MATHEMATICS

1958

SovietRxiv

Abstract

Full Text

MATHEMATICS

Yu. I. ZHURAVLEV

ON MATHEMATICAL METHODS FOR MONITORING ABSTRACT TRANSFORMERS

(Presented by Academician S. L. Sobolev, 27 VI 1958)

In engineering one has to deal with devices which we shall call **transformers**, possessing a definite "input" and a definite "output," and which function in the following way. At the "input" there arrives some physical quantity x of a definite nature. In the device a transformation of this quantity takes place, and at the "output" a quantity $f(x)$, likewise of a definite physical nature, is delivered. This is what happens when the device is operating correctly. However, one must reckon with breakdowns arising in the course of operation. In such cases the quantity $f(x)$ is delivered with distortions. It is important to know whether the device is operating correctly in each individual case. For this purpose current monitoring of the device is used during its operation; this consists in the fact that at certain instants of time predetermined values x, for which $f(x)$ is known in advance, are supplied to the input. These values are compared with the quantities delivered by the device.

In the present note we consider certain general questions connected with estimating the possibilities of monitoring. The question is that of detecting systematic faults of transformers. Random failures are, generally speaking, not captured here. In what follows we shall assume that $x \in [a, b]$ and that $f(x)$ is measurable and bounded on $[a, b]$. We shall denote transformers by the letter s. We shall consider a transformer as a "black box," i.e., we shall assume that in the process of operation we cannot obtain any information about its internal construction. We shall assume that only certain functional characteristics of the transformer are known to us.

There exists an approach[1] to the theory of monitoring that differs from the one set forth in this note. This approach may be called discrete. It is assumed that the transformer has a finite number of faults and that, for each fault, it is known into what the function $f(x)$ passes when this fault occurs. In this case, by applying methods of the algebra of logic, it is possible not only to determine whether the device is operating correctly, but also to indicate exactly which fault is taking place. Under the approach set forth below and based on the study of continuous quantities, the cardinality of the set of faults and the functional characteristics of the faults are immaterial. On the other hand, in the present case we can only obtain an answer as to whether the device is operating correctly, while which fault is taking place is not determined.

It is natural to single out and study transformers possessing the following property: from the fact that a transformer s operates correctly for the input signal x_0, it follows that with high probability it operates correctly in some neighborhood of x_0.

We now pass to the mathematical formulation of the problem.

The properties of the device s that interest us will be obtained by considering two objects associated with s: $p_s(x)$ and $A_s[\varphi, x_0]$. Here $p_s(x)$ is the probability of the following event: the device s operates correctly for the input signal x_0. We shall assume that $p_s(x)$ is measurable.

Denote by $\widetilde{L}_{[a,b]}$ the class of functions $\varphi(x)$, measurable on $[a, b]$, such that $0 \leqslant \varphi(x) \leqslant 1$ for $x \in [a, b]$.

Definition. The **control operator** $A_s[\varphi, x_0]$ of a device s is an operator defined for $\varphi \in \widetilde{L}_{[a,b]}$, $a \leqslant x_0 \leqslant b$, and possessing the following properties:

1°. $A_s[\varphi, x] = F(x) \in \widetilde{L}_{[a,b]}$.

2°. For arbitrary φ_1 and φ_2 from $\widetilde{L}_{[a,b]}$ and arbitrary $a \leqslant x_0 \leqslant b$, if $\varphi_1 \geqslant \varphi_2$, then $A_s[\varphi_1, x_0] \geqslant A_s[\varphi_2, x_0]$ on $[a, b]$.

3°. $A_s[\varphi, x_0] = F(x) \geqslant \varphi(x)$ on $[a, b]$.

4°. Let $A_s[\varphi(x), x_0] = F(x)$. Then $F(x_0) = 1$.

5°. $A_s[A_s(\varphi, x_1), x_2] = A_s[A_s(\varphi, x_2), x_1]$.

