



Soviet-era science, translated into English

PHYSICS

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1958

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Abstract

Full Text

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CYCLOTRON RESONANCE IN FILMS

(Presented by Academician L. D. Landau, 31 XI 1957)

1. Cyclotron resonance in a bulk metal, the theory of which was constructed in ^(1,2), occurs in a constant magnetic field $\mathbf{H}(H, 0, 0)$, parallel to the surface of the metal, when the frequency ω of the external alternating field is a multiple of the Larmor frequency of electron precession $\Omega = eH/mc^*$ (the notation is the same as in ^(1,2)).

In the present communication we give the results of calculating the surface impedance of a monocrystalline metal film of thickness d under conditions of a sharply anomalous character of the skin effect, i.e., when the effective penetration depth of the current $\delta_{\text{eff}} = (c^2|Z|/2\pi\omega)$ is small in comparison with all the other length parameters: the maximum orbit radius r , the mean free path l , and the film thickness d . In this case, naturally, it is assumed that the mean free path l is the largest of all these quantities, since the condition $2\pi r \ll l$ ($2\pi r/l = 2\pi/\Omega t_0 \approx 2\pi/\omega t_0 \ll 1$) ensures the possibility of resonance, while the influence of the sample dimensions can be appreciable only when $d < 2r$. Indeed, when $d > 2r$ ($d - 2r \gg \delta_{\text{eff}}$), the electrons participating in the resonance “know” nothing of the presence of the second boundary of the film, and the surface impedance of the film in this case coincides with the impedance of a bulk metal (to within terms of order $\exp(-d/\delta_{\text{eff}})$).

When $d < 2r$, that part of the electrons for which the diameter of the orbit in the magnetic field $2\rho = 2r \sin \varphi > d$ (φ is the angle between the direction of \mathbf{H} and the electron velocity \mathbf{v}) undergo collisions with both sides of the film and therefore does not participate in the resonance (diffuse scattering of the electrons is assumed). The remaining electrons, whose velocity makes a small angle with the magnetic field ($2\rho < d$), still “know” nothing of the second boundary and give the same contribution to the resonance as in a bulk sample ($d = \infty$).

Starting from this physical picture of the phenomenon, it is easy to understand the formulas given below for the current density and the surface impedance of the film, obtained by a rigorous solution of the equations of the problem.

2. The complete system of equations consists of Maxwell's equations and the kinetic equation for the addition to the Fermi distribution function.

In ⁽¹⁾ it was shown that, in order to find the tensor of the total surface impedance

$$Z_{\mu\nu} = (4\pi\omega/ic^2) \partial E_\mu(0)/\partial E'_\nu(0),$$

it is convenient to pass in the equations to Fourier components with respect to the coordinates. In this case

$$j_\mu(k) = j_\mu^{(0)}(k) - \frac{1}{\pi} \int_0^\infty q_{\mu\nu}(k; k') \mathcal{E}_\nu(k') dk', \quad (1)$$

where $j_\mu^{(0)}(k)$ is the Fourier component of the current density in a bulk metal (taking into account collisions of electrons with the boundary $z = 0$) ^(1,2), and the second term

* Recently several experimental papers have appeared ⁽³⁾, in which observation is reported of the effect predicted in ⁽¹⁾ for a number of metals (Sn, Bi, Pb, Cu).

in (1) is due to the presence of the second surface of the film $z = d$:

$$q_{\mu\nu}(k, k') = \frac{2e^2}{h^3} \int_0^\infty -\frac{\partial f_0}{\partial \varepsilon} \partial \varepsilon \int_{p_H^{\min}}^{p_H^{\max}} \frac{m}{\Omega^2} dp_H \int_0^{2\pi} v_\mu(\tau) d\tau \int_{-\infty}^\tau |v_z(\tau_1)| \cos[kd \operatorname{sgn} v_z(\tau_1) + \frac{k}{\Omega} \int_{\tau_1}^\tau v_{zd}\tau_2] d\tau_1 \int_{s(\tau_1)}^{\tau_1} \exp\left(\int_\tau^\xi \gamma d\tau_2\right) v_\nu(\xi) \cos\left[k'd \operatorname{sgn} v_z(\tau_1) + \frac{k'}{\Omega} \int_{\tau_1}^\xi v_{zd}\tau_2\right] d\xi. \quad (2)$$

