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Abstract

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PHYSICS

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DISPERSION RELATIONS FOR THE PRODUCTION OF π-MESONS

(Presented by Academician N. N. Bogolyubov on 13 VI 1958)

In the present work, following ([1,2]), we have obtained dispersion relations for the process $\pi + N \to 2\pi + N$ for two different spin and isotopic states ("non-spin-flip," isotopic spin 3/2).

We fix the reference system by means of the condition $\mathbf{p}' + \mathbf{p} = 0$ and, for simplicity, confine ourselves to the case in which the energies of the emitted mesons are equal, while the difference of their momenta $\mathbf{q}' - \mathbf{q}''(= 2\vec{\Delta} = 2\delta\mathbf{e}_\Delta)$ is constant and perpendicular to the nucleon momentum ($\vec{\Delta}\mathbf{p} = 0$). Then all four-momenta of the process can be expressed in terms of the independent quantities E, $\mathbf{p}$, $\vec{\Delta}$:

$$p = \left(\sqrt{\mathbf{p}^2 + M^2},\, \mathbf{p}\right), \qquad p' = \left(\sqrt{\mathbf{p}^2 + M^2},\, -\mathbf{p}\right),$$

$$q = (E,\, -(1+\varepsilon)\mathbf{p} + \lambda\mathbf{e}), \qquad q' = \left(\frac{E}{2},\, \frac{1-\varepsilon}{2}\mathbf{p} + \frac{\lambda}{2}\mathbf{e} + \delta\mathbf{e}_\Delta\right),$$

$$q'' = \left(\frac{E}{2},\, \frac{1-\varepsilon}{2}\mathbf{p} + \frac{\lambda}{2}\mathbf{e} - \delta\mathbf{e}_\Delta\right).$$

Here the unit vectors $\mathbf{e}$, $\mathbf{e}_\Delta$, $\mathbf{p}/|\mathbf{p}|$ form an orthogonal coordinate system. E is the energy of the incident π-meson, with respect to which the dispersion relations will be established. The quantities λ and ε are determined from the relations

$$\lambda^2 = E^2 - 4(\mu^2 + \delta^2) - (1-\varepsilon)^2\mathbf{p}^2, \qquad \varepsilon = \frac{1}{\mathbf{p}^2}\left(\frac{3}{4}\mu^2 + \delta^2\right).$$

The reaction threshold energy is obtained from the condition $\lambda^2 = 0$.

Fig. 1

Figure 1: Fig. 1

The retarded and advanced amplitudes of the process under consideration can be constructed analogously to the case of a fixed nucleon source ([3]). In the notation of article ([3]) they have the form

$$T^{\rm ret}_{\beta\alpha}(q'',q') = \frac{1}{(2\pi)^{9/2}}\int dx'\,dx''\,\langle \mathbf{p}',\sigma',t'|\frac{\delta^2 j(0)}{\delta\varphi'(x')\,\delta\varphi''(x'')}|\mathbf{p},\sigma,t\rangle e^{iq''x''+iq'x'};$$

$$T^{\rm ad}_{\beta\alpha}(q'',q') = \frac{1}{(2\pi)^{9/2}}\int dx'\,dx''\,\langle \mathbf{p}',\sigma',t'|\frac{-\delta^2 \lambda(0)}{\delta\varphi'(x')\,\delta\varphi''(x'')}|\mathbf{p},\sigma,t\rangle e^{iq''x''+iq'x'}.$$

Application of the causality condition ([4])

$$\frac{\delta j(x)}{\delta\varphi'(y)} = 0 \quad \text{for } y \preccurlyeq x; \qquad \frac{\delta \lambda(x)}{\delta\varphi'(y)} = 0 \quad \text{for } y \succcurlyeq x$$

shows that $T^{\rm ret} = 0$ for $x', x'' \preccurlyeq 0$; $T^{\rm ad} = 0$ for $x', x'' \succcurlyeq 0$.

