A METHOD FOR DETERMINING THE MEAN KINETIC ENERGY OF ELECTRONS IN THE CASE OF THE DRUYVESTEYN-DAVYDOV DISTRIBUTION

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Abstract

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A METHOD FOR DETERMINING THE MEAN KINETIC ENERGY OF ELECTRONS IN THE CASE OF THE DRUYVESTEYN-DAVYDOV DISTRIBUTION

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In probing the positive column of electric discharges in neon and in a mixture of neon and mercury ([1,2]) in a tube of radius 1.2 cm, at a neon pressure of about 1 mm Hg and at a discharge current of 2 mA, we observed that, when the semilogarithmic characteristics of the mixture, even at a partial mercury pressure of only $3 \cdot 10^{-4}$ mm Hg, had a rectilinear portion from which the electron temperature in pure neon could be estimated, the characteristics obtained differed sharply from a straight line, having a form close to a parabola. It is known that the theory of electron motion in a gas ([3]) requires that in gases for which the effective cross section does not depend on the electron velocity, the velocity distribution that is established, if elastic collisions play the principal role, should be of the Druyvesteyn-Davydov type. In view of the fact that, in our determinations, the ratio of current density to the square of the gas pressure j/p^2 is approximately three orders of magnitude smaller than in other determinations ([4]) made in neon, in which linear characteristics were obtained, we decided that, owing to the smallness of this ratio, conditions exist in which the aforementioned theory is applicable*. The calculation below makes it possible to verify, using as experimental data the points obtained in ordinary probing with a Langmuir probe, whether a Druyvesteyn-Davydov velocity distribution exists in the plasma, and also to clarify the possibility of determining the quantities of interest to us: the mean kinetic energy of the electrons V_m, the plasma potential V_1, and the number N of charge carriers in 1 cm^3.

For the electron current in a plasma to a plane or spherical probe at a lower potential than the plasma potential ([7]), if the velocity distribution is isotropic, one may write

$$I = \pi e_0 N S \int_{\sqrt{2e_0V_0/m}}^{\infty} c^3 f(c) \left(1 - \frac{2e_0 V_0}{mc^2}\right) dc. \qquad (1)$$

Here e_0 is the electron charge; c is its velocity; N is the number of electrons in 1 cm^3; S is the probe area; $V_0 = V_1 - V_s$ is the difference between the potentials of the plasma and the probe; $f(c)$ is the electron-velocity distribution function. It is easy to prove ([2]) that the same relation is valid for a cylindrical probe.

* In the work of Selig and Gierh ([5]), carried out over a wide range of j/p^2, the logarithmic probe characteristics obtained in neon were also linear. It should be noted, however, that in the presence of cumulative phenomena, which are decisive for establishing the electron-velocity distribution ([6]), the similarity laws are no longer valid. It is necessary to compare only the current densities, which for the cited authors exceed by approximately a factor of 25 the densities used by us, and this may explain the discrepancy of the results.

In the case when a velocity distribution of the Druyvesteyn-Davydov type ([8]) occurs in the region being probed, we substitute in (1), instead of the distribution function,

$$f(c) = \frac{1.94}{4\pi c_p^3} \exp\left[-\frac{1}{2}\frac{c^4}{c_p^4}\right], \tag{2}$$

where c_p is the most probable velocity. We obtain

$$I = \frac{1.94}{2mc_p^2} e_0 NS \int_{e_0V_0}^{\infty} W \exp\left[-\frac{1}{2}\frac{W^2}{W_p^2}\right] \left(1 - \frac{e_0V_0}{W}\right) dW, \tag{3}$$

where W is the electron energy and W_p is the most probable energy. Denoting

$$x = \frac{W}{\sqrt{2}W_p}; \qquad y = \frac{e_0V_0}{\sqrt{2}W_p}; \qquad \varphi(y) = \frac{e^{-y^2}}{\sqrt{\pi}}; \qquad \Phi(y) = \frac{1}{\sqrt{\pi}} \int_{-y}^{+y} e^{-x^2} dx \tag{4}$$

and integrating (3), we obtain

$$I \sim -\varphi(y) + y[1 - \Phi(y)].$$

With the aid of tables and graphs, the difference on the right-hand side can be represented in the form of the function $\varphi(y')$. The relation between y' and y is shown in Fig. 1. It can be seen that over a large portion this relation is practically linear. Therefore, in a first approximation one may put

$$y' = 0.61 + 0.958y. \tag{5}$$

Relation (5) was obtained graphically from the linear portion of the dependence curve. In this approximation one may write

$$I \sim e^{-{y'}^2},$$

i.e., the form of the semilogarithmic characteristic for this type of distribution should be very close to a parabola. Substituting y' from (5) and y from (4), and taking into account that $V_0 = V_l - V_s$, then taking logarithms, we obtain

$$\lg I = -\frac{0.199}{V_p^2} V_s^2 + \frac{1}{V_p}\left(\frac{0.398}{V_p} V_l - 0.359\right) V_s + C_0 =$$

$$= C_2 V_s^2 + C_1 V_s + C_0, \quad (6)$$

where V_p is the most probable energy of the electrons in electron-volts.

