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Abstract

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ON THE PHONON THERMAL CONDUCTIVITY OF SUPERCONDUCTORS

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As is known, there exist several mechanisms of thermal conductivity associated with the interaction of electrons, phonons, and impurity atoms. In superconductors, owing to the presence of a gap in the energy spectrum ([1]), the thermal conductivity of the lattice plays a large role. In ([2]) the electronic thermal conductivity of superconductors associated with the scattering of electrons by impurities was found. We shall now consider the lattice thermal conductivity due to the action of electrons on phonons. There is a temperature region in which this interaction mechanism is one of the principal ones ($T \gtrless (0.3—0.5)T_k$).

The kinetic equation for the phonon distribution function has the form

$$-\frac{\partial N}{\partial T}\frac{dT}{dx}u_0\frac{q_x}{q} = \left(\frac{\partial N}{\partial t}\right)_{\rm st};$$

u_0 is the velocity of sound, q is the phonon momentum.

In the Hamiltonian of the electron-phonon interaction

$$H' = \sum_{\substack{k,s,q \\ k'=k+q}} V_{kk'} a^+_{ks} a_{k's} b^+_q + \text{conj.}$$

we pass to new Fermi amplitudes by making the transformation ([3])

$$\alpha_{k0} = u_k a_{k,\,1/2} - v_k a^+_{-k,\,-1/2}; \qquad \alpha_{k1} = u_k a_{-k,\,-1/2} + v_k a^+_{k,\,1/2},$$

$$u_k^2 = \frac{1}{2}(1+\xi/\varepsilon), \qquad v_k^2 = \frac{1}{2}(1-\xi/\varepsilon).$$

The original Hamiltonian will take the form

$$H' = \sum_k V_{kk'}\left[(u_k u_{k'} - v_k v_{k'})(\alpha^+_{k0}\alpha_{k'0} + \alpha^+_{k1}\alpha_{k'1}) + \right.$$

$$+(u_k v_{k'} + u_{k'} v_k)(\alpha^+_{k0}\alpha^+_{k'1} + \alpha_{k0}\alpha_{k'1})]\, b^+_q + \text{conj.}$$

We now write the collision integral in the new amplitudes:

$$\left(\frac{\partial N}{\partial t}\right)_{\text{st}} = \int |V_{kk'}|^2 \left\{ \left(1 + \frac{\xi\xi' - \Delta^2}{\varepsilon\varepsilon'}\right)[f'(1-f)(N_q+1) - \right.$$

$$-N_q f(1-f')]\delta(\varepsilon' - \varepsilon - \hbar\omega) + \frac{1}{2}\left(1 - \frac{\xi\xi' - \Delta^2}{\varepsilon\varepsilon'}\right)[(N_q+1)ff' -$$

$$\left. -N_q(1-f)(1-f')]\delta(\varepsilon' + \varepsilon - \hbar\omega)\right\} \frac{p^2\,dp}{4\pi^2\hbar^4}\sin\vartheta\, d\vartheta\, d\varphi.$$

We write the distribution functions in the form

$$f = f_0 + g, \qquad N = N_0 - r(x) q_x \frac{\partial N}{\partial x}\frac{1}{kT}, \qquad x = \frac{\hbar\omega}{kT};$$

f_0 and N_0 are equilibrium functions, with

$$f_0 = \left(\exp\left(\frac{\varepsilon}{kT}\right) + 1\right)^{-1}, \quad \text{where } \varepsilon = \sqrt{\xi^2 + \Delta^2};$$

$\xi\xi' > 0$ for scattering processes and $\xi\xi' < 0$ for the creation of pairs of excitations (1). The addition to the distribution function of electronic excitations was found in (2) and is equal to

$$g = \frac{p_x}{m}\frac{\partial f_0}{\partial \varepsilon}\frac{\varepsilon}{T}\tau_0\frac{\xi}{|\xi|}\frac{\partial T}{\partial x}.$$

Carrying out the integration over angles, we obtain:

$$r(x)=\left[\frac{N_0u_0^2}{T}-|V|^2\tau_0\frac{Am}{q}\int_{-\infty}^{\infty}\left(1+\frac{\xi\xi'-\Delta^2}{\varepsilon\varepsilon'}\right)\left[\frac{\varepsilon+\hbar\omega}{2T}\frac{\xi}{|\xi'|}-\right.\right.$$

$$\left.-\frac{\varepsilon}{2T}\frac{\xi}{|\xi|}\right]f_0(\varepsilon)f_0(\varepsilon+\hbar\omega)\exp\left(\frac{\varepsilon}{kT}\right)\frac{\varepsilon}{|\xi'|}\,d\xi-$$

$$-D(x)|V|^2\tau_0\frac{Am}{2q}\int_{-\xi_1}^{\xi_1}\left(1-\frac{\xi\xi'-\Delta^2}{\varepsilon\varepsilon'}\right)\left[-\frac{\hbar\omega-\varepsilon}{2T}\frac{\xi'}{|\xi'|}-\frac{\varepsilon}{2T}\frac{\xi}{|\xi|}\right]$$

