EQUATIONS OF THE STEADY-STATE ASYNCHRONOUS OPERATION OF ELECTRICAL SYSTEMS

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Fig. 1

Figure 1: Fig. 1

Abstract

Full Text

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EQUATIONS OF THE STEADY-STATE ASYNCHRONOUS OPERATION OF ELECTRICAL SYSTEMS

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As a result of some accident, for example a short circuit or loss of excitation, one or several synchronous machines may fall out of synchronism and enter a regime of steady-state asynchronous operation. To study this regime it is necessary to know by what system of differential equations it is described and, in particular, it is very important to know under what conditions among these equations there will be differential equations with periodic coefficients. In this case the resulting system of differential equations should be solved with the aid of continuous- or discrete-computation calculating machines. It is, however, equally important to know under what conditions all the differential equations of the system under investigation will have only constant coefficients, since in that case their solution is easily obtained by the operational method or by the Fourier-integral method.

Consider a scheme consisting of two synchronous generators ($SG1$) and ($SG2$) and an asynchronous motor ($AM1$), connected to the terminals of $SG1$ (see Fig. 1). The designations of the instantaneous values of the longitudinal and transverse components of currents and voltages and their positive directions are given in Fig. 1.

Fig. 1

The equations for the regime of steady-state asynchronous operation that interests us are most naturally obtained as a special case of the equations of transient electromechanical processes, i.e., processes with variable and different angular velocities ω_1, ω_2, and ω_{d1} of the rotors of $SG1$, $SG2$, and $AM1$ ([1]).

Thus, in accordance with the problem posed, we shall assume that the angular velocities ω_1, ω_2, and ω_{d1} are given and constant, with ω_1 and ω_2 different from one another. We introduce the slip

$$s = \frac{\omega_1 - \omega_2}{\omega_1}, \tag{1}$$

which will also be a constant quantity.

As is known ([2–4]), the angles θ_1 and θ_2 between the longitudinal axes d_1 and d_2 of the rotors of $SG1$ and $SG2$ and the magnetic axis of phase a of the stator are determined as follows:

$$\theta_1 = \int_0^t \omega_1 dt + \theta_{10} = \omega_1 t + \theta_{10}, \qquad \theta_2 = \int_0^t \omega_2 dt + \theta_{20} = \omega_2 t + \theta_{20}, \tag{2}$$

$$\theta_2 - \theta_1 = (\omega_2 - \omega_1)t + \theta_{20} - \theta_{10} = -s\omega_1 t + \theta_{210}. \tag{3}$$

In view of the constancy of the angular velocities ω_1 and ω_2, the angular accelerations of the rotors of $SG1$ and $SG2$ are equal to zero:

$$\frac{d^2\theta_1}{dt^2} = \frac{d\omega_1}{dt} = 0, \qquad \frac{d^2\theta_2}{dt^2} = \frac{d\omega_2}{dt} = 0, \tag{4}$$

and the need to consider the equations of motion (5) disappears.

Generally speaking, after the numerical solution of one or another problem concerning the calculation only of electromagnetic transients in electric systems, the equations of motion of the machine rotors may serve to check how constant their angular velocities remain over the time interval under consideration. The initial value of the angle θ_{210}, as well as the voltages U_{1f} and U_{2f} at the terminals of the field windings of $SG1$ and $SG2$, are specified and constant.

Thus, in accordance with the notation adopted in (1), assuming that both synchronous generators have longitudinal and transverse damper windings, and considering only such commutations in which zero components of currents and voltages do not arise, we have the following system of differential equations:

$SG1:$

$$u_{1d} = -r_{c1} i_{1d} + \frac{d}{dt}\left(-L_{d1} i_{1d} + M_{f1} i_{1f} + M_{g1} i_{1g}\right) - \left(-L_{q1} i_{1q} + M_{h1} i_{1h}\right)\omega_1,$$

$$u_{1q} = -r_{c1} i_{1q} + \frac{d}{dt}\left(-L_{q1} i_{1q} + M_{h1} i_{1h}\right) + \left(-L_{d1} i_{1d} + M_{f1} i_{1f} + M_{g1} i_{1g}\right)\omega_1,$$

$$U_{1f} = r_{f1} i_{1f} + \frac{d}{dt}\left(-\frac{3}{2} M_{f1} i_{1d} + L_{f1} i_{1f} + M_{fg1} i_{1g}\right), \tag{5}$$

$$0 = r_{g1} i_{1g} + \frac{d}{dt}\left(-\frac{3}{2} M_{f1} i_{1d} + L_{g1} i_{1g} + M_{fg1} i_{1f}\right),$$

$$0 = r_{h1} i_{1h} + \frac{d}{dt}\left(-\frac{3}{2} M_{h1} i_{1q} + L_{h1} i_{1h}\right).$$

