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# Physics

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## Abstract

## Full Text

Physics

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# On the Question of the Structure of the Proton

*(Presented by Academician N. N. Bogolyubov, 28 XI 1957)*

In the work of Chambers and Hofstadter <sup>(1)</sup>, the electromagnetic structure of the proton was investigated experimentally. Of the six possible distribution functions for the density  $d(r)$  of charge in the proton, the authors consider the function

$$d(r) = d_0 r \exp(-r), \quad (1)$$

to be the most preferable, where  $r$  is the distance to the center of the proton, expressed in the corresponding units.

A theoretical investigation of the question of the structure of the proton would be most consistent if this question were considered on the basis of a rigorous field theory. However, since such a consideration encounters serious difficulties, we shall solve this question approximately, proceeding from certain phenomenological premises.

In this connection we shall make use of the well-known representation of the proton as a core surrounded by a meson cloud <sup>(2,3)</sup>. Let this cloud be formed by one meson.

From the known relativistic equation for the  $\Psi$ -wave function of a particle of mass  $m$  and spin 0 moving in a field with potential  $V(r)$  (see, for example, <sup>(4)</sup>), we have:

$$\Delta\Psi + \frac{1}{\hbar^2 c^2} [(E - V)^2 - (mc^2)^2] \Psi = 4\pi D, \quad (2)$$

where  $E = i\hbar \cdot \partial/\partial t$ ;  $D$  is the source density;  $\hbar$  and  $c$  are known constants. Assuming that the process of meson emission is completed and that the meson interacts with the core, we obtain:

$$\Delta\Psi + \frac{1}{\hbar^2 c^2} [(E - V)^2 - (mc^2)^2] \Psi = 0. \quad (3)$$

To determine the interaction potential  $V$ , we shall use I. E. Tamm's hypothesis of the "dissociation" of a meson in the field of the core into a nucleon-antinucleon

pair\*. Then, as the interaction potential of such a pair with the core, one may take the usual Yukawa potential

$$V(r) = -\frac{g_1 g_2}{r} \exp(-\varkappa r), \quad (4)$$

where  $r$  is the distance from the pointlike core to the center of the meson-pair\*\*;  
 $g_1$  is the nuclear charge of the core;  $g_2$  is the “effective” charge of the meson-pair,  
 $\varkappa = \mu c/\hbar$ ;  $\mu$  is the mass of an ordinary  $\pi$ -meson.

\* Obviously, as a result of such “dissociation,” the meson mass  $m$  increases in comparison with the mass of an ordinary  $\pi$ -meson. In the first approximation, for simplicity, we regard the core as fixed at the origin of coordinates.

\*\* We note that the linear dimensions of such a pair  $\sim \hbar/2Mc$ , where  $M$  is the nucleon mass, will be considerably smaller than the radius of the meson cloud  $\sim \hbar/\mu c$ .

We seek the solution of equation (3) in the form

$$\Psi(\mathbf{r}, t) = \exp\left(-i\frac{\varepsilon}{\hbar}t\right) \Phi(\mathbf{r}), \quad (5)$$

where  $t$  is time;  $\varepsilon$  is a constant with the dimension of energy, which for simplicity we take to be real. This means that we seek stationary solutions for our core-meson system, assuming that the violation of stationarity occurs, for example, in accordance with I. E. Tamm’s hypothesis, because of annihilation of the central core with an antinucleon arising when a pair is formed.

Introducing polar coordinates, we represent the spatial part of the function  $\Phi(\mathbf{r})$  in the form

$$\Phi(\mathbf{r}) = R(r)Y(\vartheta, \varphi). \quad (6)$$

By the usual procedure, from equation (3), taking account of (5) and (6), we obtain for the angular part  $Y(\vartheta, \varphi)$  of the function  $\Psi$  the known equation whose solutions are the spherical functions:

$$Y_{lm} = \left(\frac{2l+1}{4\pi} \frac{(l-m)!}{(l+m)!}\right)^{1/2} P_l^m(\cos \vartheta) e^{im\varphi} \quad (7)$$

(where  $l$  and  $m$  are integers:  $l = 0, 1, 2, \dots$ ;  $m = -l, \dots, -1, 0, 1, \dots, l$ ;  $P_l^m$  are the unnormalized associated Legendre functions of order  $m$ ), and the equation for the radial part  $R(r)$ :

$$\frac{d^2 R}{dr^2} + \frac{2}{r} \frac{dR}{dr} + \left[ A + \frac{2\beta\varepsilon \exp(-\varkappa r)}{\hbar c} + \frac{\beta^2}{r^2} \exp(-2\varkappa r) - \frac{l(l+1)}{r^2} \right] R = 0 \quad (8)$$

(where  $\beta = g_1 g_2 / \hbar c$ ;  $A = (\varepsilon^2 - m^2 c^4) / \hbar^2 c^2$ ).

