ON SHOCK WAVES IN MEDIA WITH AN ARBITRARY EQUATION OF STATE

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Abstract

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Straight discontinuities are considered in media whose internal energy is completely determined by the value of the specific volume and the entropy.

If the function $E = E(V, S)$ is known, then the equation

$$dE = T\,dS - p\,dV \tag{1}$$

gives two more relations between the five quantities: the specific volume V, the pressure p, the absolute temperature T, the internal energy E, and the entropy S, which determine the state of the medium. Three finite equations between V, p, T, E, S describe all the mechanical and thermal properties of the body (since reversible processes are in question). The form of the function $E = E(V, S)$ cannot be established by thermodynamic reasoning, but thermodynamics imposes a number of restrictions on the relations between V, p, T, E, S (see, for example, ([8])). What cannot be obtained by thermodynamic reasoning in the relation between p, V, S is what defines the term "arbitrary equation of state." Thus, in particular, the sign of the derivative $(\partial^2 p/\partial V^2)_S$, which substantially affects the character and properties of shock transitions, remains arbitrary. Straight discontinuities in media for which the sign of $(\partial^2 p/\partial V^2)_S$ is constant throughout the whole region have been studied in detail. (An exposition of the question, together with historical and bibliographical references, may be found in ([1–5]).)

The known methods of theoretical investigation are based on differential relations that hold along the Hugoniot adiabat, and make it possible to carry out the investigation completely when the sign of $(d^2 p/dV^2)_S$ is constant. In the case where the sign of $(\partial^2 p/\partial V^2)_S$ changes, the data supplied by these methods prove insufficient for determining the form and properties of the shock adiabats.

In the present communication additional information about shock adiabats is set forth which, together with known facts and methods, makes it possible to construct Hugoniot adiabats and to investigate the properties of the shock transition for media with an arbitrary equation of state, when the derivative $(\partial^2 p/\partial V^2)_S$ changes sign along Poisson adiabats.

Let, as a result of a process schematized by a surface of strong discontinuity, a particle pass from state 1 to state 2. The characteristics pertaining to these

Fig. 1

Figure 1: Fig. 1

Fig. 2

Figure 2: Fig. 2

states (we shall conventionally denote them by the indices 1 and 2) are connected by three relations following from the conservation laws ([2]):

$$\frac{w_1}{V_1} = \frac{w_2}{V_2} = j, \tag{2}$$

$$j^2 = \frac{p_2 - p_1}{V_1 - V_2}; \tag{3}$$

$$E_2(V_2, p_2) - E_1(V_1, p_1) = \frac{1}{2}(p_1 + p_2)(V_1 - V_2). \tag{4}$$

Here w is the velocity of the surface of discontinuity relative to the medium, and j is the density of the flow through the surface of discontinuity.

For fixed V_1, p_1 and variable V_2, p_2, (4) gives on the V-p diagram the graph of the Hugoniot adiabat with center (V_1, p_1). To (2)–(4) there must be added a condition reflecting the irreversible character of the process:

$$S_2 \geqslant S_1. \tag{5}$$

Let us compare the transition under consideration with a reversible process. We shall calculate the increment of internal energy corresponding to the shock transition, carrying the particle reversibly from state 1 to state 2, and we shall do this in two ways, as shown in Fig. 1. As a result, according to (1), we shall have:

Fig. 1

Fig. 2

$$E_2 - E_1 = \int_{S_1}^{S_2} T\Big|_{V=V_2} dS - \int_{V_1}^{V_2} p\Big|_{S=S_1} dV; \tag{6}$$

$$E_2 - E_1 = \int_{S_1}^{S_2} T\Big|_{V=V_1} dS - \int_{V_1}^{V_2} p\Big|_{S=S_2} dV. \tag{7}$$

