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Abstract

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APPROXIMATE EQUATIONS FOR THE AMPLITUDE OF PHOTON SCATTERING ON NUCLEONS

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The study of the process of photon scattering on nucleons can yield important information about the mesonic structure of the nucleon. In the present work, on the basis of dispersion relations for Compton scattering ([1]), approximate equations have been obtained for the physical amplitudes.*

1. Kinematic study of the amplitude

Let us denote the initial momenta of the nucleon and photon by p and k, and the final ones by p' and k'. From the vectors p and k, p' and k', taking into account the conservation laws for energy-momentum $p + k = p' + k'$, one can construct two independent scalar products ν and ν_1:**

$$\nu = (p + p') \cdot k, \qquad \nu_1 = k \cdot k'. \tag{1}$$

From the relativistic invariance of the process, the amplitude can be represented in the form

$$\hat{R} = \sum_i \sum_{\nu,\mu=0}^{3} \Omega_i(\nu, \nu_1)\, \bar{u}(p')\, \hat{R}^i_{\mu\nu}\, u(p)\, e'_\mu e_\nu, \tag{2}$$

where e_ν, e'_μ are the polarization vectors of the initial and final photon; $\bar{u}(p')$, $u(p)$ are spinors characterizing the nucleon in the final and initial states; $\Omega_i(\nu, \nu_1)$ are invariant functions possessing only an isotopic structure; $\hat{R}^i_{\mu\nu}$ are operators containing the spin structure of the amplitude of the process. The summation is over all independent structures.

From the conditions of relativistic and gauge invariance one can find the number of independent structures $\hat{R}^i_{\mu\nu}$ and obtain their explicit expression ([3]). In the case of a pseudoscalar meson field the number of independent structures is equal to 10. If one takes into account the invariance of the amplitude with respect to time reversal, this number is reduced to 6. As 6 independent structures we choose the following expressions $\left(\hat{R}_i = \sum_{\mu,\nu} e'_\mu \hat{R}^i_{\mu\nu} e_\nu\right)$:

$$\hat{R}_1 = \frac{1}{p\cdot k\, p\cdot k'}\left\{e\cdot e'\, p\cdot k\, p\cdot k' + (e\cdot p'\, e'\cdot p\, p\cdot k - e'\cdot p'\, e\cdot p\, p'\cdot k)\right\};$$

$$\hat{R}_2 = \frac{(k\cdot k')^{1/2}}{p\cdot k\, p\cdot k'}\left\{\hat{e}'\,(e\cdot p'\, p\cdot k - e\cdot p\, p'\cdot k) + \hat{e}\,(e'\cdot p\, p'\cdot k' - e'\cdot p'\, p\cdot k')\right\};$$

$$\hat{R}_3 = \frac{1}{(k\cdot k')^{1/2}}\left\{\hat{e}'\,(\hat{k}+\hat{k}')\,\hat{e} - \hat{e}\,(\hat{k}+\hat{k}')\,\hat{e}'\right\};$$

$$\hat{R}_4 = \frac{1}{p\cdot k\, p\cdot k'}\left\{\hat{e}'\cdot\hat{k}'\,(e\cdot p'\, p\cdot k - e\cdot p\, p'\cdot k) + \hat{e}\,\hat{k}\,(e'\cdot p'\, p\cdot k' - e'\cdot p\, p'\cdot k')\right\};$$

* Many interesting relations for Compton scattering are contained in the works ([2]).

** In the system $p + p' = 0$, the photon energy is $E = \nu/2p_0$, and the recoil momentum is $\mathbf{p}^2 = \nu_1/2$.

$$\hat{R}_5 = \frac{(k\cdot k')^{1/2}}{p\cdot k\, p\cdot k'}\{\hat{e}'(\hat{k}+\hat{k}')\,\hat{e}\, p\cdot k' + \hat{e}(\hat{k}+\hat{k}')\,\hat{e}'\, p\cdot k\} - 2\hat{R}_2;$$

$$\hat{R}_6 = \frac{(k\cdot k')^{-1/2}}{p\cdot k\, p\cdot k'}\{2(\hat{k}+\hat{k}')(e\cdot p'\, e'\cdot p\, p\cdot k + e'\cdot p'\, e\cdot p\, p\cdot k') - 2p\cdot k\, p'\cdot k[\hat{e}'(e\cdot p' + e\cdot p) + \hat{e}(e'\cdot p + e'\cdot p')]\} + \hat{R}_3;$$

$$a\cdot b = a_0 b_0 - \mathbf{ab}, \qquad \hat{a} = a_0\gamma_0 - \mathbf{a}\gamma. \tag{3}$$

The isotopic dependence of Ω_i is obvious:

$$\Omega_i = \Omega_i^1 + \Omega_i^2\tau_3 = \Omega_i^{(p)}\frac{1+\tau_3}{2} + \Omega_i^{(n)}\frac{1-\tau_3}{2}, \tag{4}$$

where $\Omega_i^{(p)}$ describes scattering on a proton, and $\Omega_i^{(n)}$, scattering on a neutron.

