Astronomy

1958

SovietRxiv

Abstract

Full Text

Astronomy

S. A. Zhevakin

On a Unified Interpretation of Various Types of Pulsational Stellar Variability

(Presented by Academician M. A. Leontovich, 9 XII 1957)

On the basis of a calculation of nonadiabatic oscillations of a multilayer discrete spherical model of a star' s envelope (in the linear approximation), we show here that all known types of pulsational stellar variability, differing from one another in the phase relations between oscillations of brightness and oscillations of the stellar radius, can be obtained from this model for different values of one of the model parameters (namely, the parameter of nonadiabaticity of the oscillations y_3 of the HeII critical-ionization zone ($^{1-4}$)). A similar interpretation was set forth by us earlier ($^{1-4}$); however, it was incomplete, since in ($^{1-4}$) we did not know of the existence, alongside long-period variables of the o Ceti type, in which **the occurrence of the epoch of maximum brightness precedes by $\sim 1/4$ of the oscillation period the epoch of maximum compression of the star**, also of such long-period variables in which **these two epochs approximately coincide** (^{5}) (here and below it is understood that the epoch of maximum compression of the star is determined from radial-velocity curves in absorption lines). We shall call this type of long-period variables, after one of its representatives, the RR Herculis type.* The same phase shift as in RR Her is also found in variables of the β Can Maj. type.

We shall call the outer part of a star, whose oscillations are not quasiadiabatic, the envelope ($^{1-4,7}$). The envelope consists of the zone of secondary critical ionization of helium, which we shall conventionally take to be the layer with $\gamma_1 - 1 = (d\ln T/d\ln\rho)_{\text{ad}} < 0.26$, and of the atmosphere of the star lying above the zone. Calculations of a number of stellar envelopes for different values of the effective temperature of the star T_{eff}, the acceleration of gravity in the envelope g, and a helium content of $15 \div 20\%$ (by number of atoms) showed that while the parameter y_3 varies strongly when the quantities g and T_{eff} change (it is larger the larger g and T_{eff} are), the ratio of the mass of the atmosphere m_a to the mass of the HeII critical-ionization zone m_3 remains almost unchanged and is approximately $m_a/m_3 = 0.88 \div 0.93$.

Calculations of nonadiabatic oscillations of various envelopes, carried out with the aid of 4- and 5-layer discrete models, showed that, for a given dependence of the opacity coefficient of the stellar envelope on density and temperature (determined by the chemical composition), the character of the nonadiabatic oscillations (i.e., the magnitude of the phase shift between the oscillations of brightness and by oscillations of the star’ s radius and the ratio of the amplitudes of these oscillations) is determined mainly by the value of the parameter y_3 and by the value of the ratio m_a/m_3, and depends little on the remaining parameters, being universal in this sense.

Figure 1 shows the phase shift ψ between the epochs of maximum brightness and minimum stellar radius (solid lines)* and the ratio d of the amplitude of the radiation flux at the exit from the star to the maximum amplitude of the radiation flux, attained before entry into the ionized zone, as a function of $\lg y_3$ for three values of m_a/m_3 (0.822; 0.895; 0.930)**. The dotted line shows those parts of the curves for which the total dissipation of the energy of the star’ s oscillations is positive and which therefore are not realized. The curves were obtained from a calculation of nonadiabatic oscillations of three specific four-layer spherical envelope models with a hydrogen content of 85.7% and helium content of 14.3%. However, in accordance with what was said above, these curves may be regarded as universal, i.e., applicable also to models of other envelopes of the same chemical composition.

Taking into account that, in moving along the variables of the “major sequence” (8) from spectra of class A to spectra of class M, the accelerations of gravity in the atmospheres decrease and that, as follows from the theory of stellar atmospheres, the zone of critical ionization of He II must in this case descend into the depths of the star, into denser layers, and the parameter y_3 must decrease, moving correspondingly in Fig. 1 from right to left, we obtain the following sequence of different types of stellar behavior:

1) $\lg y_3 \gtrsim 0.54$. The zone cannot(2–4) create the negative dissipation necessary for exciting self-oscillations of the star—the star does not pulsate.

2) $-0.1 \lesssim \ln y_3 \lesssim 0.54$. Oscillations arise with a phase shift $\psi \approx +90^\circ$, characteristic of Cepheids and RV Tauri variables***.

3) $-0.25 \lg \lesssim y_3 \lesssim -0.1$. Here two cases may occur****.

a) The amplitude and phase of the radiation flux are relatively stable. This case is realized if $m_a/m_3 > (m_a/m_3)_{\text{cr}}$*****, and the fluctuation changes

---

* Usually, when speaking of the phase relations between oscillations of brightness and radial velocity in long-period variables, one means the relations characteristic of *o* Ceti, and not of RR Her. This circumstance apparently indicates not so much the relative rarity of the latter type of pulsational variable as the fact that the phase relations were first investigated in the bright variable *o* Ceti (6), and for many years, because of the difficulties of studying them in long-period variables, *o* Ceti remained the only long-period variable well studied in this respect.

of the parameter y_3 (into whose nature we do not enter here) are substantially smaller than the width of the interval Δy_3, in which the transition occurs

---

* For Cepheids $\psi \approx +90°$, for long-period variables of the Ceti type $\psi \approx -90°$. The convention for measuring the angles is shown in Fig. 1.

