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Abstract

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ON THE SO-CALLED "PERIOD–LUMINOSITY RELATION" FOR CEPHEIDS

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The revision of the zero point of the period–luminosity relation for classical Cepheids, arrived at as a result of Baade' s observations with the 200-inch telescope (^2), showed how unreliable the previous statistical determinations of the zero point of the period–luminosity relation from proper motions and trigonometric parallaxes had been, and how excessively optimistic the authors of these early works were in estimating the error limits of the zero-point values they found (^1).

In the theory of stellar variability that we constructed by 1953 ([3-]11), we encountered the difficulty that the former value of the zero point of Shapley' s period –luminosity relation for classical Cepheids proved to be incompatible with the theoretical value*. Baade' s revision of the zero point removed this difficulty. **However, one more difficulty remained in our theory. Namely, the theory led to the conclusion that the period–luminosity relation in the form of a curve or of a sufficiently narrow band on the plane of variable periods (P) and median luminosity (L_0), contrary to the assertions of observational astronomers, does not exist, and that instead of a curve there should be a broad band corresponding, for a given period, to a scatter of luminosity values of $1^m.2$. This circumstance (or the equivalent circumstance–that Cepheids, according to our theory, should occupy in the spectrum–luminosity diagram not a narrow but a broad band) was discussed as a** shortcoming** of our theory in (^6, ^8, ^9).

In December 1957, at the 90th meeting of the American Astronomical Society, Sandage reported on his investigation of the period–luminosity relation for Cepheids (^15). According to Sandage, it turned out that a given value of the period corresponds to a scatter in luminosity of $1^m.2$, i.e. instead of a period–luminosity **curve** there is in reality a **band** $1^m.2$ wide. Thus, the conclusion obtained from theory concerning a luminosity scatter of $1^m.2$ unexpectedly for us proved to be confirmed by Sandage' s results, based on observational material.

In Fig. 1 the results of Sandage for galactic Cepheids are reproduced (^15). Two groups of Cepheids, one of which consists of Cepheids of class C, and the

Fig. 1

Figure 1: Fig. 1

other of Cepheids of classes A and B according to Eggen' s classification (^16, ^17)***,

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* The theory of stellar variability, in conjunction with Shapley' s zero point, led to excessively small masses of classical Cepheids, of the order of $1\,M_\odot$ (^9).

** With Baade' s value of the zero point, the theory of stellar variability leads to masses of classical Cepheids about ~ 1.6 times smaller than follows from the mass–luminosity law for main-sequence stars, which is in agreement both with the period–density relation (^12, ^13) and with Tissen' s work (^14).

*** According to (^17), class C includes galactic Cepheids possessing a relatively symmetric light curve, close to sinusoidal, in which the time between the onset of minimum and maximum light is $0.37\,P$; if this time is greater than $0.37\,P$, then Cepheids possessing the main asymmetric light curve (as in δ Cep) belong to class A, while Cepheids possessing an asymmetric light curve with a secondary maximum (as in η Aquil) belong to class B. Cepheids of class C have smaller amplitudes of light and radial-velocity variations than Cepheids of classes A–B ([16-]17).

have, for a given value of the period, different luminosities (the regions of Cepheids with the corresponding values of the periods in days (d) are marked in Fig. 1 by strokes). If separate period-luminosity relations are introduced for Cepheids of class C and of classes A-B, then the scatter of the luminosity values in these relations can be reduced.

Fig. 1

We shall now show how the theory of stellar variability can explain the results obtained by Sandage. To this end let us consider the graphs in Fig. 2, which give the dependence on the parameter y_3 of the nonadiabaticity of the oscillations of the HeII critical-ionization zone of a variable star of the quantity ψ (solid curves) and of the quantity d (dash-dot curves). The quantity ψ is the lag of the phase of maximum brightness behind the phase of maximum compression of the star (for Cepheids $\psi \simeq +90^\circ$, for long-period variables of the Mira Ceti type $\psi \simeq -90^\circ$); d is the ratio of the amplitude of the radiation flux at the exit from the star to the amplitude of the radiation flux at the entrance into the ionized zone. The curves are drawn for different values of the ratio of the mass of the atmosphere m_a to the mass of the zone m_3 ([11])*. It is seen from Fig. 2 that variability of the Cepheid type corresponds to a wide interval of values of $\lg y_3$:

$$-0.1 < \lg y_3 < 0.4$$

Fig. 2

Figure 2: Fig. 2

(we use the middle curve for $m_a/m_3 = 0.894$). We shall now see that the width of this interval also determines that scatter of the luminosities of Cepheids which can exist at a given period P.