Definition. The **reliability** of a converter s is

$$N = \frac{1}{b-a} \int_{[a,b]} p_s(x)\, dx.$$

Put

$$A_s[p_s(x), x_1] = P_{x_1}(x), \qquad A_s[P_{x_1}(x), x_2] = P_{x_1 x_2}(x), \dots$$

$$\dots, \quad A_s[P_{x_1,\dots,x_{n-1}}(x), x_n] = P_{x_1,\dots,x_n}(x).$$

Definition. The **reliability** of a converter s under the condition that the circuit operates correctly for the input signals $x_1, \dots, x_n$ is

$$N_{x_1,\dots,x_n} = \frac{1}{b-a} \int_{[a,b]} P_{x_1,\dots,x_n}(x)\, dx.$$

In what follows, by a device s we shall mean a pair $(p_s(x), A_s[\varphi, x_0])$ (notation $s = (p_s(x), A_s[\varphi, x_0])$).

Devices s_1 and s_2 for which $p_{s_1}(x) = p_{s_2}(x)$, $A_{s_1}[\varphi, x_0] = A_{s_2}[\varphi, x_0]$, will be called **isomorphically controllable** and will not be distinguished.

A converter $(p_s(x), A_s[\varphi, x_0])$ is called **finitely controllable** if for every $\varepsilon > 0$ there is an $n(\varepsilon)$ such that there exists a sequence $x_1, \dots, x_{n(\varepsilon)}$ for which

$$1 - N_{x_1,\dots,x_{n(\varepsilon)}} \leqslant \varepsilon.$$

We shall formulate sufficient conditions for finite controllability of converters.

Definition. The control operator $A_s[\varphi, x_0]$ is called **stable with respect to reliability** if for an arbitrary function $\varphi \in \widetilde{L}_{[a,b]}$, any $\varepsilon > 0$, and arbitrary $x \in [a, b]$, there is a $\delta(\varepsilon)$ such that, if $|x - x_0| < \delta(\varepsilon)$, then $|F(x) - F(x_0)| < \varepsilon$.

Theorem 1. A converter $s = (p_s(x), A_s[\varphi, x_0])$ whose control operator is stable with respect to reliability is finitely controllable.

Theorem 2. A control operator $A_{s'}[\varphi, x_0]$ which maps the function $\varphi(x) \equiv 0$ on $[a, b]$ into a function continuous at the point x_0 is stable with respect to reliability and, consequently, the converter $s' = (p_{s'}(x), A_{s'}[\varphi, x_0])$ is finitely controllable.

Let us study converters whose control operators have the following form. Consider a family of functions $\{f(x - x_0)\}$, $-\infty \leqslant x \leqslant +\infty$, $a \leqslant x_0 \leqslant b$, such that:

1°. $f(x - x_0)$ is differentiable on $(-\infty, +\infty)$.

2°. $f(x - x_0)$ is monotonically increasing for $x \leqslant x_0$.

3°. The straight line $x = x_0$ is an axis of symmetry for $f(x - x_0)$.

4°. $f(0) = 1$, $f(-\infty) = -\infty$.

For the converters under study,

$$A_s[\varphi, x_0] = \max[\varphi, f(x - x_0)] = A_{x_0}(\varphi, f).$$

Obviously, the control operator $A_{x_0}(\varphi, f)$ is reliable-stable. Let S_f be the set of all converters of the form $[p_s(x), A_{x_0}(\varphi, f)]$.

For each $s \in S_f$ define the number $n_s(\varepsilon)$ as follows:

1°. There exists a sequence $x_1, x_2, \dots, x_{n_s(\varepsilon)}$ such that

$$1 - N_{x_1,x_2,\dots,x_{n_s(\varepsilon)}} \leq \varepsilon.$$

2°. For no $k < n_s(\varepsilon)$ does there exist a sequence $x_1, \dots, x_k$ such that

$$1 - N_{x_1,\dots,x_k} \leq \varepsilon.$$

Let

$$n(\varepsilon) = \max_{s \in S_f} n_s(\varepsilon).$$

Estimate the number $n(\varepsilon)$. Put $x - a = t$.