Referring further the equations of $IM1$ and of the line L to coordinate axes rigidly connected with the rotor of $SG1$, we obtain the following system of equations for $IM1$:

$IM1:$

$$u_{1d} = r_{\ 1} i_{\ 1d} + \frac{d}{dt}\left(L_{c11} i_{\ 1d} + L_{ad1} i_{1pd}\right) - \left(L_{c11} i_{\ 1q} + L_{ad1} i_{1pq}\right) \omega_1,$$

$$u_{1q} = r_{\ 1} i_{\ 1q} + \frac{d}{dt}\left(L_{c11} i_{\ 1q} + L_{ad1} i_{1pq}\right) + \left(L_{c11} i_{\ 1d} + L_{ad1} i_{1pd}\right) \omega_1,$$

$$0 = r_{p1} i_{1pd} + \frac{d}{dt}\left(L_{p11} i_{1pd} + L_{ad1} i_{\ 1d}\right) - \left(L_{ad1} i_{\ 1q} + L_{p11} i_{1pq}\right)(\omega_1 - \omega_{\ 1}), \quad (6)$$

$$0 = r_{p1} i_{1pq} + \frac{d}{dt}\left(L_{p11} i_{1pq} + L_{ad1} i_{\ 1q}\right) + \left(L_{ad1} i_{\ 1d} + L_{p11} i_{1pd}\right)(\omega_1 - \omega_{\ 1}).$$

Next we write the equations of Kirchhoff' s first law at points 1 and 2. Let us note that at point 2 they must be written not only taking into account that the positive directions of the currents in $SG2$ and in the line L are opposite, but also, chiefly, taking into account that the equations of the line L are referred to axes rigidly connected with the rotor of $SG1$, while the equations of $SG2$ are referred to axes rigidly connected with its rotor.

Node 1:

$$i_{1d} - i_{\ 1d} - i_{\ell d} = 0, \qquad i_{1q} - i_{\ 1q} - i_{\ell q} = 0. \quad (7)$$

Node 2:

$$i_{\ell d} + i_{2d} \cos(s\omega_1 t - \theta_{210}) + i_{2q} \sin(s\omega_1 t - \theta_{210}) = 0,$$

$$i_{\ell q} - i_{2d} \sin(s\omega_1 t - \theta_{210}) + i_{2q} \cos(s\omega_1 t - \theta_{210}) = 0. \quad (8)$$

SG2:

Fig. 2

Figure 2: Fig. 2

$$u_{2d} = -r_{c2}i_{2d} + \frac{d}{dt}\left(-L_{d2}i_{2d} + M_{f2}i_{2f} + M_{g2}i_{2g}\right) - \left(-L_{q2}i_{2q} + M_{h2}i_{2h}\right)\omega_2,$$

$$u_{2q} = -r_{c2}i_{2q} + \frac{d}{dt}\left(-L_{q2}i_{2q} + M_{h2}i_{2h}\right) + \left(-L_{d2}i_{2d} + M_{f2}i_{2f} + M_{g2}i_{2g}\right)\omega_2,$$

$$U_{2f} = r_{f2}i_{2f} + \frac{d}{dt}\left(-\frac{3}{2}M_{f2}i_{2d} + L_{f2}i_{2f} + M_{fg2}i_{2g}\right), \tag{9}$$

$$0 = r_{g2}i_{2g} + \frac{d}{dt}\left(-\frac{3}{2}M_{f2}i_{2d} + L_{g2}i_{2g} + M_{fg2}i_{2f}\right),$$

$$0 = r_{h2}i_{2h} + \frac{d}{dt}\left(-\frac{3}{2}M_{h2}i_{2q} + L_{h2}i_{2h}\right).$$

