Relaxation Interaction of Shock Waves with the Burning Zone

![Fig. 1](figure)

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Fig. 1

Figure 1: Fig. 1

Abstract

Full Text

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Relaxation Interaction of Shock Waves with the Burning Zone

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It was shown experimentally by one of the authors that in certain explosive mixtures, when a shock wave passes through the burning zone, its amplification occurs. Figure 1 gives, for a benzene-air mixture, the dependence of the amplification coefficient k^2 on the composition of the mixture α.

Fig. 1

Let us investigate how the kinetics of combustion will affect the structure and strength of a shock wave as it passes through the burning zone. We shall characterize the structure of the shock wave by the wave width b, the pressure amplitude $\overline{\Delta p}$ and the density ρ at the leading front of the wave, by the impulse I, and by the propagation velocity in the initial mixture D.

Let us introduce into consideration the diffusion-kinetic relaxation time t_r—the time during which the reaction conditions instantaneously changed by the shock wave do not cause substantial changes in the diffusion and heat fluxes in the reaction zone. When the statistical equilibrium is disturbed by the shock wave, the transition to the new equilibrium state takes place in a finite time. The time for establishment of the new temperature (relaxation of the energy of translational motion of the molecules) is of the order of 10^{-9} sec, whereas the time for macroscopic equalization of temperature and concentration in space as a result of kinetics, diffusion, and thermal conductivity is immeasurably greater. During this time incomplete statistical equilibrium is observed.

Calculations show that t_r is of the same order as the reaction time τ (10^{-3}–10^{-5} sec), determined as the ratio of the width of the burning zone l to the normal flame propagation velocity λ.

Thus, during the time t_r, within the reaction zone there is an increase in temperature and pressure. Changes in the reaction zone produce corresponding (transmitted with the speed of sound) changes in the gas parameters near the burning zone.

If the state of the reaction zone is characterized by the change in impulse

$$I_r = \int_0^\tau \Delta p \, dt,$$

then the state of the gas near the burning zone will be determined by the corresponding change in the quantity of motion; in this case the total impulse and the total quantity of motion of the gas will remain unchanged (external forces are absent).

After the relaxation time has elapsed, the excess impulse in the reaction zone is transformed into the impulse of a shock wave running forward and into an equal but oppositely directed impulse of a shock wave running backward.

If Q is the mass heat-producing capacity of the fuel, expressed

in mechanical units, w is the reaction rate referred to a unit mass of gas, $\mathfrak{M}$ is the mass rate of heat release; then

$$Q = \int_0^\tau \mathfrak{M} \, dt = Q \int_0^\tau w \, dt,$$

whence

$$\int_0^\tau w \, dt = 1.$$

To describe the state in the shock wave, we shall take as the origin of time the moment when the leading front of the shock wave and the flame front coincide. Then any state in the shock wave behind the flame front can be determined by two parameters: t and t'. The time t' fixes the moment at which the flame front passes through the given state in the shock wave, while t is the time of variation of this state, beginning from the moment $t = t'$, in the course of propagation of the shock wave. Thus, $\rho = \rho(t, t')$, $p = p(t, t')$, $w = w(t, t')$, etc.

Let $\Delta\rho(t, t') = \rho(t, t') - \rho_0$ denote the excess density in the shock wave relative to the initial gas. During the time dt', an excess mass of gas will pass through the flame front,

$$\lambda \Delta\rho(t, t') \, dt'.$$

The amount of heat released by this mass as a result of the chemical reaction occurring during the time dt will be

$$\Delta q = \lambda \Delta\rho(t, t') \mathfrak{M}(t, t') \, dt \, dt'.$$

The total amount of heat released during the relaxation time in the reaction zone is determined by the formula

$$q = \lambda \int_0^\tau dt' \int_{t'}^\tau \Delta\rho(t,t')\mathfrak{M}(t,t')\, dt = \lambda Q \int_0^\tau dt' \int_{t'}^\tau \Delta\rho(t,t')w(t,t')\, dt. \quad (1)$$

All the excess heat released during the time τ in the reaction zone as the shock wave passes goes to increase the internal energy of the gas in the reaction zone,

$$q = \lambda \int_0^\tau dt' \int_{t'}^\tau c_v \frac{\partial \Delta\rho T(t,t')}{\partial t}\, dt =$$

$$= \frac{\lambda}{\gamma - 1} \int_0^\tau dt' \int_{t'}^\tau \frac{\partial \Delta p(t,t')}{\partial t}\, dt = \frac{\lambda}{\gamma - 1} \int_0^\tau \Delta p(\tau, t')\, dt', \quad (2)$$

where $\gamma = c_p/c_v$; ΔT and Δp are the changes of temperature and pressure in the reaction zone. Substituting into (2) the expression for q from (1) and taking into account that

$$\int_0^\tau \Delta p\, dt' = I_r$$

is the additional impulse of the shock wave generated by combustion, we obtain

$$Q(\gamma - 1) \int_0^\tau dt' \int_{t'}^\tau \Delta\rho(t,t')w(t,t')\, dt = I_r. \quad (3)$$