Having determined (for example, by the least-squares method) the coefficients C_2 and C_1 of the experimentally obtained parabola, one can calculate the most probable energy V_p, as well as the plasma potential V_l. In addition, there must exist a point α with respect to which the representation

$$\log I = f[(V_s - \alpha)^2] = f(v_s^2)$$

gives a straight line.

The value of α is obtained by substituting in (6)

$$V_s = v_s + \alpha$$

and setting equal to zero the term of first degree with respect to v_s. We obtain

$$\alpha = V_l - 0.902 V_p.$$

If the line $\log I = f(v_s^2)$ is a straight line, we have proof that the Druyvesteyn-Davydov distribution really exists in the region being probed.

After determining, in the first approximation, the potential of the probed region, we obtain the quantity V_0, and consequently also y, for each point of determination, and we can apply corrections, taking into account the curved dependence in Fig. 1. Drawing a vertical through the corresponding point to its intersection with the straight line (5), and drawing through the point of intersection a straight line parallel to the abscissa axis to its intersection with the true curve of dependence, we graphically obtain Δy and, correspondingly, ΔV_s. Adding these corrections to the experimentally obtained values V_s, we obtain a new series of values V_s', with the aid of which the coefficients of the parabola are calculated in the second approximation. The operation is repeated until the desired accuracy is achieved in determining the limits to which V_p, V_1, and a' converge.

Fig. 1. Course of the functions $\varphi(y)$ and $y'(y)$

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As in the Maxwellian case, the electron current is determined by extrapolating the ion current. The choice of points that must be used in determining the coefficients of the parabola in the first approximation presents certain difficulties, since the potential of the region is not seen directly on the semilogarithmic characteristic. Therefore preliminary trials are sometimes necessary either in order not to include in the calculation values obtained at a probe potential more positive than the plasma potential, or in order not to omit some points lying at a lower potential, which would considerably reduce the accuracy.

When the approximation in determining the potential of the region becomes satisfactory, the number of carriers per 1 cm^3 can be calculated. Indeed, for $V_0 = 0$, from (3), by integration, we obtain

$$I = \frac{1.94}{8} e_0 N S c_p,$$

whence, from the calculated most probable velocity and from the measured current at the potential of the region, N is determined.

In applying this method to experimental results obtained under various conditions, from 2 to 4 approximations were needed to obtain the potential of the region with an accuracy up to 1 V and the electron energy with an error not exceeding 3%.

In Fig. 2, A is shown an example of a semilogarithmic probe characteristic obtained in the positive column of neon at a pressure of 1.31 mm Hg and a discharge current of 1.5 mA. Curve B reproduces the section of extrapolation of the ion current used to determine the electron current. Curve C is $\log I = f(v_s^2)$ before introducing the correction by means of Fig. 1 (points a) and after introducing correction (6). The course of the function is linear within the experimental errors. The value of the most probable electron energy determined by the method described is $V_p = 6.7$ eV.

To check this result, we compare it with the mean kinetic energy of the electrons, which for our operating conditions follows from the application of Schottky' s theory ([9]). The path that we used coincides with that followed by Engel and Steenbeck ([10]), with the difference that, instead of the Maxwell distribution function, the Druyvesteyn distribution function was applied.

Druivesteyn–Davydov. Taking the values of the various constants for neon from (10) and putting $p = 1.31$ mm Hg, $R = 1.2$ cm, we obtained the equation

$$1.84 \cdot 10^2 \frac{1}{V_p^{5/2}} \int_{21.5}^{\infty} (V - 21.5) V \exp\left[-\frac{1}{2}\frac{V^2}{V_p^2}\right] dV = 1.$$

Fig. 2

Figure 2: Fig. 2

Here V is the electron energy; V_p is their most probable energy in electron-volts. Carrying out a graphical integration for several values of V_p, we obtained the value satisfying the equation, $V_p = 6.9$ eV.

Fig. 2. Probe characteristic obtained in neon at a pressure of 1.31 mm Hg and at a discharge-current density of $3.34 \cdot 10^{-4}$ A/cm^2. A—semilogarithmic characteristic of the electron current, B—ion current, C—logarithm of the electron current as a function of $(V_s - \alpha)^2$: a—before introducing the correction, b—after introducing the correction

If, however, one assumes that the electron velocity distribution is Maxwellian and determines the most probable energy from the approximately linear segment between 54 and 47 V (relative to the anode), then one obtains $V_p = 3.4$ eV. Applying the same diffusion theory under the assumption of a Maxwellian distribution, in our conditions we obtain the equation

$$4.27 \cdot 10^2 V_p^{1/2} \left(1 + \frac{21.5}{2V_p}\right) e^{-21.5/V_p} = 1,$$

whose solution is $V_p = 2.6$ eV.

The very good agreement in the first assumption and the unsatisfactory agreement in the second is an argument in favor of applying the described method of processing a semilogarithmic characteristic of this type.

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Note: Figure translations are in progress. See original paper for figures.