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Abstract

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ON THE THEORY OF THE BUBBLE CHAMBER

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1. The phenomenon, discovered by Glaser in 1952 ([1]), of the boiling of a superheated liquid under the action of radiation and the formation of individual vapor bubbles along the tracks of charged particles was subsequently observed in many liquids with very different properties. At present one may assert that this phenomenon must be based on some general nature of the interaction of fast charged particles with a metastable liquid.

In the experiments carried out up to the present time, only the number of bubbles per unit length of track g has been studied quantitatively. In papers ([2,3]) the first data are given on the dependence of g on $\beta = v/c$ and on the parameters of the liquid p and T. A satisfactory theory explaining the phenomenon itself and the observed dependences is still lacking.

Below an attempt is made to explain the available experimental data, based on the idea that the formation of visible bubbles along the track is the result of the entry of subcritical embryos into the region of substantial localization of the energy lost by the charged particle. (In a moderately superheated liquid, in the absence of boiling, there are always vapor bubbles of subcritical dimensions, whose distribution changes comparatively little up to a time of the order of the intrinsic boiling time.) The consequence of this is an increase of the radius of the embryo to a size exceeding the critical radius R_k. We shall define the region of localization by considering, for definiteness, fast heavy particles ($\chi = Z_0 e^2/\hbar v \ll 1$; $Z_0 e$ is the charge of the particle) and a liquid whose molecules are formed by light atoms.

2. We shall use the fact that the influence of such a particle in distant collisions is equivalent to the influence of a perturbing potential whose center moves rectilinearly with constant velocity $\mathbf{v}$ ([4]). Characterizing the position of the nucleus of an atom of the substance by a definite value r (we neglect the polarization-field effect) and taking into account that r is large in comparison with the atomic dimensions a, for the transition probability we have

$$w_{0n} = \frac{1}{\hbar^2} \left| \int_{-\infty}^{\infty} (\mathbf{d})_{0n} \cdot \mathbf{E}(r)\, e^{\frac{i}{\hbar}(E_n - E_0)t}\, dt \right|^2 . \quad (1)$$

Here $\mathbf{E}(r)$ is the electric field created by the moving particle at a distance r from the trajectory ([5]); $\mathbf{d}$ is the dipole moment.

Effectively, transitions occur in the region of impact parameters bounded above by the adiabaticity parameter

$$p \sim \hbar v/\sqrt{1-\beta^2}\, I \tag{2}$$

(I is a certain mean ionization potential).

Multiplying w_{0n} by $E_n - E_0$ and summing over all states of the discrete and continuous spectra. Restricting the consideration to values of r,

smaller than p, and making use of the known summation theorem [(6)], for the energy released in a unit volume we obtain

$$q_1 = 2NZZ_0^2 e^4/mc^2\beta^2 r^2, \tag{3}$$

where N is the number of atoms of the substance per unit volume, and Z is the atomic number.

In the region $r > a$, a substantial contribution to the localization of energy losses will be made by the slowing down of secondary electrons that have acquired, in a collision, an energy $\gg 2I$. For $\chi \ll 1$ these electrons are produced in the region $r \ll a$, and the probability of their production per unit path with energy ε_0 is determined by the expression

$$d\Phi(\varepsilon_0) \simeq \frac{L}{\beta^2}\frac{d\varepsilon_0}{\varepsilon_0^2}; \qquad L = \frac{2\pi NZZ_0^2 e^4}{mc^2} \tag{4}$$

((4) loses accuracy for ε_0 close to I).

Most secondary electrons, with the exception of δ-electrons with energy close to the maximum (which are not important for our consideration), are scattered through an angle $\simeq \pi/2$. Accordingly, the problem may be regarded as the slowing down of electrons with the energy spectrum (4), emitted from the line $r = 0$ at right angles to it.

The slowing down of the secondary electrons will be accompanied by scattering, which after passage through a layer of a certain thickness will lead to a diffusive character of the motion. For the boundary of transition between the regions of directed and diffusive motion, following Bethe, we choose the condition that $\overline{\cos\theta}$ decrease by a factor e.