Let us establish certain symmetry properties of the functions $\Omega_i(\nu, \nu_1)$. With the aid of the S-matrix formalism, the matrix element of Compton scattering can be written in the form [1]

$$\langle\gamma'|s|\gamma\rangle = \frac{i}{\sqrt{4k_0k_0'}}\int e^{i(k'x-ky)}\left\langle \Phi_{p'}\left|\frac{\delta j_\nu(y)}{\delta A_\mu(x)}\right|\Phi_p\right\rangle dx\,dy, \tag{5}$$

$$\langle\gamma'|s|\gamma\rangle = i\frac{(2\pi)^4}{\sqrt{4k_0k_0'}}\delta(p+k-p'-k')\,R,$$

where $A_\mu(x)$ is the electromagnetic-field operator; $j_\mu(x) = -i\dfrac{\delta S}{\delta A_\mu(x)}S^+$; $|\Phi_p\rangle$ is the state vector of the nucleon.

From (5) one can establish that

$$R(p,k,p',k') = R^*(p',-k,p,-k'). \tag{6}$$

Substituting (3) into (2) and taking (6) into account, we obtain the following important property of $\Omega_i(\nu,\nu_1)$:

$$\Omega_i(\nu,\nu_1) = \Omega_i^*(-\nu,\nu_1), \tag{7}$$

or

$$\operatorname{Re}\Omega_i(\nu,\nu_1) = \operatorname{Re}\Omega_i(-\nu,\nu_1), \qquad \operatorname{Im}\Omega_i(\nu,\nu_1) = -\operatorname{Im}\Omega_i(-\nu,\nu_1).$$

2. Dispersion relations for the relativistic amplitudes Ω_i. Using the analyticity property of the amplitude R in the upper half-plane of the variable ν [4], we have

$$\operatorname{Re}\Omega_i(\nu,\nu_1) = \frac{1}{\pi}P\int_{-\infty}^{\infty}\frac{\operatorname{Im}\Omega_i(\nu',\nu_1)}{\nu'-\nu}\,d\nu'. \tag{8}$$

The region of negative ν in (8) can be eliminated with the aid of (7). The method of excluding the region $0<\nu<2M\mu+\mu^2-\nu_1$ for the amplitude R is given in work [1]. In this region the Hermitian part of the amplitude R, D, is written in the form

$$D = \sum_i \bar u(p')\hat R_i u(p)\Omega_i^0 = -\bar u(p')\{4(\mu^2M+\mu\varepsilon_p)\hat R_1 +$$

$$+(2\mu^2M+\mu\varepsilon_p)\hat R_4' + \frac{1}{4\nu_1^{1/2}}(2\hat\mu M + e\tau_p)^2\hat R_5 + \frac{\nu_1^{1/2}}{2}\mu^2\hat R_6\}u(p),$$

$$\tau_p = \frac{1+\tau_3}{2}, \qquad \tau_n = \frac{1-\tau_3}{2}, \qquad \hat\mu = \mu_p'\tau_p + \mu_n'\tau_n, \tag{9}$$

where μ'_p and μ'_n are the anomalous magnetic moments of the proton and neutron.

Taking (7) and (9) into account, expression (8) is written as follows:

$$\operatorname{Re}\Omega_i(\nu,\nu_1) = \Omega_i^0 + \frac{1}{\pi} P \int_{2M\mu+\mu^2-\nu_1}^{\infty} \left(\frac{1}{\nu'-\nu} + \frac{1}{\nu'+\nu} \right) \operatorname{Im}\Omega_i(\nu',\nu_1)\, d\nu'. \quad (10)$$

3. **Dispersion relations for physical amplitudes.** The derivation of dispersion relations for the relativistic amplitudes is an intermediate stage. In order to obtain dispersion relations for the physical amplitudes $M_i(\nu,\nu_1)$, we represent the Compton-scattering amplitude, expanded in three-dimensional structures $\hat{r}_i$ (all calculations below are carried out in the center-of-mass system):

$$\hat{R} = \sum_{i=1}^{6} M_i(\nu,\nu_1)\, \hat{r}_i, \quad (11)$$

where

$$\hat{r}_1 = i\left[\vec{\sigma}(\mathbf{e}\times\mathbf{n})\,\mathbf{e}'\cdot\mathbf{n} - \vec{\sigma}(\mathbf{e}'\times\mathbf{n}')\,\mathbf{e}\cdot\mathbf{n}'\right], \quad \hat{r}_2 = \mathbf{en}\,\mathbf{e}'\mathbf{n}, \quad \hat{r}_3 = i\vec{\sigma}(\mathbf{e}'\times\mathbf{n}')\times(\mathbf{e}\times\mathbf{n}),$$