** The graphs for $m_a/m_3 = 0.895$ were published earlier ([2]). We note that in ([1]) graphs were constructed for 6 values of m_a/m_3, obtained from a rough calculation on two "stitched" one-layer models.

*** In this case the phase shift for Cepheids predicted by the theory is not exactly $+90°$, but may be, depending on the value of the parameter of the stellar envelope, either less than or greater than $+90°$, lying within the limits $50° < \psi < 130°$ (see Fig. 1). Just such limits of the values of ψ are given by observations of Cepheids. In this connection we note that the explanation, repeated from time to time [see, for example, ([9], [10])], of the phase shift in Cepheids by the role of radiation pressure must be recognized as untenable. In addition, this role of the latter is small in comparison with the role of gas pressure; furthermore, the explanation is refuted by the fact that, even admitting the dominant role of radiation pressure, we would obtain a shift ψ only within the limits $0 < \psi \leq 90°$. The values $\psi > 90°$, observed in many Cepheids, cannot in principle be explained by the role of radiation pressure. Some other arguments against this explanation were indicated in ([1]).

**** In preceding works ([1–4]) only case b) was considered.

***** $(m_a/m_3)_{\rm cr}$ is the critical value at which the transition from positive ψ to negative is maximally sharp (for a model with 14.3% helium $(m_a/m_3)_{\rm cr} = 0.895$ (Fig. 1), and increases with increasing helium content). The existence of envelopes for which $m_a/m_3 < (m_a/m_3)_{\rm cr}$ is indicated by the presence of Cepheids with $\psi > +90°$. Therefore one should expect the existence of variables in which maximum brightness coincides with maximum radius. The absence of such variables is explained, apparently, by the fact that m_a/m_3 can be only slightly smaller than the critical value, as a result of which in this case, for $-0.25 \lesssim \lg y_3 \lesssim -0.1$, irregular oscillations of brightness arise instead of stable ones; see b).

transition from values $\psi \approx +90°$ to values $\psi \approx -90°$. The latter is favored by such stellar envelopes for which m_a/m_3 is noticeably greater than $(m_a/m_3)_{\rm cr} = 0.895$ (for example, $m_a/m_3 = 0.930$, see Fig. 1).

A comparatively stable shift of the phase ψ, according to Fig. 1, will be close to zero. We obtain the phase relations between the oscillations of the luminosity and of the stellar radius observed in long-period variables of the RR Her type ([5]) and in variables of the β CaM type: the maximum luminosity occurs at the moment of greatest compression of the star.* According to Fig. 1, for the same relative amplitudes of the oscillations of the stellar radius (assuming linearity),

Fig. 1

Figure 1: Fig. 1

the amplitude of the luminosity oscillations $\delta L/L$ in RR Her stars should be $A \approx 2$ times smaller than in variables of the δ Ceti type, and in stars of the β CaM type $A = 3 \div 4$ times smaller than in Cepheids.

Fig. 1

These conclusions are confirmed by observations. If, for stars of the δ Ceti type, the change in brightness in visual magnitudes is on the average about 5^m, and in bolometric magnitudes about 1^m (^{14}), then for stars of the RR Her type the change in visual stellar magnitude is on the average $2^m.5$ (^{5}), and the bolometric change, correspondingly, should be of the order of $0^m.5$. If, because of the lack of sufficiently reliable data on the absolute magnitudes of RR Her stars, we make instead the assumption—plausible from the theoretical point of view—of equality of the relative amplitude of the radius oscillations $\delta r/r$ in stars of the δ Ceti type and of the RR Her type, then it follows from the data just given that $\delta L/L$ in RR Her variables is ~ 2 times smaller than in stars of the δ Ceti type

Table 1

Stars	A
γ Ped	1.8
ξ_1 CaM	5.4
δ Ceti	1.9
β Cep	3.0
σ Scor	3.8

[for $1^m/2.5 = \lg(1 + 0.43/1 - 0.43)$, $0^m.5/2.5 = \lg(1 + 0.22/1 - 0.22)$, $A = 0.43/0.22 \approx 2$]. For stars of the β CaM type, for which there are fairly reliable data on absolute magnitudes, the value of A can be obtained more securely. According to observational data for 5 stars of the β CaM type ($^{15-21}$), ···

---

* Such phase relations are obtained in Eddington' s theory (^{11}). In connection with this, some authors (^{12}) suppose that variability of the β CaM type can be described by Eddington' s theory. This, however, is not so. Eddington' s theory in the form (^{11}) (i.e., without involving an ionization zone) can explain (and incorrectly at that) only the phase, but not the amplitude, relations between the oscillations of the luminosity and radial velocity of stars of the β CaM type or of the RR Her type. The amplitude of the luminosity oscillations for a given observed value of the amplitude of the radial velocity (more precisely, the corresponding value of $\delta r/r$) yielded by Eddington' s theory, with the currently reliably adopted dependence of the coefficient of stellar opacity on temperature

and density (13), is $10 \div 20$ times larger than the observed one. A complete explanation of the phase and amplitude relations between the oscillations of luminosity and radial velocity in stars of the β CaM type and the RR Her type can be given by the theory of maintenance of self-oscillations of stars by the He II ionization zone (Fig. 1).

converting the radial velocities v_l into radial velocities v_r by the formula $v_r = {}^{24}/_{17} v_l$, we estimated the value of $\delta r/r$ for these stars and obtained Table 1.