Fig. 2

In order to establish the connection between the scatter of the values of the parameter y_3 and the scatter of the values of the mean luminosity L_0, let us suppose that: a) the variable stars have homologous internal structures; b) the envelopes of the variable stars contain one and the same amount of helium; c) the degree of ionization of HeII does not depend on the gas pressure and is determined only by the temperature; d) the envelopes of the variable stars are built according to a polytrope of index n. Then, for a given period,

$$P \sim \sqrt{R^3/M}$$

$$R \sim \sqrt[3]{M}, \quad (1)$$

where R and M are the radius and mass of the star. The temperature in the envelope of the star can be represented by the formula $T \sim \frac{M}{R}u$, where u is an Emden function of index n.

In consequence of the assumption of the independence of ionization from pressure, the mean temperature of the critical-ionization zone, in passing from one star to another, must remain the same. Hence for the quantity u in

* Fig. 2 differs from Fig. 1 of paper ([11]) in that, as a result of a refinement of the calculation, the dotted parts of the curves corresponding to positive dissipation of the energy of the star' s oscillations begin at somewhat different values of the parameter y_3; moreover, in ([11]) d denoted the ratio of the amplitude of the radiation flux at the exit from the star not to the amplitude of the radiation flux at the entrance into the zone (as in the present paper), but to the maximum value of the radiation flux reached at some distance before the entrance into the ionized zone (the boundaries are still determined by the condition $\gamma_1 - 1 - (d\ln T/d\ln\rho)_{\text{ad}} < 0.26$).

zone we have

$$u \sim \frac{R}{M}. \quad (2)$$

Taking into account that in the envelope of the star the temperature gradient is almost constant and is proportional to the acceleration of gravity g, by virtue of (1), (2) we find

$$\frac{dT}{dr} \sim g \sim \frac{M}{R^2} \sim M^{1/3}, \tag{3}$$

$$\rho \sim \frac{M}{R^3} u^n \sim M^{-2n/3}, \tag{4}$$

where ρ is the mean density of the HeII critical ionization zone. The nonadiabaticity parameter of the oscillations of the zone y_3, other conditions being equal (chemical composition, period, etc.), is proportional to the luminosity L_0 and inversely proportional to the mass of the zone $\Delta M = 4\pi r^2 \rho d$ (5), where $d \sim \frac{1}{dT/dr} \sim M^{-1/3}$ is the thickness of the zone. Hence, by virtue of (1), (3), (4), we have

$$y_3 \sim \frac{L_0}{\Delta M} \sim \frac{L_0}{\rho R^2 d} \sim M^{\frac{2n-1}{3}} L_0. \tag{5}$$

In the spirit of the theory of evolution of stars of constant mass, it is natural to assume that the luminosity of the star L_0 does not depend on mass and that the masses of Cepheids, at least within their individual groups (short-period, long-period, etc.), are approximately the same (the latter assumption was advanced by S. A. Melnikov (18)). Then (5) becomes

$$y_3 \sim L_0. \tag{6}$$

It follows from (6) that the interval of values $\Delta \lg L_0$ corresponding to the interval of values $\Delta \lg y_3 = 0.5$, in which, according to the graphs of Fig. 2, Cepheid-type variability occurs ($-0.1 < \lg y_3 < 0.4$), is equal to $\Delta \lg L_0 = \Delta \lg y_3 = 0.5$. Passing to stellar magnitudes, we obtain the interval of values Δm in which, for a given period P, Cepheid-type variability occurs:

$$\Delta m = 2.5\, \Delta \lg L_0 = 2.5 \cdot 0.5 = 1^m 25$$

in complete agreement with the value 1^m2 obtained by Sandage from observational data.

