta \neq k, l;\ \alpha < \beta)$ is as follows. We examine the functions $A_{\alpha\beta}$ $(\alpha, \beta = 1, \ldots, n;\ \alpha, \beta \neq k', l';\ \alpha < \beta)$ and determine whether one of them contains a single variable as a term. If the variable x is contained in the function A_{ps}, then we take system (II) No. $p-s$ and, substituting in it the variable $X_{ps} = x$, try to solve it with respect to the functions $Y_{\alpha\beta}$ $(\alpha, \beta = 1, \ldots, n;\ \alpha, \beta \neq p, s;\ \alpha < \beta)$.

3. Let us consider the case of separating a longitudinal-transverse contact. From Fig. 1c it is seen that between the complete conductivities of the multipoles

between A, X, and Y there is the relation

$$\begin{aligned} Y_{k'\beta} + X_{k(n+1)} Y_{(n+1)\beta} &= A_{k\beta}, && \beta = 1, \ldots, n;\ \beta \neq k, k'; \\ Y_{\alpha\beta} + X_{k(n+1)} Y_{\alpha k'} Y_{(n+1)\beta} + X_{k(n+1)} Y_{\alpha(n+1)} Y_{k'\beta} &= A_{\alpha\beta}, && \alpha, \beta = 1, \ldots, n;\ \alpha, \beta \neq k, k';\ \alpha < \beta; \end{aligned} \tag{III}$$

We shall call this system of equations system No. k. There are n such systems.

The method for determining $X_{k(n+1)}$ and $Y_{\alpha,\beta}$ $(\alpha, \beta = 1, \ldots, n+1;\ \alpha, \beta \neq k;\ \alpha < \beta)$ is as follows. We examine the functions $A_{\alpha\beta}$ $(\alpha, \beta = 1, \ldots, n;\ \alpha, \beta \neq k';\ \alpha < \beta)$. If, in some group $A_{k\beta}$ $(\beta = 1, \ldots, n;\ \beta \neq k, k')$, there is a common variable x in some of the functions of the group, then we take system (III) No. k, substitute into it $X_{k(n+1)} = x$, and try to solve it with respect to the functions $Y_{\alpha\beta}$ $(\alpha, \beta = 1, \ldots, n+1;\ \alpha, \beta \neq k;\ \alpha < \beta)$. To carry out the synthesis of the circuit, it is necessary to successively extract contacts until unit or zero functions $Y_{\alpha\beta}$ are obtained $(\alpha, \beta = 1, \ldots, q;\ q \geq n;\ \alpha < \beta)$ (for the multi-terminal network $A'^{\prime\ldots\prime}$).

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B. Method of synthesizing multi-terminal nonrepeating circuits. Since in this case a contact of each type may occur only once, the functions $A''^{\cdots\prime}_{\alpha\beta}$ cannot contain variables already extracted. This substantially simplifies the synthesis method. Further simplification can be obtained by using the following theorem:

Theorem 2. *When r longitudinal and transverse contacts are extracted from a multi-terminal network A, the contact conductivities of the resulting multi-terminal network $A''^{\cdots\prime}$ (with r primes) do not depend on the order in which the r contacts are extracted.*

Consequently, these r contacts may be extracted in any order.

It may happen that the process of extracting contacts, in an attempt to synthesize a nonrepeating circuit and in examining all variants of extracting longitudinal-transverse contacts, cannot be carried through to the end. This will indicate that a nonrepeating circuit of the multi-terminal network under consideration is unrealizable.

C. Examples. Let us consider examples of the synthesis of multi-terminal and two-terminal circuits.

Example 1. Let Boolean functions satisfying conditions (2) be given:

$$A_{12} = a + bd, \quad A_{13} = ad + ab + ac + bcd, \quad A_{23} = c + ab + ad.$$

Let us try to synthesize a three-terminal circuit, without restricting ourselves to nonrepeating circuits. Examining the functions $A_{\alpha\beta}$, we note that the function A_{23} contains the single variable c as a term. In connection with this, we try to solve system (II) No. 2—3 under the assumption $X_{23} = c$. The solution may have the form

$$Y_{12'} = a + bd = A'_{12}; \quad Y_{13'} = ad + ab = A'_{13}; \quad Y_{2'3'} = ab + ad = A'_{23}.$$

Noting that a is a single variable, we apply system (II) No. 1—2. The solution is

$$X_{12} = a, \quad Y_{1'2'} = bd = A''_{12}; \quad Y_{1'3} = ad = A''_{13}; \quad Y_{2'3} = ab = A''_{23}.$$

And so on. Third stage: (I) No. 1;

$$X_{11'} = d; \quad Y_{1'2} = b = A'''_{12}; \quad Y_{1'3} = a = A'''_{13}; \quad Y_{23} = ab = A'''_{23}.$$

Fourth stage: (I) No. 2;

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$$X_{22'} = b; \quad Y_{12'} = 1 = A_{12}^{(4)}; \quad Y_{13} = a = A_{13}^{(4)}; \quad Y_{2'3} = a = A_{23}^{(4)}.$$

Fifth stage: (I) No. 3;

$$X_{33'} = a; \quad Y_{12} = 1; \quad Y_{13'} = 1; \quad Y_{23'} = 1.$$

We have obtained constants corresponding to a three-terminal network whose terminals are shorted to one another. Taking into account the order of extraction of the contacts and the terminals from which they were extracted, we construct the circuit of Fig. 2.