At this point we shall make the assumption that, for complex values of E, the functions $T^{\rm ret}_{\beta\alpha}(q'',q') = T^{\rm ret}_{\beta\alpha}(E)$ and $T^{\rm ad}_{\beta\alpha}(q'',q') = T^{\rm ad}_{\beta\alpha}(E)$, respectively in the upper and lower half-planes, are analytic. Some discus-

of the analytic properties of the process amplitude were recently carried out in [5].

Consideration of the behavior of the difference $T^{\rm ret}(E) - T^{\rm ad}(E)$ for real values of E gives, under the condition $\mathbf{p}^2 + \delta^2 < \frac{1}{2}M\mu - \mu^2$, the spectrum shown in Fig. 1.

Fig. 1

For $E = \pm E_1^0$ and $E = \pm E_2^0$, where

$$E_1^0 = \frac{-{}^5/_2\mu^2 - 2(\mathbf{p}^2+\delta^2)}{2\sqrt{\mathbf{p}^2+M^2}}, \qquad E_2^0 = \frac{-{}^1/_2\mu^2 - 2(\mathbf{p}^2+\delta^2)}{2\sqrt{\mathbf{p}^2+M^2}}, \qquad (*)$$

$T^{\rm ret}(E)\,T^{\rm ad}(E)$ acquires delta-function singularities. The continuous spectrum begins at the values $E = \pm E_k$, where

$$E_k = \frac{2M\mu - {}^3/_2\mu^2 - 2(\mathbf{p}^2+\delta^2)}{2\sqrt{\mathbf{p}^2+M^2}}.$$

Since the values of $T^{\text{ret}}(E)$ and $T^{\text{ad}}(E)$ coincide on a finite segment of the real axis, the set of functions $T^{\text{ret}}(E)$ and $T^{\text{ad}}(E)$ constitutes a single analytic function possessing poles and cut lines on the real axis. Application of Cauchy' s theorem to this function leads to a dispersion relation for the process $\pi + N \to \pi' + \pi'' + N'$. In doing so we shall assume that at high energies the amplitude of the process decreases as $1/E$ or faster. If, as $E \to \infty$, the amplitude decreases more slowly than $1/E$, then dispersion relations can likewise be written down directly, but to determine the contribution of the pole at infinity one must use the subtraction method, known from the dispersion relations for elastic scattering.

Integration over negative energies can be eliminated by applying the relation

$$A_{\beta\alpha}(-E) = -P_{s's}\overset{*}{A}_{\beta\alpha}(E).$$

$\overset{*}{A}_{\beta\alpha}$ is the complex-conjugate matrix element of the anti-Hermitian part $A_{\beta\alpha}$ of the amplitude of the process; $P_{s's}$ permutes the spin and isotopic indices of the initial and final nucleon states. The integral in the interval $-E_k < E < E_k$ can be calculated explicitly.

To obtain now the dispersion relations for specific physical processes, one must take into account the isotopic and spin structure of the amplitude. With regard to the isotopic structure, one should bear in mind that from the six possible permutations of the three τ-matrices $\tau_\rho, \tau_{\rho'}, \tau_{\rho''}$ ($\equiv \tau'\tau''\tau$) one can construct four independent expressions, so that the amplitude has the form

$$T_{\beta\alpha} = \langle t'|(\tau'\tau''+\tau''\tau')\tau A_{\sigma'\sigma}+(\tau'\tau''-\tau''\tau')\tau B_{\sigma'\sigma}+(\tau'\tau+\tau\tau')\tau'' G_{\sigma'\sigma}+(\tau'\tau-\tau\tau')\tau'' D_{\sigma'\sigma}|t\rangle,$$

where $T_{\beta\alpha} = T^{\text{ret}}_{\beta\alpha}(E)$; t, t' and σ', σ denote, respectively, the values of the isotopic and ordinary spins of the nucleon in the initial and final states. Investigation of the ordinary spin structure shows that $T_{\beta\alpha}$ has the following form:

$$T_{\beta\alpha} = \langle\sigma'|\{A_{t't} + q'_\mu\gamma_\mu B_{t't} + q''_0\gamma_0 G_{t't}\}\gamma_5|\sigma\rangle.$$

Thus we obtain 12 independent dispersion relations, which correspond to 12 different eigenstates of the ordinary and isotopic spins of the system.