Substituting (6) and (7) in turn into (4) and taking (5) into account, we obtain the restrictions imposed by condition (5) on the change of the mechanical characteristics in the shock wave:

$$\left\{\frac{p_1+p_2}{2}(V_1-V_2)-\int_{V_2}^{V_1} p\bigg|_{S=S_1} dV\right\} \geqslant 0; \tag{8}$$

$$\left\{\frac{p_1+p_2}{2}(V_1-V_2)-\int_{V_2}^{V_1} p\bigg|_{S=S_2} dV\right\} \geqslant 0. \tag{9}$$

To clarify the properties of shock transitions, a geometrical interpretation of inequalities (8), (9) is useful (see Fig. 2). In (4) only inequality (8) is indicated. Let us note that, for $(\partial p/\partial S)_V > 0$ and $V_1 > V_2$, a necessary and sufficient condition for the entropy not to decrease is the fulfillment of inequality (9), while inequality (8) will be satisfied as a consequence of (9); for $V_1 < V_2$, (9) will be a consequence of (8). For $(\partial p/\partial S)_V < 0$ the inequalities exchange roles. Since the whole range of variation of V in which the shock adiabat is defined is of interest, it is necessary to keep both inequalities in mind.

Let us pose the question: can the Hugoniot adiabat have more than one common point with the Poisson adiabat? The answer to this question is given by the following proposition:

Theorem 1. *If the Hugoniot adiabat with center (V_1, p_1) has, with the Poisson adiabat $S = S_2$, a common point (V_2, p_2) and, moreover, on the adiabat*

of Poisson there are points (V_i, p_i) satisfying the condition

$$\frac{p_1+p_i}{2}(V_1-V_i)=\frac{p_1+p_2}{2}(V_1-V_2)+\int_{V_i}^{V_2} p\bigg|_{S=S_2} dV, \tag{10}$$

then the shock adiabat will pass through the point (V_i, p_i); they will have no other common points. The point (V_1, p_1) will lie on the shock adiabats with centers (V_2, p_2) and (V_i, p_i).

Indeed, since, by assumption,

$$E_2-E_1=\frac{p_1+p_2}{2}(V_1-V_2)$$

and, according to (1),

$$\int_{V_i}^{V_2} p\bigg|_{S=S_2} dV = E_i - E_2,$$

Fig. 3

Figure 3: Fig. 3

it follows from (10) that

$$E_i - E_1 = \frac{p_1 + p_i}{2}(V_1 - V_i).$$

Assuming that the shock adiabat passes through a point of the Poisson adiabat that does not satisfy condition (10), we arrive at a contradiction.

If the points (V_1, p_1) and (V_2, p_2) coincide, then instead of (10) we shall have

$$\frac{p_1 + p_i}{2}(V_1 - V_i) = \int_{V_i}^{V_1} p\bigg|_{S=S_1} dV. \tag{11}$$

If the Poisson adiabat is a segment of a straight line on some interval, then, as is seen from (11), the Hugoniot adiabat with center on this segment coincides with the latter on the interval under consideration.

Relying on Theorem 1, one can decide the question of the possibility of compression and rarefaction shock waves. When everywhere $(\partial^2 p/\partial V^2)_S > 0$ (or $(\partial^2 p/\partial V^2)_S < 0$), the answer for the entire region is unambiguous and is given by Zemplén' s theorem. If, however, the second derivative $(\partial^2 p/\partial V^2)_S$ changes sign, then an unambiguous answer for the entire region cannot be given. At the center $(\partial p/\partial V)_\Gamma = (\partial p/\partial V)_S < 0$, and, if the first nonzero higher derivative is $(\partial^n p/\partial V^n)_S$, then $(\partial S/\partial V)_\Gamma = ... = (\partial^n S/\partial V^n)_\Gamma = 0$,

$$\left(\frac{\partial^{n+1} S}{\partial V^{n+1}}\right)_\Gamma = -\frac{1}{2T}\left(\frac{\partial^n p}{\partial V^n}\right)_S.$$

Fig. 3

For a given initial state, these relations determine the relative position of the Hugoniot and Poisson adiabats at the center; Theorem 1 makes it possible to judge their mutual position away from the center. It must be borne in mind that, as is known ([2]), the Hugoniot adiabat cannot intersect the straight lines $V = V_1$, $p = p_1$ outside the center (V_1, p_1).