Example 2. Consider an example of the synthesis of a contact nonrepeating three-terminal network. Let the Boolean functions be given:

$$A_{12} = bed+abeh+ceh+fh+acde+adf+bcdf; \quad A_{13} = h+ad+bcd+bdef; \quad A_{23} = f+ce+abe+bdeh.$$

We apply system (II), Nos. 1-3, under the assumption $X_{13} = h$. The solution (unique in this case) has the form $Y_{1'3'} = ad + bcd + bdef = A'_{13}$; $Y_{1'2} = bde + acde + adf + bcdf = A'_{12}$; $Y_{23'} = f + ec + abe = A'_{23}$. And so on. 2nd stage: (I),

Fig. 2
Fig. 2

Fig. 3
Fig. 3

Fig. 4
Fig. 4

No. 1; $X_{11'} = d$; $Y_{1'2} = be + ace + af + bcf = A''_{12}$; $Y_{1'3} = a + bc + bef = A''_{13}$; $Y_{23} = f + ec + abe = A''_{23}$. 3rd stage: (II), Nos. 1-3; $X_{13} = a$; $Y_{1'3'} = bc + bef = A'''_{13}$; $Y_{1'2} = be + bcf = A'''_{12}$; $Y_{23'} = f + ec = A'''_{23}$. 4th stage: (I), No. 1; $X_{11'} = b$; $Y_{1'2} = e + cf = A_{12}^{(4)}$; $Y_{1'3} = c + ef = A_{13}^{(4)}$; $Y_{23} = f + ec = A_{23}^{(4)}$. 5th stage: (II), Nos. 1-2; $X_{12} = e$; $X_{1'2'} = cf = A_{12}^{(5)}$; $Y_{1'3} = c = A_{13}^{(5)}$; $Y_{2'3} = f = A_{23}^{(5)}$. 6th stage: (I), No. 1; $X_{11'} = c$; $Y_{1'2} = f = A_{12}^{(6)}$; $Y_{1'3} = 1 = A_{13}^{(6)}$; $Y_{23} = f = A_{23}^{(6)}$. 7th stage: (I), No. 2; $X_{22'} = f$; $Y_{12'} = 1$; $Y_{2'3} = 1$; $Y_{13} = 1$.

Using the selected contacts, we construct the circuit of Fig. 3.

Example 3. Consider an example of synthesis of a contactless two-terminal network. Let the Boolean function be given as

$$A_{12} = abc + abfn + acemn + aefm + bcdm + bdfmn + cden + def.$$

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We apply system (III), No. 1, under the assumption $X_{13} = a$. The solution is $Y_{1'2} = bcdm + bdfmn + cden + def = A'_{12}$; $Y_{23} = bc + bfn + cemn + efm + DA'_{12} = A'_{23}$. The symbol $DF[x \ldots y]$ means that from the function F one may take any part in which the variables $x, \ldots, y$ do not occur. We leave arbitrary $Y_{1'3} = \Pi_{13} = A'_{13}$ (with the proviso that subsequently for $Y_{1'3}$ only values satisfying inequalities (2) will be taken). 2nd stage: set $Y_{1'3} = A'_{13} = d\Pi'_{13}$, solve system (I), No. 1; $X_{11'} = d$; $Y_{1'2} = bcm + bfmn + cen + ef = A''_{12}$; $Y_{23} = bc + bfn + cenm + efm + DA'_{12}[d] = A''_{23}$; $(DA'_{12}[d] = 0)$; $Y_{1'3} = \Pi_{13} = A'_{13} = m + \Pi'_{13}$. 3rd stage: (II), Nos. 1-3; $X_{13} = m$; $Y_{1'2} = cen + ef = A'''_{12}$; $Y_{23'} = bc + bfn = A'''_{23}$; $Y_{1'3'} = \Pi'_{13} = A'''_{13} = e\Pi'''_{13}$. 4th stage: (I), No. 1; $X_{11'} = e$; $Y_{1'2} = cn{+}f = A^{(4)}_{12}$; $Y_{23} = bc{+}bfn = A^{(4)}_{23}$; $Y_{1'3} = \Pi'''_{13} = A^{(4)}_{13} = b\Pi^{(4)}_{13}$. 5th stage: (I), No. 3; $X_{33'} = b$; $Y_{12} = cn + f = A^{(5)}_{12}$; $Y_{23'} = c + fn = A^{(5)}_{23}$; $Y_{13'} = \Pi^{(4)}_{13} = A^{(5)}_{13} = n + \Pi^{(5)}_{13}$. 6th stage: (II), Nos. 1-3; $X_{13} = n$; $Y_{1'2} = f = A^{(6)}_{12}$; $Y_{23'} = c = A^{(6)}_{23}$; $Y_{1'3'} = \Pi^{(5)}_{13} = A^{(6)}_{13} = f\Pi^{(6)}_{13}$. 7th stage: (I), No. 1; $X_{11'} = f$; $Y_{1'2} = 1 = A^{(7)}_{12}$; $Y_{23} = c = A^{(7)}_{23}$; $Y_{1'3} = \Pi^{(6)}_{13} = A^{(7)}_{13} = c\Pi^{(7)}_{13}$. 8th stage: (I), No. 3; $X_{33'} = c$; $Y_{13'} = 1$; $Y_{23'} = 1$; $Y_{12} = 1$.

Using the selected contacts, we construct the circuit of Fig. 4.

It follows from the last example that synthesis can also be carried out in the case where not all functions $A_{\alpha\beta}$ are specified, but only some part of them.

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Note: Figure translations are in progress. See original paper for figures.

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