The value of r corresponding to an energy loss in the first region from ε_0 to ε can be represented in the form

$$r = \frac{1}{\omega}\int_{\varepsilon}^{\varepsilon_0} \overline{\cos\theta}\,\frac{1}{B}\,\varepsilon\,d\varepsilon, \qquad B = \ln\frac{\varepsilon}{I}\sqrt{\frac{e}{2}}, \qquad \omega = 2\pi N Z e^4. \tag{5}$$

For $\overline{\cos\theta}$ we have the expression

$$\overline{\cos\theta} = \exp\left\{-\frac{1}{\omega}\int_{\varepsilon}^{\varepsilon_0}\frac{K}{B}\,\varepsilon\,d\varepsilon\right\}, \tag{6}$$

$$K = N\int_0^{\pi}\sigma(\vartheta)(1-\cos\vartheta)\,d\Omega \tag{7}$$

($\sigma(\vartheta)d\Omega$ is the differential effective cross section for scattering of electrons by atoms of the substance [(8)]).

Transforming (7), with accuracy up to a factor ~ 1 under the logarithm sign, permits (6) to be reduced to the form

$$\overline{\cos\theta} = \exp\left\{-\frac{Z}{4}\int_{\varepsilon/\varepsilon_0}^{1}\frac{A}{B}\,\frac{1}{(\varepsilon/\varepsilon_0)}\,d\left(\frac{\varepsilon}{\varepsilon_0}\right)\right\}, \qquad A = \ln\frac{8\hbar^2 me}{e^4 Z I_s}.$$

If one neglects the weak dependence of A/B on ε_0, apart from $\varepsilon/\varepsilon_0$, it follows from this that $\overline{\cos\theta} \simeq \varphi(\varepsilon/\varepsilon_0)$ and, in particular, a decrease of $\overline{\cos\theta}$ by a factor e approximately corresponds to a definite value $\varepsilon'/\varepsilon_0$.

As a consequence of this,

$$r \simeq \varepsilon_0^2\psi(\varepsilon/\varepsilon_0). \tag{8}$$

Hence

$$\varepsilon/\varepsilon_0 \simeq \psi_1(r/\varepsilon_0^2). \tag{9}$$

For the density of energy losses we have

$$q = \frac{1}{2\pi r}\int_{\varepsilon_{0\,\min}}^{\varepsilon_{0\,\max}}\left(-\frac{d\varepsilon}{dr}\right)d\Phi(\varepsilon_0). \tag{10}$$

Using (9), after transformation we obtain

$$q_2 \cong \frac{L}{4\pi r^2\beta^2}\left\{\psi_1\left(\frac{r}{\varepsilon_{0\,\max}^2}\right) - \psi_1\left(\frac{r}{\varepsilon_{0\,\min}^2}\right)\right\}. \tag{11}$$

Considering the region r, where secondary electrons with $\varepsilon_{0\,\min}$ practically no longer contribute to the energy losses, while the losses of the fastest electrons are still small, we find, in accordance with (9),

$$q_2 \cong L/4\pi r^2\beta^2. \qquad (12)$$

For the second region it is necessary to consider the slowing down during diffusion of electrons emitted from a cylindrical surface of radius

$$r' = \varepsilon_0^2\psi(\varepsilon'/\varepsilon_0). \qquad (13)$$

We shall use the "age" equation with τ, defined as

$$\tau = \frac{1}{3\omega}\int_{\varepsilon}^{\varepsilon'}\frac{1}{KB}\,\varepsilon\,d\varepsilon \cong \frac{4\varepsilon_0^4}{\omega^2 Z}\int_{\varepsilon/\varepsilon_0}^{\varepsilon'/\varepsilon_0}\frac{1}{AB}\left(\frac{\varepsilon}{\varepsilon_0}\right)^3 d\left(\frac{\varepsilon}{\varepsilon_0}\right). \qquad (14)$$

The solution of this equation, taking into account the initial condition $\tau = 0$; $f = \text{const}\cdot\delta(r - r_a)$, leads to the expression for the distribution function

$$f = \text{const}\,\frac{1}{4\pi\tau}\exp\left(-\frac{{r'}^2 + r^2}{4\tau}\right)J_0\left(\frac{ir'r}{2\tau}\right)$$

(J_0 is the Bessel function of zero order).