In Table 1, A shows how many times the reduced amplitude of the luminosity oscillations of the indicated stars is smaller than that of the Cepheid δ Cep, η Aquil (for δ Cep $\delta r/r = 0.053$, $\Delta m_\delta = 0^m.58$ (14); for η Aquil $\delta r/r = 0.053$, $\Delta m_\delta = 0.55$ (14)). Taking into account the inaccuracy of the data, the agreement of the tabular values of A with the theoretical value $A = 3 \div 4$ should be regarded as good.

b) The amplitude and phase of the radiation flux are not stable. This case is realized when the fluctuation changes of the parameter y_3 are of the order of, or greater than, the width of the interval Δy_3 discussed in a). The realization of this case is favored by such stellar envelopes for which Δy_3 is minimal (the latter will occur at $m_a/m_3 \approx 0.895$). Then, for stars for which $\lg y_3$ falls in the interval $-0.25 \lesssim \lg y_3 \lesssim -0.1$, chaotic changes in the phase shift of the radiation flux should be observed, approximately within the range from -90 to $+90^\circ$, and changes in the amplitude of the radiation flux by a factor of $3 \div 4$ (see Fig. 1)—the star' s brightness will undergo irregular changes (1–4). The semiregular variables of the types AF Cygni and RS Cancri (22) are especially well suited to the indicated interpretation.

4) $-0.4 \lesssim \lg y_3 \lesssim -0.25$. Oscillations arise with a phase shift $\psi \approx -90^\circ$, characteristic of variables of the type Ceti (1–4).

5) $\lg y_3 \lesssim -0.4$. The envelope cannot create negative dissipation sufficient to excite self-oscillations of the star (1–4).

It should be emphasized that, whereas the interpretation of variability of the Cepheid type, the β CMa type, the RR Her type, and the Ceti type requires no assumptions, the interpretation set forth here of irregular and semiregular variability is not so reliable, since it is based on the still hypothetical assumption of the existence of fluctuation changes of the parameter y_3 (see, however, (1,2)).

Scientific-Research Radiophysical Institute
at Gorky State University
named after N. I. Lobachevsky

Received
4 XII 1957

REFERENCES

[1] S. A. Zhevakin, *Astron. zhurn.*, **31**, 141, 335 (1954).
[2] S. A. Zhevakin, *DAN*, **99**, 353 (1954).
[3] S. A. Zhevakin, Proceedings of the 4th Conference on Cosmogony, Publishing House of the Academy of Sciences of the USSR, 1955, p. 625.
[4] S. A. Zhevakin, Collection in Memory of A. A. Andronov, Publishing House of the Academy of Sciences of the USSR, 1955, p. 629.
[5] R. Sanford, *Ap. J.*, **111**, 270 (1950).
[6] A. H. Joy, *Ap. J.*, **63**, 281 (1926).
[7] S. A. Zhevakin, *DAN*, **99**, 217 (1954).
[8] S. Payne-Gaposchkin, S. Gaposchkin, *Variable Stars*, Cambridge, Mass., 1938.
[9] B. V. Kukarkin, *Astron. zhurn.*, **13**, 103 (1936).
[10] P. N. Kholopov, V. P. Tsesevich, article "Variable Stars," *BSE*, 2nd ed.
[11] A. S. Eddington, *Internal Constitution of the Stars*, Cambridge, 1926.
[12] C. de Jager, *Bull. Astron. Inst. Netherl.*, **12**, 81 (1953).
[13] E. Vitense, *Zs. f. Astrophys.*, **28**, 81 (1951).
[14] E. Pettit, S. B. Nicholson, *Ap. J.*, **78**, 320 (1933).
[15] D. H. McNamara, *Ap. J.*, **122**, 95 (1955).
[16] A. D. Williams, *Publ. Astron. Soc. Pacif.*, **66**, 25 (1954).
[17] O. Struve, ibid., **67**, 135 (1955).
[18] M. F. Walker, ibid., **65**, 49 (1953).
[19] O. Struve, D. H. McNamara, S. M. Kung, C. Beimer, *Ap. J.*, **118**, 39 (1953).
[20] A. Blaauw, M. P. Savedoff, *Bull. Astron. Inst. Netherl.*, **12**, 69 (1953).
[21] O. Struve, H. D. McNamara, V. Zebergs, *Ap. J.*, **112**, 122 (1955).
[22] B. V. Kukarkin, P. P. Parenago, *Variable Stars*, 1937.

Note: Figure translations are in progress. See original paper for figures.