As a first approximation, let us adopt a piecewise-linear approximation of the reaction rate in time. In the interval $t' \le t \le t' + \tau$, $w = \beta(t - t')$. For $t \le t'$, i.e., up to the moment when the flame front passes through the state of the shock wave with density $\rho(t,t')$, the reaction rate w is equal to zero; then it increases continuously over the time τ and at $t = t' + \tau$ instantaneously falls to zero. From the identity

$$\int_{t'}^{t'+\tau} w\, dt = 1$$

we determine $\beta = 2/\tau^2$. Hence,

$$w = \frac{2}{\tau^2}(t - t').$$

If τ_b is the residence time of the burning zone in the shock wave, then, depending on whether $\tau \gg \tau_b$ or $\tau \ll \tau_b$, the laws of heat release in the shock wave are substantially different. In the first case, during the passage of the wave through the flame front the reaction does not have time to go to completion, and the wave receives only part of the chemical energy, which goes to increasing the momentum of the wave. In the second case the reaction is completed entirely within the time τ.

In accordance with these two cases, the general integral (3) splits into two integrals:

$$I_r = Q(\gamma-1)\int_0^{\tau_b} dt' \int_{t'}^{\tau_b} \Delta\rho(t,t')\, w(t,t')\, dt; \qquad \tau \gg \tau_b, \tag{4}$$

$$I_r = Q(\gamma-1)\int_0^{\tau} dt' \int_{t'}^{\tau} \Delta\rho(t,t')\, w(t,t')\, dt; \qquad \tau \ll \tau_b. \tag{5}$$

Substituting into (4) and (5) $w = \dfrac{2}{\tau^2}(t-t')$ and $\rho = \bar\rho + (\rho_0 - \bar\rho)\dfrac{t'}{\tau_b}$, and carrying out the integration, we obtain

$$I_r = \frac{Q(\gamma-1)(\bar\rho-\rho_0)\tau_b^3}{4\tau^2}; \qquad I_r = \frac{Q(\gamma-1)(\bar\rho-\rho_0)}{3}\,\frac{\tau}{\tau_b}\left(\tau_b - \frac{1}{4}\tau\right). \tag{6}$$

$$\tau \gg \tau_b \qquad\qquad \tau \ll \tau_b$$

By the definition of the momentum of the shock wave, $I = \int_0^b \rho v\, dx$, where v is the gas velocity in the shock wave. Substituting v and ρ in I, we then have $I = \frac{1}{2}D^2\tau_b(\bar\rho - \rho_0)$. Expressing from this $\bar\rho - \rho_0$ in terms of I, D, and τ_b, and substituting into (6), we obtain

$$I_r = \frac{(\gamma-1)Q}{2D^2}\,\frac{\tau_b^2}{\tau^2} I; \qquad I_r = \frac{2(\gamma-1)Q}{3D^2}\,\frac{\tau^2}{\tau_b^2}\left(\frac{\tau_b}{\tau} - \frac{1}{4}\right) I. \tag{7}$$

$$\tau \gg \tau_b \qquad\qquad \tau \ll \tau_b$$

The total momentum of the wave I^* after the passage of the shock wave through the burning zone is composed of the intrinsic momentum I and the additional (relaxation) momentum I_r:

$$I^* = \left(1 + \frac{(\gamma-1)Q}{2D^2}\,\frac{\tau_b^2}{\tau^2}\right) I, \qquad I^* = \left[1 + \frac{2(\gamma-1)Q}{3D^2}\,\frac{\tau^2}{\tau_b^2}\left(\frac{\tau_b}{\tau} - \frac{1}{4}\right)\right] I. \tag{8}$$

$$\tau \gg \tau_b \qquad\qquad \tau \ll \tau_b$$

In (8), I is the momentum of the shock wave that has passed through the burning zone. If ξ is the refraction coefficient, then $I = \xi I_0$, where I_0 is the momentum of the incident wave. The momentum I' of the reflected wave is obviously equal to $(1-\xi)I_0$. Thus, for shock waves that have passed through the burning zone, we have $I^* = k_i \xi I_0$, and for reflected waves $I^* = k_i(1-\xi)I_0$, where k_i is the coefficient of relaxation amplification of the shock-wave momentum,

$$k_i = 1 + \frac{(\gamma-1)Q}{2D^2}\frac{\tau_b^2}{\tau^2}; \qquad k_i = 1 + \frac{2(\gamma-1)Q}{3D^2}\frac{\tau}{\tau_b}\left(1 - \frac{1}{4}\frac{\tau}{\tau_b}\right). \tag{9}$$