Let us determine $\overline{r^2}$ for electrons of one age. Omitting the intermediate calculations, we write the final result $\overline{r^2} = {r'}^2 + 4\tau$. The quantity $r = (\overline{r^2})^{1/2}$ may be regarded as the effective distance over which the electron energy changes from ε' to ε. Then, taking (13) and (14) into account, one can write

$$r^2 \cong \varepsilon_0^4\xi(\varepsilon/\varepsilon_0). \qquad (15)$$

Hence

$$\varepsilon/\varepsilon_0 \cong \xi_1(r^2/\varepsilon_0^4). \qquad (16)$$

The density of energy losses in the region under consideration is determined by an expression analogous to (10). Using (16), we find

$$q_3 \cong \frac{L}{4\pi r^2\beta^2}\left\{\xi_1\left(\frac{r^2}{\varepsilon_{0\,\max}^4}\right) - \xi_1\left(\frac{r^2}{\varepsilon_{0\,\min}^4}\right)\right\}.$$

The same arguments as in the transition from (11) to (12) make it possible to simplify this expression:

$$q_3 \cong L/4\pi r^2\beta^2. \tag{17}$$

The total density of energy losses by secondary electrons is

$$q' = q_2 + q_3 = \alpha' \frac{NZZ_0^2 e^4}{mc^2} \frac{1}{\beta^2 r^2} \qquad (\alpha' \sim 1). \tag{18}$$

The total amount of energy localized in a unit volume at a distance r from the track, for $a < r < \rho$, is determined by the expression:

$$q = q_1 + q' = 3\alpha \frac{NZZ_0^2 e^4}{mc^2} \frac{1}{\beta^2 r^2} \quad (\alpha \sim 1). \tag{19}$$

3. The processes occurring in matter when a fast charged particle passes through it are associated either with the formation of ions, or with the release of heat, or with both simultaneously. Their macroscopic effectiveness obviously depends in an essential way on the magnitude of q.

Let us consider the region where $q \leq q_0$ (q_0 is some fixed value of the density). From the results obtained it follows that the dimensions of this region are inversely proportional to β,

$$r_0 = \gamma/\beta. \tag{20}$$

Hence an important conclusion follows: all processes will occur similarly in the volume of a cylinder around the track whose radius varies as $1/\beta$. We note that the limiting dimensions of the region where ionization or excitation can in principle take place will increase as β increases.

4. Visible vapor bubbles along the track of a charged particle are formed as a result of nuclei entering a cylindrical volume of radius $R = r_0 + R_{\text{eff}}$, where R_{eff} is the effective radius of nuclei being transformed through R_k ($R_{\text{eff}} \leq R_k$). Then the number of particles per unit length of track is determined as

$$g \simeq N_{\text{eff}}\pi \left(R_{\text{eff}} + \gamma/\beta\right)^2, \tag{21}$$

where N_{eff} is the number of nuclei per unit volume of liquid whose radius lies within the limits $R_{\text{eff}} < R < R_k$.

(21) in fact takes into account the purely thermal action of the passing particle. Ions can promote bubble growth only under the condition that they are symmetrically distributed around the nucleus, and therefore their positive role can appear only when the center of the bubble falls inside the region (20). Consequently, if ions make some contribution to the formation of

visible bubbles, then an analogous expression with $R_{\text{eff}} = 0$ should be added to (21). We note that a term in g independent of β can appear only in those special cases when $\gamma/\beta \sim R_{\text{eff}}$.

Expression (21) qualitatively describes the existing experimental facts. Indeed, in a sufficiently superheated liquid $r_0 \gg R_{\text{eff}}$, and (21) becomes

$$g \simeq N_{\text{eff}} \gamma_1 \frac{NZZ_0^2}{\beta^2}.$$

A similar character of the dependence on β was observed in work ([2]). The presence of a velocity-independent term in g was first described in ([3]). The observed exponential character of the dependence of g on T ([3]) and $p_\infty - p$ ([2]) is evidently associated with N_{eff}.

Finally, the absence of boiling in xenon ([7]) can be easily explained by the small value of γ (large q_0 because of the delocalization of part of the energy due to scintillation), and also by the small magnitude of N_{eff} (large R_{eff}—a pure liquid).

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Note: Figure translations are in progress. See original paper for figures.