In formula (2) and in $j_\mu^{(0)}(k)$, through $s(\tau)$ (see ^(1,2)) is denoted the nearest root preceding τ of one of the following equations (see ⁽⁵⁾):

$$\frac{1}{\Omega} \left| \int_{s(\tau)}^\tau v_{zd}\tau_2 \right| = 0; \quad d \quad (\text{for the other notation see } (1,2)).$$

Formulas (1), (2) are exact and are valid for any angle of inclination of **H** relative to the film and arbitrary relations between the parameters entering these formulas. For $d \rightarrow \infty$, $j_\mu(k) \rightarrow j_\mu^{(0)}(k)$.

The calculation of $j_\mu(k)$ in the general case appears difficult. We give the final result of the calculation by the stationary-phase method for the case when $\delta_{\text{eff}}(\sim 1/|k|) \ll d$, $\min(v/\Omega)$, $l/(1 + \omega t_0)$ (anomalous skin effect), $d < c(p_y^{\max} - p_y^{\min})/eH = 2r$ (for the quadratic dispersion law $\varepsilon(\mathbf{p}) = p^2/2m$ these inequalities correspond to $\delta_{\text{eff}} \ll d$, r , $l/(1 + \omega t_0)$; $d < 2r$):

$$j_{\mu}(k) = \frac{3\pi}{4} \left\{ \frac{a_{\mu\nu}}{k} \mathcal{E}_{\nu}(k) - \frac{(a_{\mu\nu} - c_{\mu\nu})}{\pi} \int_0^{\infty} \frac{\mathcal{E}_{\nu}(k') dk'}{(k+k')\sqrt{kk'}} \right. \\ \left. - \frac{2}{\pi^2} c_{\mu\nu} \int_0^{\infty} \frac{\ln(k/k')}{k^2 - k'^2} \mathcal{E}_{\nu}(k') dk' \right\} + \frac{a_{\mu\nu}}{k} \mathcal{E}_{\nu}(k') O\left(\frac{1}{\sqrt{kr}}\right), \quad (3)$$

where

$$a_{\mu\nu} = \frac{16e^2}{3h^3} \left\{ \int_{\varphi_1}^{\varphi_2} \frac{n_{\mu}n_{\nu}}{K} \frac{d\varphi}{1 - \exp(-2\pi\bar{\gamma})} + \int_{\varphi_2}^{\pi+\varphi_1} \frac{n_{\mu}n_{\nu}}{K} d\varphi \right\}; \\ c_{\mu\nu} = \frac{4e^2}{3h^3} \left\{ \int_{\varphi_1}^{\varphi_2} \frac{n_{\mu}n_{\nu}}{K} (3 + \exp(-2\pi\bar{\gamma})) + 3 \int_{\varphi_2}^{\pi+\varphi_1} \frac{n_{\mu}n_{\nu}}{K} d\varphi \right\}; \quad (4) \\ 2\pi\bar{\gamma} = 2\pi i \frac{\omega}{\Omega} + \frac{2\pi\nu_0}{\Omega}, \quad \nu_0 \equiv \frac{1}{t_0} = \frac{1}{2\pi} \int_0^{2\pi} \frac{d\tau}{t_0(p)},$$

and the points φ_{α} , $\pi + \varphi_{\alpha}$ are determined from the equation $c|p_y(\tau_1) - p_y(\tau_2)| = eHd$ ($v_z(\tau_{\alpha}) = 0$); moreover, for $\varphi_1 < \varphi < \varphi_2$, $\pi + \varphi_1 < \varphi < \pi + \varphi_2$ the electron trajectory is located entirely inside the film, while in the remaining cases the electron collides with one of the film surfaces at each revolution (see Fig. 1). For simplicity only one convex centrally symmetric Fermi surface is considered, although the result is readily written also in the general case of a nonconvex surface. In the presence of several Fermi surfaces (several bands), $a_{\mu\nu}$ and $c_{\mu\nu}$ are sums of expressions analogous to (4) over all bands. When $d \geq 2r$, $\varphi_2 \equiv \pi + \varphi_1$ and (3) coincides with formula (5.13) for $A_{\mu\nu}$ and $C_{\mu\nu}$ in a bulk metal⁽¹⁾. For the quadratic dispersion law and $\nu_0 = \text{const}$

$$a_{xx}^{yy} = \frac{\sigma}{l} \left\{ \frac{2\varphi_0 \pm \sin 2\varphi_0}{\pi(1 - \exp(-2\pi\bar{\gamma}))} + 1 - \frac{2\varphi_0}{\pi} \pm \sin 2\varphi_0 \right\}; \quad a_{xy} = 0; \quad (5) \\ \frac{\sigma}{l} = \frac{ne^2}{p} = \frac{16\pi me^2 \zeta}{3h^3}; \quad \varphi_0 = \arcsin\left(\frac{d}{2r}\right).$$