$$\tau \gg \tau_b \qquad\qquad \tau \ll \tau_b$$

The coefficient $k = \dfrac{\Delta p^*}{\Delta p}$ of relaxation amplification of the amplitude of the shock wave is accordingly determined by the formulas

$$k = \frac{I^*}{I} = k_i; \qquad \tau \gg \tau_b \qquad k = 1 + \frac{I_r}{I}\frac{\tau_b}{\tau} = 1 + \frac{2(\gamma-1)Q}{3D^2}\left(1 - \frac{1}{4}\frac{\tau}{\tau_b}\right). \qquad \tau \ll \tau_b \tag{10}$$

For values $\Delta\bar{p}/p_0 \ll 1$, the amplification coefficient does not depend on the number of passages of the shock wave through the combustion zone:

$$k = 1 + \frac{(\gamma-1)Q}{2c_0^2}\frac{\tau_b^2}{\tau^2}; \qquad \tau \gg \tau_b \qquad k = 1 + \frac{2(\gamma-1)Q}{3c_0^2}\left(1 - \frac{1}{4}\frac{\tau}{\tau_b}\right), \qquad \tau \ll \tau_b \tag{11}$$

where c_0 is the speed of sound in the initial medium. For $\tau/\tau_b \ll 1$,

$$k = k_{\max} = 1 + \frac{2(\gamma-1)Q}{3c_0^2},$$

and maximum amplification is obtained; if $\tau/\tau_b \gg 1$, then $k = k_{\min} = 1$ and there is no amplification. The amplification coefficient of the shock-wave impulse, as follows from (9), has a maximum at $\tau = \tau_b$ and is close to unity both for $\tau/\tau_b \gg 1$ and for $\tau/\tau_b \ll 1$.

The phenomenon of maximum amplification of the shock-wave impulse when the condition $\tau = \tau_b$ is fulfilled (relaxational "capture" of the shock wave by the combustion zone) may be called impulse resonance.

The refraction coefficients are determined from the condition for transition of a shock wave from a medium with temperature T_0, density ρ_0, and pressure

p_0 into a medium with temperature T, density ρ, and the same pressure p_0, or conversely. The interface between the media is then a contact discontinuity. If the temperature T of the medium is maintained by flame propagation, then T is determined from the condition $Q = c_p(T - T_0)$. In the absence of flame propagation the contact discontinuity decays because of thermal conduction.

For weak shock waves propagating from the medium (T_0, p_0, ρ_0) into the medium (T, p_0, ρ), the refraction coefficient is

$$\xi_- = \frac{2}{1 + \sqrt{T/T_0}};$$

for waves propagating in the opposite direction,

$$\xi_+ = \frac{2\sqrt{T/T_0}}{1 + \sqrt{T/T_0}}.$$

For real explosive mixtures, in particular for benzene with air, the quantity $\xi_+\xi_- < 1$ is close to unity. Consequently, if $k_\pm$ denotes the amplification coefficient of the shock wave running between two walls of a closed chamber during one cycle, then

$$k_\pm = k^2 \xi_+ \xi_-.$$

Under resonant conditions the value $\xi_+\xi_-$ may be neglected and $k_\pm = k^2$. In the absence of resonant capture and when $\tau \to 0$, hence when the combustion zone in the limit becomes infinitely thin,

$$k_\pm = k_{\max}^2.$$

In this case, as was to be expected, the amplitude of the shock wave is amplified maximally, while the impulse of the wave is not changed.

It is of interest to consider the dependence of k on the coefficient of excess air α of the combustible mixture. In this case the heat capacity of the mixture is

$$Q_{\text{mix}} = \frac{\alpha Q}{1 + \alpha Z},$$

if $\alpha \ll 1$, and

$$Q_{\text{mix}} = \frac{Q}{1 + \alpha Z},$$

if $\alpha \gg 1$, where Z is the amount of air required for the complete combustion of 1 kg of fuel. For a mixture of benzene with air, $k_{\max}$ varies within the limits from 2.5 to 2.8 with the corresponding change of α from 1.2 to 0.65. As experiment shows (Fig. 1), for the same mixture the maximum value $k^2 = 1.6$ shifts toward richer mixtures ($\alpha = 0.8$), which corresponds to a greater reaction rate, i.e., to a smaller τ. According to calculations, for a mixture of benzene with air at $\alpha = 0.8$ such a maximum $k^2 = 1.6$ is obtained at $\tau/\tau_b = 1$. Moving from the point $\alpha = 0.8$ toward leaner or richer mixtures, we obtain a smaller value of k.

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Note: Figure translations are in progress. See original paper for figures.