If, instead of the assumption that there is no relation between the stellar mass and luminosity, we were to assume that $L_0 \sim M^3$, then for $n = 3$, according to (5), we would obtain a scatter of luminosities $\Delta m = 0^m 80$. Thus, one should expect that the true scatter must lie within the limits $0^m 8 < \Delta m < 1^m 25$; moreover, since a strong dependence of luminosity on mass of the type $L_0 \sim M^3$

is hardly satisfied for Cepheids, it is most likely that the scatter should be close to its limit 1^m25.

Since the nonadiabaticity parameter of the oscillations of the ionized zone y_3 is proportional to the luminosity, and since, as y_3 increases, when $\lg y_3$ approaches the value 0.4, the amplitude of the self-oscillations of Cepheids must approach zero (for at $\lg y_3 \simeq 0.4$ the dissipation of the energy of the star' s oscillations becomes zero, and for $\lg y_3 > 0.4$ the curves of Fig. 2 are drawn with a dotted line, which means that self-oscillations are impossible because of the positive dissipation of the energy of the star' s oscillations), then, for a given period, Cepheids possessing greater luminosity must have a smaller amplitude of self-oscillations than Cepheids of the same period with smaller luminosity, i.e., they must belong to Eggen' s class C. Conversely, Cepheids of relatively lower luminosity must have a larger amplitude of self-oscillations and therefore belong to Eggen' s classes A–B. This explains the

Sandage' s result that class C Cepheids have greater luminosities than class A–B Cepheids of the same period (Fig. 1). It also follows from the theory, again in agreement with observations, that Cepheids of greater luminosity must be bluer than Cepheids of smaller luminosity of the same period.

In paper (19), Arp, who found a scatter in luminosity of $B \sim 1^m$ among Cepheids with a given value of the period in globular clusters, raises the question: are we dealing with several discrete lines on the period–luminosity diagram (each corresponding to its own value of the zero point), or with a broad band?—and for a number of reasons he inclines toward the first assumption.

However, since the scatter in luminosity found by Arp for Cepheids in globular clusters does not exceed the "natural" scatter of $1^m.25$, it follows from what has been said above that these Cepheids constitute a homogeneous group in the sense that conditions a), b) are fulfilled for them. Therefore, in the present case there is hardly any basis for speaking of a multiplicity of zero points, as Arp does and as other authors do in a number of analogous cases. In reality we have a broad band.

Since the scatter in luminosity for Cepheids in globular clusters (population II) and for classical Cepheids (population I) of identical periods is, for both populations as a whole, about 2^m, i.e., exceeds the "natural" scatter, it follows that conditions a), b) for the two populations as a whole are not fulfilled, i.e., these populations cannot be combined into a homogeneous group. Therefore on the $P - L^0$ diagram we shall have for them two different bands (superposed one upon the other). If desired, two discrete zero points could be assigned to the mean lines of these bands.

If it is possible, in order to reduce the scatter in luminosity, to introduce different zero points within a homogeneous group of Cepheids for individual subgroups characterized by one feature or another (for example, belonging to Eggen' s classes C and A–B), then these zero points cannot be discrete.

It is evident from Fig. 2 that Cepheids with a phase shift $\psi = 50 \div 60^\circ$ should have greater luminosities (since $y_3 \sim L_0$) than Cepheids with a phase shift $\psi > 90^\circ$ of the same period. Observational data (20) show that such a correlation does indeed exist. Therefore, in order to reduce the scatter in luminosity, it would be possible to divide Cepheids into subgroups with different zero points, being guided not by the degree of asymmetry of the light curve or by the amplitudes of the light and radial-velocity variations, but by the magnitude of the phase shift between the light variations and the radial-velocity variations.

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Note: Figure translations are in progress. See original paper for figures.