(an analogous form is taken by $c_{\mu\nu}$; here the Ox axis is chosen along the direction of the vector \mathbf{H}).

In that range of frequencies and magnetic fields where

$$|1 - \exp(-2\pi\bar{\gamma})| \ll 1$$

(i.e., near resonance and in a strong magnetic field ($2\pi|\bar{\gamma}| \ll 1$), terms not containing $[1 - \exp(-2\pi\bar{\gamma})]^{-1}$ may be neglected (if φ_0 is not small).

The final formulas for the surface impedance of the film will be written only for a quadratic dispersion law, limiting ourselves, in the case of an arbitrary dispersion law, to general remarks*. For $\nu_0 = \text{const}$

$$Z_{xx}^{(\text{pl})} = Z_{yy}^{(\text{mass})} \left\{ \frac{2\varphi_0 \pm \sin 2\varphi_0}{\pi} \right\}^{-1/s}; \quad Z_{xy}^{(\text{pl})} = 0; \quad (6)$$

$$Z_{(\text{mass})} = Z_0 \{1 - \exp(-2\pi\bar{\gamma})\}^{-1/s};$$

Fig. 1. Fermi surface $\varepsilon(\mathbf{p}) = \zeta$

Z_0 is the impedance of a bulk metal at $H = 0$ (6). If $\varphi_0 \ll 1$, then in calculating the surface impedance it is necessary to retain all terms in $j_\mu(k)$ (including the terms with $c_{\mu\nu}$).

3. An analysis of formulas (3)–(6) shows that in a film of thickness $d < 2r$ there is a peculiar anisotropy of the surface impedance with respect to the polarization of the incident wave (which is absent in a bulk metal): $Z_{xx} \neq Z_{yy}$. This anisotropy is connected with the fact that the relative contribution of the “resonant” electrons to the current density is different for $\mathbf{j} \parallel \mathbf{H}$ and $\mathbf{j} \perp \mathbf{H}$, owing to collisions of some of the electrons (not participating in the resonance) with the sides of the film.

This anisotropy is also manifested in the fact that Z_{xx} at $d = 2r$ ($\varphi_2 = \pi + \varphi_1$) is continuous together with its first two derivatives with respect to H , whereas Z_{yy} is continuous, but its derivative with respect to H has a finite jump. **If the Fermi surface $\varepsilon(\mathbf{p}) = \zeta$ is nonconvex (or splits into several surfaces), then the derivative $\partial Z_{yy}/\partial H$ must have as many jumps as the quantity $|p_y(\tau_1) - p_y(\tau_2)|$ has extrema on the Fermi surface***.

It also follows from formulas (4) that if $2r - d \sim d$, then the resonance at the effective mass corresponding to the central section ($p_x = 0$) of a convex Fermi surface must be absent, since the electrons on this section necessarily undergo collisions with the sides of the film. In general, resonance at extremal effective masses is suppressed if the orbit diameter for the corresponding electrons is greater than d . At the same time, resonance at the effective masses at supporting points of the Fermi surface (1, 2) ($p_x = p_x^{\text{max}}, p_x^{\text{min}}$) should persist in the film.

The indicated effect of the influence of the shape of the specimen on cyclotron resonance can in principle facilitate harmonic analysis of resonance curves—

* In this case the formulas for the impedance near resonance and in strong magnetic fields can be obtained without difficulty from (3), (4), if one takes into account that in this case the complex tensor $Z_{\mu\nu}$ is reduced to its principal axes by a rotation of the coordinate system. We note that in the general case $Z_{\mu\nu}$ cannot be reduced to its principal axes. This circumstance is not taken into account in (4).