Lemma. The equation

$$y \int_0^y f(t)\,dt = (1-\varepsilon)y$$

has no more than one positive root.

Theorem 3. If the equation

$$y \int_0^y f(t)\,dt = (1-\varepsilon)y$$

has no positive roots, then $n(\varepsilon) = 1$.

Theorem 4. If the equation

$$y \int_0^y f(t)\,dt = (1-\varepsilon)y$$

has a positive root y_1, then

$$\left[\frac{b-a}{2y_1}\right] \leq n(\varepsilon) \leq \left[\frac{b-a}{2y_1}\right] + 1.$$

Example. Let

$$A_{x_0}(\varphi, f) = \max[\varphi,\, 1 - (x - x_0)^{2k}].$$

Then

$$\left[\frac{b-a}{2\sqrt[2k]{(2k+1)\varepsilon}}\right] \leq n(\varepsilon) \leq \left[\frac{b-a}{2\sqrt[2k]{(2k+1)\varepsilon}}\right] + 1.$$

Definition. The **strong reliability** $\overline{N}$ of a converter s is called

$$\max_{x \in [a,b]} |1 - p_s(x)|.$$

Definition. The **strong reliability** $\overline{N}_{x_1,\dots,x_n}$ of a converter s, under the condition that it operates correctly on the input signals $x_1, \dots, x_n$, is called

$$\overline{N}_{x_1,\dots,x_n} = \max_{x \in [a,b]} |1 - P_{x_1,\dots,x_n}(x)|.$$

Definition. A converter is called **finitely controllable in the strong sense** if for every $\varepsilon > 0$ there is an $m(\varepsilon)$ such that

$$1 - \overline{N}_{x_1,\dots,x_{m(\varepsilon)}} \leq \varepsilon.$$

Consider the set S_f, and for each $s \in S_f$ define the number $\overline{n}_s(\varepsilon)$ as follows:

1°. There exists a sequence $x_1, x_2, \dots, x_{\overline{n}_s(\varepsilon)}$ such that

$$1 - \overline{N}_{x_1,x_2,\dots,x_{\overline{n}_s(\varepsilon)}} \leq \varepsilon.$$

2°. For no $k < \overline{n}_s(\varepsilon)$ does there exist a sequence $x'_1, \dots, x'_k$ such that

$$1 - \overline{N}_{x'_1,\dots,x'_k} \leq \varepsilon.$$

Let

$$s = (p_s(x), A_{x_0}[\varphi, f]),$$

where $p_s(x) \in \widetilde{L}_{[a,b]}$. Estimate the number $\overline{n}_s(\varepsilon)$. The equation

$$f(x - a) = 1 - \varepsilon$$

has two roots x_1 and x_2 for $0 < \varepsilon \leq 1$. Let

$$|x_1 - x_2| = \delta(\varepsilon).$$

Denote by $M_{p_s(x)}(\varepsilon)$ the set of all points x

from $[a, b]$ for which $p_s(x) < 1 - \varepsilon$, and let $\mu(\varepsilon)$ be the Lebesgue measure of this set.

Theorem 5.

$$\left[\frac{\mu(\varepsilon)}{\delta(\varepsilon)}\right] \leq \bar{n}_s(\varepsilon) \leq \left[\frac{b-a}{\delta(\varepsilon)}\right] + 1,$$

where $\bar{n}_s(\varepsilon)$ may be any integer within the limits indicated here.

If $p_s(x)$ is piecewise monotone on $[a, b]$, then the following holds:

Theorem 6.

$$\lim_{\varepsilon \to 0} \frac{\bar{n}_s(\varepsilon)}{\mu(\varepsilon)/\delta(\varepsilon)} = 1.$$

Moscow State University
named after M. V. Lomonosov

Received
24 VI 1958

References

1. I. A. Chegis, S. V. Yablonskii, *Tr. Matem. inst. im. V. A. Steklova AN SSSR*, **51**, 270 (1958).

Note: Figure translations are in progress. See original paper for figures.