Line L:

$$u_{2d}\cos(s\omega_1 t - \theta_{210}) + u_{2q}\sin(s\omega_1 t + \theta_{210}) =$$

$$= u_{1d} - r_l i_{ld} - L_{l1}\frac{di_{ld}}{dt} + L_{l1}i_{lq}\omega_1, \tag{10}$$

$$-u_{2d}\sin(s\omega_1 t - \theta_{210}) + u_{2q}\cos(s\omega_1 t - \theta_{210}) =$$

$$= u_{1q} - r_l i_{lq} - L_{l1}\frac{di_{lq}}{dt} - L_{l1}i_{ld}\omega_1.$$

Thus, we have obtained 20 linear equations with 20 unknowns: u_{1d}, u_{1q}, i_{1d}, i_{1q}, i_{1f}, i_{1g}, i_{1h}, u_{2d}, u_{2q}, i_{2d}, i_{2q}, i_{2f}, i_{2g}, i_{2h}, i_{ld}, i_{lq}, i_{lpd}, i_{lpq}, i_{nd}, i_{nq}. Of the 20 equations, 4 differential equations (8) and (10) contain periodic coefficients, and 4 equations (7) and (8) are algebraic. The total order of the system of differential equations is 16. A number of the unknowns (i_{ld}, i_{lq}, u_{1d}, u_{1q}, etc.) can quite simply be eliminated. The resulting system is quite amenable to solution on continuous-computation machines of the corresponding types and, of course, on digital computers of any type.

Fig. 2

Hence we conclude that the equations for the regime of steady asynchronous operation in systems having two or more synchronous machines will always contain periodic coefficients. Their solutions, i.e., expressions for the currents and voltages, are represented by an infinite series of harmonics, the frequencies of which, as is known, can also be established from elementary considerations, by decomposing each of the pulsating magnetic fields arising in the single-axis windings (excitation and damper windings) of synchronous machines into two fields rotating with the pulsation frequency in opposite directions relative to these windings.

If SG2 is replaced by an infinite-bus system (Fig. 2), whose voltage frequency is ω_2, then the problem is substantially simplified. Equations (8) and (9) drop out, since at node 2 the infinite-bus system maintains a constant voltage. In this case, as indicated in (3), it is necessary to put

$$u_{2d} = 0, \qquad u_{2q} = U = \text{const.} \tag{11}$$

Then equations (10) are rewritten in the form

$$U \sin(s\omega_1 t - \theta_{210}) = u_{1d} - r_l i_{ld} - L_{l1}\frac{di_{ld}}{dt} + L_{l1} i_{lq}\omega_1,$$

$$U \cos(s\omega_1 t - \theta_{210}) = u_{1q} - r_l i_{lq} - L_{l1}\frac{di_{lq}}{dt} - L_{l1} i_{ld}\omega_1. \tag{12}$$

In this particular case we have 13 linear equations (5), (6), (7), and (12), with constant coefficients, containing, naturally, 13 un-

known functions: u_{1d}, u_{1q}, i_{1d}, i_{1q}, i_{1f}, i_{1g}, i_{1h}, i_{1d}, i_{1q}, i_{1pd}, i_{1pq}, i_d, i_q. Of these, 2 equations are algebraic. The total order of the resulting system of differential equations is equal to 11.

Thus, in the case of a circuit consisting of a single synchronous generator connected by a transmission line to infinite buses, the differential equations for the steady asynchronous operating regime do not contain periodic coefficients.

The steady asynchronous operating regime will be described by a particular solution of this system of linear differential equations. It is easy to see that in this case all currents and voltages will have, in addition to a constant component, also terms of the form $A_{1m}\sin(s\omega_1 t + \alpha_1)$ and $A_{2m}\cos(s\omega_1 t + \alpha_2)$. Passing, by the known formulas, from the d, q components to phase currents and voltages,

$$i_a = \text{Re}\{i_1 e^{j\theta_1}\} = \text{Re}\{(i_{1d} + j i_{1q}) e^{j(\omega_1 t + \theta_{10})}\}, \tag{13}$$

we obtain that, for example, the currents in the stators $SG1$, $AM1$ and in the line L will have components with frequencies ω_1, $(1-s)\omega_1$ and $(1+s)\omega_1$, which can also be established directly from physical considerations.

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CITED LITERATURE

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Note: Figure translations are in progress. See original paper for figures.