$$\left.\times f_0(\varepsilon)f_0(\hbar\omega-\varepsilon)\frac{\varepsilon'}{|\xi'|}\,d\xi\right]\left[|V|^2\frac{m^2A}{q}\int_{-\infty}^{\infty}\left(1+\frac{\xi\xi'-\Delta^2}{\varepsilon\varepsilon'}\right)\times\right.$$

$$\times f_0(\varepsilon)f_0(\varepsilon+\hbar\omega)\exp\left(\frac{\varepsilon}{kT}\right)\frac{\varepsilon'}{|\xi'|}\,d\xi+D(x)|V|^2\frac{m^2A}{2q}\times$$

$$\left.\times\int_{-\xi_1}^{\xi_1}\left(1-\frac{\xi\xi'-\Delta^2}{\varepsilon\varepsilon'}\right)f_0(\varepsilon)f_0(\hbar\omega-\varepsilon)\frac{\varepsilon'}{|\xi'|}\,d\xi\right]^{-1}\frac{dT}{dx},$$

$$D(x)=\begin{cases}1, & x\geq 2b,\\ 0, & x<2b,\end{cases}\quad b=\frac{\Delta}{kT},\quad \xi_1=\sqrt{(\hbar\omega-\Delta)^2-\Delta^2},\quad A=\frac{1}{4\pi^2\hbar^4}.$$

The integrals appearing in the numerator vanish, since the integrand in each of them is odd with respect to ξ. Taking this into account, passing to integration over ε, and approximately putting in each of the integrands $L\varepsilon'\varepsilon/|\xi'||\xi|\simeq 2$; $L=[1\pm(\xi\xi'-\Delta^2)/\varepsilon\varepsilon']$ (this gives an error of order $\Delta^2(\hbar\omega)^2/4(\varepsilon\varepsilon'\mp\Delta^2)\varepsilon\varepsilon'$, i.e. $\sim 10^{-1}$ for Q at $\Delta/kT=1.5$), we obtain:

$$r=N_0\left[m^2|V|^2\frac{AkT}{q}\int_b^{\infty}\frac{\exp z\,dz}{(\exp z+1)(\exp(z+x)+1)}+\right.$$

$$\left.+D(x)m^2|V|^2\frac{AkT}{q}\int_b^{x-b}\frac{dz}{(\exp z+1)(\exp(x-z)+1)}\right]^{-1}\frac{dT}{dx}.$$

Integrating, we obtain:

$$r\simeq\frac{\text{const}}{T^2}\left[2x-2\ln(\exp(b+x)+1)(\exp b+1)^{-1}+\right.$$

$$\left.+D(x)\left(2b-x+2\ln(\exp(x-b)+1)(\exp b+1)^{-1}\right)\right]^{-1}\frac{dT}{dx}.$$

The heat flux of the lattice is equal to

$$Q = \sum_q N_0 u_0 \frac{q_x}{q} u_0 q = -\frac{(kT)^4}{6\pi^2\hbar^3 u_0^3} \int_0^\infty \frac{x^4 r(x) \exp x}{(\exp x - 1)^2} dx.$$

Substitution of the function $r(x)$ found gives:

$$Q = BT^2 \left[\int_0^{2b} \frac{x^4 \exp x \, dx}{(\exp x - 1)^2 \left[2x - 2\ln\left(\exp(b+x)+1\right)\left(\exp b + 1\right)^{-1}\right]} + \right.$$

$$\left. + \int_{2b}^\infty \frac{x^4 \exp x \, dx}{(\exp x - 1)^2 \left[x + 2b - 2\ln\left(\exp(b+x)+1\right)\left(\exp(x-b) + 1\right)^{-1}\right]} \right] \frac{dT}{dx};$$

$$B = \frac{3k^2\hbar}{4c^2\Omega_0 m}.$$

(The notation is the same as in (4).)

The well-known formula for the lattice thermal conductivity of a normal metal (4) is obtained from this if one sets $b = \Delta/kT = 0$.

An approximate calculation of the integrals (with an accuracy up to 0.05) leads to the following result:

$$\varkappa = -\frac{Q}{dT/dx} = BT^2 F(T);$$

$$F(T) = -8b^4(e^b - 1)^{-1} - 8b^3(e^b - 1)^{-1} + 6\xi(3)(e^b + 1) -$$

$$-3(e^b + 1)\sum_s \frac{1}{s^3}\exp(-2bs)(4b^2s^2 + 4bs + 2) + 6\xi(4)(e^b - 1) -$$

$$-(e^b - 1)\sum_s \frac{1}{s^4}\exp(-2bs)(8b^3s^3 + 12b^2s^2 + 12bs + 6) + 32b^3(e^{2b} - 1)^{-1} +$$

$$+a^4 \sum_s \{s \exp(-2bs) - \mathrm{Ei}[-s(2b - a)]\} + 6\sum_s \frac{1}{s^3}\exp(-2bs),$$

$$a \simeq 2b - 0.16, \qquad \xi(s) = \sum_{n=1}^\infty n^{-s}.$$

The formula satisfactorily describes the experimental data given in (5).

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named after V. I. Lenin

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CITED LITERATURE

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Note: Figure translations are in progress. See original paper for figures.