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Abstract

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THEORY OF ELASTICITY

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ON ONE PARTICULAR SOLUTION OF THE GENERAL EQUATIONS OF THE THEORY OF IDEAL PLASTICITY IN CYLINDRICAL COORDINATES

(Presented by Academician Yu. N. Rabotnov on 1 VIII 1958)

We shall write the general equations of the theory of ideal plasticity in cylindrical coordinates under the Mises plasticity condition in the form

$$\begin{gathered}\frac{\partial\sigma_\rho}{\partial\rho}+\frac{1}{\rho}\frac{\partial\tau_{\rho\theta}}{\partial\theta}+\frac{\partial\tau_{\rho z}}{\partial z}+\frac{\sigma_\rho-\sigma_\theta}{\rho}+K_\rho=0,\\ \frac{\partial\tau_{\rho\theta}}{\partial\rho}+\frac{1}{\rho}\frac{\partial\sigma_\theta}{\partial\theta}+\frac{\partial\tau_{\theta z}}{\partial z}+\frac{2\tau_{\rho\theta}}{\rho}+K_\theta=0,\\ \frac{\partial\tau_{\rho z}}{\partial\rho}+\frac{1}{\rho}\frac{\partial\tau_{\theta z}}{\partial\theta}+\frac{\partial\sigma_z}{\partial z}+\frac{\tau_{\rho z}}{\rho}+K_z=0;\end{gathered}\tag{1}$$

$$(\sigma_\rho-\sigma_\theta)^2+(\sigma_\theta-\sigma_z)^2+(\sigma_z-\sigma_\rho)^2+6(\tau_{\rho\theta}^2+\tau_{\theta z}^2+\tau_{\rho z}^2)=6k^2;\tag{2}$$

$$\frac{\varepsilon_\rho^p}{2\sigma_\rho-\sigma_\theta-\sigma_z}=\frac{\varepsilon_\theta^p}{2\sigma_\theta-\sigma_z-\sigma_\rho}=\frac{\varepsilon_z^p}{2\sigma_z-\sigma_\rho-\sigma_\theta}=\frac{\gamma_{\rho\theta}^p}{3\tau_{\rho\theta}}=\frac{\gamma_{\theta z}^p}{3\tau_{\theta z}}=\frac{\gamma_{\rho z}^p}{3\tau_{\rho z}},\tag{3}$$

where $\sigma_\rho, \tau_{\rho\theta}, \ldots$ are the components of stress; $K_\rho, \ldots$ are body forces; $\varepsilon_\rho^p, \gamma_{\rho\theta}^p, \ldots$ are the components of the rate of plastic deformation.

In what follows we shall assume the elastic deformations to be negligibly small in comparison with the plastic ones. Suppose that a temperature field is prescribed in the body; then, assuming that the influence of temperature reduces to volumetric expansion of the material, put

$$\varepsilon_\rho=\varepsilon_\rho^p+\alpha T,\qquad \varepsilon_\theta=\varepsilon_\theta^p+\alpha T,\qquad \varepsilon_z=\varepsilon_z^p+\alpha T,\tag{4}$$

$$\gamma_{\rho\theta} = \gamma^p_{\rho\theta}, \qquad \gamma_{\theta z} = \gamma^p_{\theta z}, \qquad \gamma_{\rho z} = \gamma^p_{\rho z},$$

where $\varepsilon_\rho, \gamma_{\rho\theta}, ...$ are the components of the rate of deformation; T is the temperature; α is the coefficient of linear expansion.

It is known that

$$\varepsilon_\rho = \frac{\partial u}{\partial \rho}, \qquad \varepsilon_\theta = \frac{u}{\rho} + \frac{1}{\rho}\frac{\partial v}{\partial \theta}, \qquad \varepsilon_z = \frac{\partial w}{\partial z},$$

$$\gamma_{\rho\theta} = \frac{\partial v}{\partial \rho} - \frac{v}{\rho} + \frac{1}{\rho}\frac{\partial u}{\partial \theta}, \qquad \gamma_{\theta z} = \frac{1}{\rho}\frac{\partial w}{\partial \theta} + \frac{\partial v}{\partial z}, \qquad \gamma_{\rho z} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial \rho} \tag{5}$$

where u, v, w are the components of the velocity of displacement.

Assume that none of the components depends on the angle θ. Further assume that

$$K_\rho = K_\rho(\rho), \qquad K_\theta = 0, \qquad K_z = \text{const.} \tag{6}$$

Thus, along the ρ-axis the action is allowed, for example, of centrifugal forces, and along the z-axis the action of gravity.

Suppose that $T = T(\rho)$, $\alpha = \alpha(T)$, $k = k(T)$. The last of these means the dependence of the yield limit on temperature. It is obvious that the temperature may also depend on time, which will enter as a parameter.