$$\hat{r}_4 = i\vec{\sigma}(\mathbf{n}'\times\mathbf{n})\,\mathbf{e}'\cdot\mathbf{e}, \quad \hat{r}_5 = i\vec{\sigma}(\mathbf{e}\times\mathbf{e}'), \quad \hat{r}_6 = \mathbf{e}\cdot\mathbf{e}', \quad \mathbf{n} = \mathbf{k}/|\mathbf{k}|, \quad \mathbf{n}' = \mathbf{k}'/|\mathbf{k}'|.$$

Substituting (3) into (2) and comparing with (11), we establish the relation between M_i and Ω_i:

$$M_i(\nu,\nu_1) = \sum_{j=1}^{6} c_{ij}(\nu,\nu_1)\, \Omega_j(\nu,\nu_1). \quad (12)$$

One can also obtain the inverse relations:

$$\Omega_i(\nu,\nu_1) = \sum_{j=1}^{6} b_{ij}(\nu,\nu_1)\, M_j(\nu,\nu_1). \quad (13)$$

Taking into account (10), (12), and (13), one may write the dispersion relations for the physical amplitudes M_i:

$$\operatorname{Re} M_i(\nu,\nu_1) = \sum_j c_{ij}(\nu,\nu_1)\, \Omega_j^0 +$$

$$+ \sum_{j,k} \frac{1}{\pi} P \int_{2M\mu+\mu^2-\nu_1}^{\infty} d\nu' \left(\frac{1}{\nu'-\nu} + \frac{1}{\nu'--\nu} \right) c_{ij}(\nu,\nu_1) b_{jk}(\nu',\nu_1) \operatorname{Im} M_k. \quad (14)$$

Because of the unwieldiness of the matrices c_{ik}, b_{ik}, formulas (14) are not written out explicitly. However, in the energy region where terms of order $(k_0/M)^2$ may be neglected, the matrices c_{ik} and b_{ik} are simplified, and, retaining the leading terms of the expansion, we obtain approximate equations for the physical amplitudes, which are not presented here for lack of space.

4. **The unitarity condition.** The dispersion relations (14) relate the Hermitian and anti-Hermitian parts of the reaction amplitude. The unitarity condition $S^+S = 1$, written in the one-meson approximation, allows us to express the anti-Hermitian part of Compton scattering through photoproduction amplitudes. As a result, (14) acquire the meaning of equations. From $S^+S = 1$ and $S = 1 + R$, in the one-meson approximation we have

$$\langle\gamma'|R + R^+|\gamma\rangle = -\langle\gamma'|R^+|\pi\rangle\langle\pi|R|\gamma\rangle; \tag{15}$$

$|\pi\rangle$ characterizes the nucleon and meson in the intermediate state with momenta p'', q and other quantum numbers.

Taking into account (5) and the definition of the photoproduction amplitude [5]

$$\langle\pi|R|\gamma\rangle = i\frac{(2\pi)^4}{\sqrt{4k_0q_0}}\,\delta(p + k - p'' - q)\,T, \tag{16}$$

where

$$T = i\vec{\sigma}\cdot\mathbf{e}\,F_1 + \vec{\sigma}\cdot\mathbf{m}\,\vec{\sigma}(\mathbf{n}\times\mathbf{e})\,F_2 + i\vec{\sigma}\cdot\mathbf{n}\,\mathbf{m}\cdot\mathbf{e}\,F_3 + i\vec{\sigma}\cdot\mathbf{m}\,\mathbf{m}\cdot\mathbf{e}\,F_4, \quad \mathbf{m} = \mathbf{q}/|\mathbf{q}|,$$

after simple calculations we obtain the relation between $\operatorname{Im} M_i$ and F_i:

$$\operatorname{Im} M_2 = (4\pi)^{-2}\lambda\int d\Omega\left\{\frac{m_x}{n'_x}F_{12} + \beta F_{21} + \frac{m_x}{n'_x}F_{13} + \beta F_{31} - F_{22} + \frac{n^2_{z'}m^2_y - m^2_x}{n^2_{z'}}F_{23} +\right.$$

$$\left.+(\alpha n_z - \beta m_z)F_{32} + \alpha\left[F_{14} + F_{41} + n'_zF_{33} + \mathbf{n}'\cdot\mathbf{m}\,F_{34} + \mathbf{n}\cdot\mathbf{m}\,F_{43} + F_{44}\right]\right\};$$