** Analogously to what occurs for the static conductivity of films ⁽⁵⁾.

*** It is of interest to note that when the curve $\varepsilon = \zeta$, $v_z = 0$ is flat and lies in the plane $p_z = 0$ (this may occur, for example, when one of the principal axes of the crystal coincides with the Oz axis), then, measuring only the position of the kink on the curve $Z_{yy}(H)$, one can, from $c(p_y^{\max} - p_y^{\min}) = eHd$, find the magnitude of the central diameter

$$p_y^{\max} - p_y^{\min} \equiv 2\rho \quad (p_x = p_z = 0),$$

perpendicular to the direction \mathbf{H} . By rotating \mathbf{H} in the plane of the film, one can determine all the central diameters in the section $p_z = 0$ ($v_z = 0$) and, consequently, find the shape of the contour of this section.

...ones (of course, in this case the film must be single-crystalline; the admissible mosaic angle ψ must not exceed $v_0/\omega \ll 1$), since in a film it is possible to suppress a part of the resonance minima. For example, if at the value of the magnetic field corresponding to the fundamental ($\Omega \simeq \omega$) resonance on some extremal mass, $2\rho < d$, while for $\Omega \simeq \omega/q$ ($q = 2, 3, \dots, \ll \omega/2\pi v_0$), then the resonance at all harmonics, except the fundamental ($\omega \simeq \Omega$), must be absent. On the other hand, in a film with $d < 2r$ a new resonance frequency appears (together with its harmonics), which is absent in a massive metal and which corresponds to the value of the effective mass at $\varphi = \varphi_1, \varphi_2$ ($m(\varphi_1) = m(\varphi_2)$). Indeed, for $\omega = q\Omega(\varphi_\alpha) = qeH/cm(\varphi_\alpha)$ and $v_0 \rightarrow 0$ the first term in $a_{\mu\nu}$ diverges logarithmically. This means that, for $\xi = 2\pi qv_0/\omega \ll 1$, $a_{\mu\nu}$ has a logarithmic singularity in ξ , and therefore the impedance has a resonant character, although this resonance will be strongly "smeared out." We do not give exact formulas, since, in view of the weak character of the singularity, it is necessary in $a_{\mu\nu}$ to retain the nonresonant part, as a result of which very cumbersome expressions are obtained.

If the section $p_x = p_x^{(0)}$ (corresponding to the points φ_1 and φ_2) corresponds to an extremal value of the effective mass with respect to φ , then the resonance at the passing value of H will be more "sharp" than in the preceding case. The formulas for the resonance value of the impedance in this case can be obtained from the corresponding formulas of works ^(1,2) by multiplying $Z_\alpha^{(\text{res})}$ by $\sqrt[3]{2}$ (this factor appears because the integration in (4) goes on one side of the point φ_1 ($p_x \geq p_x^{(0)}$, see Fig. 1).

All the conclusions given above are valid so long as the angle of inclination of the magnetic field relative to the surface of the film does not exceed δ_{eff}/l ($\ll d/l$), i.e., just as in a massive metal. At larger angles of inclination the resonance is absent (for details see ^(2,7)).

Taking into account the nondiffuse character of electron scattering at the film boundaries will not change the results, since in ⁽⁷⁾ it is shown that, for arbitrary (but not specular) reflection of electrons from the surface of a metal, the formulas for the surface impedance near resonance and in a strong field

$(1 - \chi \gg |\tilde{\gamma}|(\delta_{\text{eff}}/r)^{1/2})$ do not depend on the electron reflection coefficient χ . This is due to the fact that the contribution to the resonance is made only by those electrons which do not collide at all with the surface of the metal.

In the case of specular reflection (which, incidentally, is of academic interest, since electrons are scattered from the boundary practically in a diffuse manner), the leading term of the asymptotics $j_\mu(k) \sim \gamma \tilde{\phi}_{\mu\nu} \delta_\nu(k) / \sqrt{kr}$ also changes monotonically with the magnetic field (see (7)), and resonance appears only in terms of the next order in $(1/kr)^{1/2} \sim (\delta_{\text{eff}}/r)^{1/2} \ll 1$.

In conclusion, I take this opportunity to express my gratitude to I. M. Lifshitz and M. I. Kaganov for discussion of the results obtained.

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Received
30 X 1957

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Note: Figure translations are in progress. See original paper for figures.

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