We shall obtain the sought solution by assuming

$$\tau_{\rho z} = m_1\rho + \frac{m_2}{\rho}, \qquad \tau_{\rho\theta} = \tau_{\rho\theta}(\rho), \qquad u = u(\rho), \qquad v = v(\rho), \tag{7}$$

where m_1, m_2 are constants.

Obviously, from (7), (5), and (3) it follows that $\tau_{\theta z} = 0$.

Taking the adopted assumptions into account, we write the equilibrium equations (1) in the form

$$\frac{\partial \sigma_\rho}{\partial \rho} + \frac{\sigma_\rho - \sigma_\theta}{\rho} + K_\rho = 0, \qquad \frac{d\tau_{\rho\theta}}{d\rho} + \frac{2\tau_{\rho\theta}}{\rho} = 0, \qquad \frac{\partial \sigma_z}{\partial z} = -(2m_1 + K_z). \tag{8}$$

From (8) we obtain

$$\tau_{\rho\theta} = \frac{C_1}{\rho^2}, \qquad \sigma_z = -(2m_1 + K_z)z + \sigma_z^*(\rho) \qquad (C_i = \text{const},\ i = 1, 2, ...). \tag{9}$$

From (3), (4), (5) we obtain

$$\frac{du}{d\rho} + \frac{u}{\rho} + \frac{\partial w}{\partial z} - 3\alpha T = 0.$$

Suppose that

$$\frac{du}{d\rho} + \frac{u}{\rho} - 3\alpha T = C_2, \qquad \frac{\partial w}{\partial z} = -C_2;$$

hence we find

$$u = \frac{1}{\rho}\left[\int (3\alpha T + C_2)\rho \, d\rho + C_3\right], \qquad w = -C_2 z + w^*(\rho). \tag{10}$$

Next, from (3) and (5) we find

$$\sigma_z = \frac{1}{2}(\sigma_\rho + \sigma_\theta) - \frac{1}{2} f(\rho)(\sigma_\rho - \sigma_\theta), \qquad f(\rho) = \frac{3(C_2 + \alpha T)}{du/d\rho + u/\rho}. \tag{11}$$

Substituting expression (11) into the plasticity condition (2), we obtain

$$\sigma_\rho - \sigma_\theta = F(\rho), \qquad F(\rho) = \pm \frac{2\sqrt{3}\sqrt{k^2 - \tau_{\rho\theta}^2 - \tau_{\rho z}^2}}{\sqrt{3 + f(\rho)^2}}. \tag{12}$$

From (8) and (12) we find

$$\sigma_\rho = \varphi(\rho) + \psi(z), \qquad \varphi(\rho) = -\int \left(\frac{F(\rho)}{\rho} + K\right) d\rho + C_4, \tag{13}$$

$$\sigma_\theta = \varphi(\rho) - F(\rho) + \psi(z).$$

From (11), (13), and (9) we obtain

$$\sigma_z = -(2m_1 + K_z)z + \varphi(\rho) - \frac{1}{2}F(\rho)(1 + f(\rho)), \qquad \psi(z) = -(2m_1 + K_z)z.$$

To determine the components v and w, we shall have the equations

$$\frac{\gamma_{\rho\theta}}{\tau_{\rho\theta}} = \frac{\gamma_{\rho z}}{\tau_{\rho z}} = \chi(\rho), \qquad \chi(\rho) = \frac{3(du/d\rho - \alpha T)}{2\sigma_\rho - \sigma_\theta - \sigma_z}$$

or

$$\frac{dv/d\rho - v/\rho}{\tau_{\rho\theta}} = \frac{dw^*/d\rho}{\tau_{\rho z}} = \chi(\rho).$$

Hence it is easy to find

$$v = \rho\left(\int \frac{\chi(\rho)\tau_{\rho\theta}}{\rho}\,d\rho + C_5\right), \qquad w = -C_2 z + \int \chi(\rho)\tau_{\rho z}\,d\rho + C_6.$$

The solution obtained corresponds to a screw-like plastic flow of an ideally plastic material between approaching rough cylindrical surfaces and contains many known particular solutions of the plane and axisymmetric problems of the theory of ideal plasticity. It includes, for example, Prandtl' s well-known solution for the compression of a plastic mass by two rough plates ([1]), the solution of Carathéodory and Schmidt ([2]) (see also ([3,4])), Hill' s solution on the extrusion of a plastic mass from a contracting rough sleeve ([5]), and the author' s solution ([6]), generalizing all these solutions to the case of the action of body forces and a temperature field.

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Note: Figure translations are in progress. See original paper for figures.