In what follows we give only two of them, namely for the transition from the state with total isotopic spin $^3/_2$ to the two possible states with isotopic spin $^3/_2$, the isotopic spin of the two-π-meson system being respectively 2 and 1. The value of the nucleon spin is not changed.

$$D^{11}_{\frac{3}{2};2}(E) = \frac{1}{\pi} P \int_{E_k}^{\infty} \left\{ \frac{A^{11}_{\frac{3}{2};2}(E')}{E'-E} + \frac{\overset{*}{A}{}^{11}_{\frac{3}{2};2}(E')}{E'+E} \right\} dE' + \left(1 + \frac{E_2^0}{p_0}\right) \frac{i\sqrt{10p^2}\, g}{4(p-q')_0 p_0} \times$$

$$\times \left\{ \frac{a\,[D_1^1(-{}^3/_4E_2^0) - D_2^1(-{}^3/_4E_2^0)] + b\,[D_1^2(-{}^3/_4E_2^0) + D_2^2(-{}^3/_4E_2^0)]}{E_2^0 + E} + \right.$$

$$\left. + \frac{a\,[D_1^1({}^3/_4E_2^0) - D_2^1({}^3/_4E_2^0)] + b\,[D_1^2({}^3/_4E_2^0) D_2^2({}^3/_4E_2^0)]}{E_2^0 - E} \right\},$$

$$D^{11}_{\frac{3}{2};1}(E) = \frac{1}{\pi} P \int_{E_k}^{\infty} \left\{ \frac{A^{11}_{\frac{3}{2};1}(E')}{E'-E} - \frac{1}{3} \frac{4\overset{*}{A}{}^{11}_{\frac{1}{2};1}(E') + 5\overset{*}{A}{}^{11}_{\frac{3}{2};1}(E')}{E'+E} \right\} dE' +$$

$$+ \left(1 + \frac{E_1^0}{p_0}\right) \frac{i\sqrt{2p^2}\, g}{2(q+p_0)p_0} \left\{ \frac{cD_1^2(0) + dD_2^2(0)}{E_1^0 + E} \right\}.$$

We note that the beginning of the continuous spectrum E_k still lies in the unobservable region (see Fig. 1), allowance for which leads to integral equations between $D_{\beta\alpha}$ and $A_{\beta\alpha}$. The second relation contains the contribution of the amplitude $A_{\frac{1}{2};1}$, which corresponds to total isotopic spin ${}^1/_2$ and to the isotopic spin of the emitted mesons equal to 1. The coefficients are determined by the expressions

$$a = -(1-\varepsilon)M, \qquad b = (1+\varepsilon)\{2p_0(p-q')_0 - p_0E_2^0 - 2M^2\} +$$

$$+ E_2^0(p-q')_0 - (E_2^0)^2 + \mu^2,$$

$$c = -(1+\varepsilon)M, \qquad d = -(1-\varepsilon)\{M^2 + p_0(p+q)_0\} + (1+\varepsilon)p^2 +$$

$$+ \varepsilon E_1^0 p_0 + E(p+q)_0 - (E_1^0)^2 + \mu^2;$$

g is the meson charge (for questions concerning its determination see ([6])); D_j^i is the Hermitian part of the amplitude for elastic scattering of π-mesons by a nucleon; j denotes two different isotopic states, and i has a meaning analogous to that for the ordinary spin. The energies contained in these amplitudes lie in the unobservable region of elastic scattering and are determined through

(*). These amplitudes can be calculated with the aid of dispersion relations for elastic scattering.

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Note: Figure translations are in progress. See original paper for figures.