If in the region $V_2 \leq V \leq V_1$ the Hugoniot adiabat has more than one common point with one of the Poisson adiabats, then in this region the derivative $(\partial^2 p/\partial V^2)_S$ changes sign along the Hugoniot adiabat.

The converse assertion is false.

Let us note also the following properties of shock adiabats.

Fix the point (V_1, p_1) and draw through it a shock adiabat and a ray. If this ray intersects the shock adiabat at the points (V_2, p_2), (V_3, p_3), then the points (V_k, p_k) $(k = 1, 2, 3)$ will lie simultaneously on each of the shock adiabats with center (V_k, p_k).

The converse assertion is also true: if three points simultaneously lie on each of the shock adiabats with centers at these points, then the latter lie on one straight line.

Construct shock adiabats with centers at the points (V_i, p_i) $(V_B \leq V_i \leq V_A)$ of the Poisson adiabat $S = S_1$ (Fig. 3). The corresponding compression branch of each shock adiabat will intersect the tangent constructed at the center at the point (V, p). The totality of such points forms a continuous curve defined by the equations

$$p - p_i(V_i) - \left(\frac{\partial p}{\partial V}\right)_{V=V_i} (V - V_i) = 0,$$

$$E(V, p) - E[V_i, p_i(V_1)] + \frac{1}{2}(V - V_i)[p + p_i(V_i)] = 0. \qquad V_B \leq V_i \leq V_A.$$

This curve intersects each of the horizontals $p = \text{const}$ at only one point.

Along the Hugoniot adiabat

$$\left[T\left(\frac{\partial S}{\partial p}\right)_V - \frac{V_1 - V}{2}\right]\left(\frac{\partial p}{\partial V}\right)_\Gamma + \left[T\left(\frac{\partial S}{\partial V}\right)_p - \frac{p - p_1}{2}\right] = 0.$$

Since $(\partial S/\partial p)_V$ and $(\partial S/\partial V)_p$ have the same sign, $(\partial p/\partial V)_\Gamma < 0$ in the rarefaction region $(V_1 < V)$ when $(\partial p/\partial S)_V > 0$, and in the compression region $(V_1 > V)$ when $(\partial p/\partial S)_V < 0$. In the compression region when $(\partial p/\partial S)_V > 0$ (rarefaction when $(\partial p/\partial S)_V < 0$), the derivative $(\partial p/\partial V)_\Gamma$ may be either negative or positive. In particular, a case is in principle possible in which at the terminal point

$$\left(\frac{\partial S}{\partial p}\right)_V = \frac{V_1 - V}{2T},$$

and the corresponding compression branch of the shock adiabat will have the form shown in Fig. 4. This case is regarded in ([6]) as unlikely; in ([6]) it is asserted that the equality

$$\left(\frac{\partial S}{\partial p}\right)_V = \frac{V_1 - V}{2T}$$

Fig. 4

Figure 4: Fig. 4

cannot occur at the terminal point, but the reasoning contains an inaccuracy.

Fig. 4

For the media under consideration, for a known initial state and a prescribed velocity of the discontinuity surface or gas velocity, relations (2)–(4) do not always uniquely determine the shock transition.

It is easy to see that what has been set forth above is also applicable to solids, when the elementary work of the internal forces is represented in the form of a monomial $X\,dy$. The propagation of instantaneous disturbances in media with a nonlinear dependence of stress on strain $\sigma = \sigma(e)$, when everywhere $(\partial\sigma/\partial e) > 0$, $\partial\sigma/\partial S = 0$, and $\partial^2\sigma/\partial e^2$ changes sign, was first considered in ([7]).

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Note: Figure translations are in progress. See original paper for figures.