$$\operatorname{Im} M_6 = (4\pi)^{-2}\lambda\int d\Omega\,\{F_{11} - \mathbf{n}\cdot\mathbf{m}\,F_{12} - \mathbf{n}'\cdot\mathbf{m}\,F_{21} + n'_zF_{22} +$$

$$+ m_y^2[F_{14} + F_{41} + n'_z F_{23} + n'_z F_{32} + n'_z F_{33}\, \mathbf{n}'\cdot\mathbf{m}\, F_{34} + \mathbf{n}\cdot\mathbf{m}\, F_{43} + F_{44}]\};$$

$$n_{x'}^2 \operatorname{Im} M_1 - n'_z \operatorname{Im} M_3 + \operatorname{Im} M_5 = (4\pi)^{-2}\lambda \int d\Omega\{-F_{11} + \mathbf{n}\cdot\mathbf{m}\, F_{12} + \mathbf{n}'\cdot\mathbf{m}\, F_{21} -$$
$$- m_x^2 F_{14} - m_y^2 F_{41} - n'_z F_{22} + n'_x m_x \beta F_{23} - n'_z m_y^2 F_{32} + n'_x m_x F_{24} + n'_x m_x m_y^2 F_{34}\};$$

$$\operatorname{Im} M_3 - n'_z \operatorname{Im} M_5 = (4\pi)^{-2}\lambda \int d\Omega\{n'_z F_{11} - \mathbf{n}'\cdot\mathbf{m}\, F_{12} - \mathbf{n}\cdot\mathbf{m}\, F_{21} +$$
$$+ n'_z m_y^2 F_{14} - n'_x m_x \beta F_{41} + F_{22} + m_y^2 F_{23} + n_{x'}^2 \beta^2 F_{32} - n_{x'}^2 \beta F_{31} - n_{x'}^2 \beta m_y^2 F_{34}\};$$

$$2 \operatorname{Im} M_1 + n'_z \operatorname{Im} M_3 + \operatorname{Im} M_5 = (4\pi)^{-2}\lambda \int d\Omega \left\{-F_{11} + \left(\beta - m_x \frac{n'_z}{n'_x}\right) F_{12} - \right.$$
$$- \left(\beta n'_z - \frac{m_x}{n'_x}\right) F_{21} - m_x \frac{n'_z}{n'_x} F_{13} - \beta n'_z F_{31} - \mathbf{n}'\cdot\mathbf{m}\, \frac{m_x}{n'_x} F_{14} - \mathbf{n}\cdot\mathbf{m}\, \beta F_{41} + n'_z F_{22} +$$
$$+ \left(n'_z m_y^2 + \mathbf{n}\cdot\mathbf{m}\, \frac{m_x}{n'_x}\right) F_{23} + \left(n'_z m_y^2 + \beta\, \mathbf{n}'\cdot\mathbf{m}\right) F_{32} +$$
$$\left. + \frac{m_x}{n'_x} F_{24} + \beta F_{42} + \alpha n_{x'}^2 F_{33} + \alpha n_{x'}^2 \left(\beta F_{34} + \frac{m_x}{n'_x} F_{43}\right)\right\};$$

$$\operatorname{Im}(M_3 + M_4) = (4\pi)^{-2}\lambda \int d\Omega \left\{-\frac{m_x}{n'_x} F_{12} - \beta F_{21} + F_{22} + m_y^2 (F_{23} + F_{32}) + \right.$$
$$\left. + m_y^2 F_{33} + m_y^2 \left(\beta F_{34} + \frac{m_x}{n'_x} F_{43}\right)\right\}. \tag{17}$$

Here $n'_x = \sin\theta$, $n'_z = \cos\theta = \mathbf{n}\cdot\mathbf{n}'$, $m_x = \sin\theta' \cos\varphi'$, $m_y = \sin\theta' \sin\varphi'$, $m_z = \mathbf{n}\cdot\mathbf{m} = \cos\theta'$, $\mathbf{n}'\cdot\mathbf{m} = \cos\theta'' = \cos\theta\cos\theta' + \sin\theta\sin\theta'\cos\varphi'$, $d\Omega =$

$$= \sin\theta'\, d\theta'\, d\varphi', \quad \beta = \frac{(\mathbf{m}\times\mathbf{n}')_y}{n'_x}, \quad \alpha = \frac{1}{n'_x}\left[m_x m_z + \frac{n'_z}{n'_x}(m_y^2 - m_x^2)\right], \quad F_{ik} =$$
$$= F_i^+(\mathbf{n}'\cdot\mathbf{m}) F_k(\mathbf{n}\cdot\mathbf{m}), \quad \lambda = \frac{\omega E_k}{W}.$$

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Note: Figure translations are in progress. See original paper for figures.