For the existence of bound solutions it is necessary that  $A < 0$ , i.e., that  $\varepsilon < mc^2$ . Then as  $r \rightarrow \infty$  the function  $R(r)$  will behave like  $\exp(\pm \rho/2)$ , where  $\rho = 2r/r_0$ ,  $r_0 = (\sqrt{-A})^{-1}$ . Obviously, we must take the minus sign in the exponent. Therefore we shall seek the solution of equation (8) in the form

$$R(r) = \exp(-\rho/2)f(\rho). \quad (9)$$

The equation for  $f(\rho)$  has the form

$$f'' + \left(\frac{2}{\rho} - 1\right) f' + \left\{ \frac{1}{\rho} (r_0 B e^{-\alpha \rho} - 1) + \frac{1}{\rho^2} [\beta^2 e^{-2\alpha \rho} - l(l+1)] \right\} f = 0, \quad (10)$$

where  $B = \beta \varepsilon / \hbar c$ ,  $\alpha = \varkappa r_0 / 2$  (primes denote differentiation with respect to  $\rho$ ). Expanding in this equation  $\exp(-\alpha \rho)$  and  $\exp(-2\alpha \rho)$  in known series, we seek, in accordance with (6), solutions of equation (10) in the form

$$f(\rho) = \rho^j w(\rho), \quad w(\rho) = \sum_{k=0}^{\infty} a_k \rho^k. \quad (11)$$

Proceeding in the usual way, we obtain:

$$j = -\frac{1}{2} \pm \sqrt{\left(l + \frac{1}{2}\right)^2 - \beta^2}; \quad (12)$$

$$a_1 = \left[ \frac{1}{2} + \frac{r_0(\varkappa \beta^2 - B)}{2(j+1)} \right] a_0,$$

$$a_k = \frac{1}{k(k+2j+1)} \left\{ a_{k-1} [k+j+r_0(\varkappa \beta^2 - B)] + r_0^2 \varkappa \sum_{\nu=0}^{k-2} (-1)^\nu a_{k-2-\nu} \frac{(r_0 \varkappa)^\nu}{(\nu+1)!} \left( \frac{B}{2^{\nu+1}} - \frac{\beta^2 \varkappa}{\nu+2} \right) \right\} \quad (k \geq 2). \quad (13)$$

Near  $r = 0$ , where one may restrict oneself to the initial terms of the expansions  $\exp(-\chi r)$  and  $\exp(-2\chi r)$ , equation (8) can be reduced to a degenerate hypergeometric equation, and its solution will have a form analogous to (9), but  $\omega(\rho)$  here must be replaced by a degenerate hypergeometric function (of Pochhammer).

In the case of arbitrary values of  $\rho$ , it is not difficult to see that, owing to the first term of formula (13), the  $k$ -th term of series (11), as  $k \rightarrow \infty$ , will behave approximately as  $(d/k!) \rho^k$  ( $d = \text{const}$ ), i.e., as  $\rho \rightarrow \infty$  the sum of this series

will grow as  $\exp \rho$ . Therefore it is necessary to terminate the increasing part of series (11) at some  $n$ -th term, taking

$$n + j + r_0(\chi\beta^2 - B) = 0. \quad (14)$$

Then, instead of the increasing part of the series, we obtain a polynomial of degree  $(n-1)$ , while  $R(\rho)$  with the remaining (infinite) part of series (11) decreases rapidly as  $\rho \rightarrow \infty$ .

From (14) it is not difficult to find  $\varepsilon$ :

$$\varepsilon = mc^2 \frac{\beta^3(\mu/m) \pm (n+j)\{[\beta^2 + (n+j)^2] - \beta^4(\mu/m)^2\}^{1/2}}{\beta^2 + (n+j)^2}, \quad (15)$$

and then also  $r_0$  as a function of  $n$ ,  $j$ ,  $\beta$ , and  $m$ .

Thus, the wave function of a meson with mass  $m$ , forming a cloud around the nucleus, has the form

$$\Psi(\mathbf{r}, t) = \exp\left(-i\frac{\varepsilon}{\hbar}t\right) Y_l^m(\vartheta, \varphi) R(r), \quad (16)$$

where

$$R(r) = e^{-\rho/2} \rho^j \sum_{k=0}^{\infty} a_k \rho^k. \quad (17)$$

From the normalization conditions

$$a_0 \simeq \left[ \frac{2(m^2 c^4 - \varepsilon^2)^{1/2}}{\hbar c} \right]^{j+1/2} [\Gamma(2j+3)]^{-1/2} \quad (l=1, 2).$$

Let us note that for  $\beta^2 > 0$ , according to (12),  $j$  will be positive (i.e.,  $R(r)$  will not have a pole at zero) only when  $l > 1$ . Thus the lowest state of the system under consideration will be the  $P$ -state.

The dependence of the meson-cloud density on distance agrees approximately with experiment for  $2j = 1$ . Hence  $\beta = g_1 g_2 / \hbar c \simeq 1.118$ .

In conclusion I consider it my duty to express deep gratitude to Acad. N. N. Bogolyubov for valuable consultations.

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## CITED LITERATURE

1. E. E. Chambers, R. Hofstadter, *Phys. Rev.*, **103**, No. 5, 1454 (1956).
2. E. Fermi, *Lectures on  $\pi$ -Mesons and Nucleons*, Moscow, 1956.
3. E. Fermi, *Elementary Particles*, Moscow, 1952.
4. H. Bethe, *Lectures on the Theory of the Atomic Nucleus*, Moscow, 1949.
5. P. Gombás, *The Problem of Many Particles in Quantum Mechanics*, Moscow, 1952.
6. V. I. Smirnov, *A Course of Higher Mathematics*, 2, Moscow, 1954, p. 139.

*Note: Figure translations are in progress. See original paper for figures.*

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