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**Abstract**

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# ON THE FOCUSING OF BEAMS OF CHARGED PARTICLES BY HIGH-FREQUENCY FIELDS

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1. As is known, for the focusing of rectilinear beams of charged particles one may use periodic electrostatic and magnetostatic fields, equivalent to a sequence of converging and diverging lenses ([1]). A focusing effect also occurs under the action of a rapidly varying electromagnetic field of a definite structure, as is not difficult to verify by considering a rectilinear beam focused by a periodic static field in a reference frame moving uniformly along the beam with velocity $v$, different from the mean particle velocity $v_z$. The focusing is preserved, but it is now accomplished not in a static field, but in the field of an inhomogeneous electromagnetic wave*. In addition to such focusing systems, there also exist specifically high-frequency ones, which have no static analogues and cannot be reduced to static ones by any transformation of reference frames.

2. By focusing of rectilinear beams of charged particles in the most general sense of the word we shall understand the confinement of their trajectories within a certain cylindrical region of radius $R_M$.

Let a beam of charged particles ($e$ is the charge, $m$ the mass, $\eta = e/m$), flying along the $z$ direction with mean velocity $v_z$ ($v_z^2 = 2|\eta||V_z|$), be subjected to the action of an electromagnetic field $\mathbf{E}(\mathbf{r}, t) = \mathbf{E}(\mathbf{r})e^{i\omega t}$, $\mathbf{H}(\mathbf{r}, t) = \mathbf{H}(\mathbf{r})e^{i\omega t}$. In the nonrelativistic approximation the equation of motion of a single particle inside the beam is written in the form

$$\ddot{\mathbf{r}} = \eta \mathbf{E}'(\mathbf{r}, t) + \frac{\eta}{c}[\dot{\mathbf{r}}\mathbf{H}] + \eta \mathbf{E}^{(q)}(\mathbf{r}, t), \tag{1}$$

where $\mathbf{r}$ is the radius vector, and $\mathbf{E}^{(q)}(\mathbf{r}, t)$ is the field strength of the space charge. We shall assume that the beam retains a quasi-cylindrical form and that the particle density in it is sufficiently small. Then the distortions introduced by the beam into the high-frequency field may be neglected, and, for an estimate of $\mathbf{E}^{(q)}$, one may restrict oneself to the two-dimensional static formula

$$\mathbf{E}^{(q)} = -\nabla \varphi^{(q)}, \qquad \varphi^{(q)} = -\frac{Q}{R_M^2} R^2, \tag{2}$$

where $Q$ is the charge per unit length, "smeared" over the cross section of the beam with constant density.

3. Equation (1) can be substantially simplified if one is interested only in the motion of a particle averaged over the period of the high-frequency field ([3]). Let us write the function $\mathbf{r}(t)$ as the sum of a slowly varying (on the scale of the period of oscillations of the external field) function $\mathbf{r}_0(t)$ and a rapidly–

* One of the possible variants of such focusing systems (using a slow symmetric wave of type $\mathrm{TM}_{10}$) was considered in ([2]) as applied to linear accelerators.

the oscillating function $\mathbf{r}_1(t)$. Assuming the latter to be, in absolute value, considerably smaller than the beam radius $R_M$,

$$|\mathbf{r}_1| \ll R_M \tag{3}$$

and neglecting in (1) terms of order $|\dot{\mathbf{r}}_0|/c$ and $|\mathbf{r}_1|/R_M$, after averaging over the period of the high-frequency field we arrive at the following equation for $\mathbf{r}_0(t)$:

$$\ddot{\mathbf{r}}_0(t) = -\nabla\Phi; \qquad \Phi = \frac{\eta^2}{4\omega^2}|\mathbf{E}|^2 - \left|\eta\varphi^{(q)}\right|. \tag{4}$$

**4.** Thus, the time-averaged force acting on a particle in the beam is potential. Consequently, in order to focus the beam it is necessary to create in the scalar field $\Phi$ a potential "trough" (a two-dimensional potential well) extending along $z$. In order that particles which on the $z$-axis have radial velocity $v_R$ ($v_R^2 = 2|\eta|V_R$) should not be able to go beyond the boundary of the beam ($R = R_M$), it is necessary that the condition $\Phi|_{R=R_M} > |\eta|V_R$ be fulfilled, which, together with condition (3), makes it possible to determine the limits for the amplitude of the external field

$$\frac{\omega^2}{|\eta|}R_M \gg |\mathbf{E}|_{R=R_M} > 2\omega\left(\frac{V_R + \left|\varphi^{(q)}\right|_{R=R_M}}{|\eta|}\right)^{1/2}. \tag{5}$$

**5.** As an example, let us consider the focusing of a rectilinear paraxial beam in the field of a symmetric transverse-electric wave ($\mathrm{TE}_{10}$) propagating in a circular waveguide with perfectly conducting walls. The only nonzero azimuthal component of the electric-field strength we write in the form $E_\theta = 2E_0 J_1(\varkappa R)e^{i\omega t - ihz}$, where $J_1(\varkappa R)$ is the Bessel function, $h^2 = k^2 - \varkappa^2$, $\varkappa = 3.8/R_B$, $R_B$ is the radius of the waveguide.

Figure 1 schematically shows the relief of the potential $\Phi$ (without taking into account the space-charge field). The averaged equation of the radial motion of a particle in the plane perpendicular to the beam axis, in accordance with (4), has the form

Fig. 1

Figure 1: Fig. 1

Fig. 2

Figure 2: Fig. 2

$$\ddot{R}_0 + \frac{2\eta^2 E_0^2 \varkappa}{\omega^2} J_1(\varkappa R_0) J_1'(\varkappa R_0) = \frac{2Q\eta}{R_M^2} R_0. \tag{6}$$

For motion in the paraxial region, expanding the Bessel functions in a series and retaining in (6) terms of order $\varkappa R_0$, we obtain

$$\ddot{R}_0 + \Omega^2 R_0 = 0, \qquad \Omega^2 = \frac{\eta^2 E_0^2 \varkappa^2}{2\omega^2} - \frac{2\eta Q}{R_M^2}. \tag{7}$$

This is the usual oscillator equation, whose solution is bounded for a real natural frequency ($\Omega^2 > 0$). The mean trajectory does not go beyond the boundary of a cylinder of radius $R_M$ under the condition

$$E_0 > \frac{2\omega}{(\varkappa R_M)\sqrt{|\eta|}} \left( V_R + \frac{I}{v_z} \right)^{1/2}, \tag{8}$$

where $I$ is the absolute magnitude of the current in the beam ($I = |Q| v_z$).

For the case of a purely electron beam, in Fig. 2 there are plotted, on the basis of inequalities (3) and (8), the dependences of the limiting values of the field strength as functions of $\varkappa R_M$. The following parameter values were adopted: $\omega = 2\pi \cdot 10^{10}$ Hz, $k/\varkappa = 1.2$, $(V_R + I/v_z) \sim 1$ V. Although equation (7) for thick beams $(\varkappa R_M) \sim 1$ becomes no longer applicable, nevertheless

directly from Fig. 1 it is clear that focusing is in principle possible up to values $xR_M \simeq 1.8$.

6. In an analogous way one can verify that focusing occurs in almost all waves of the transverse-electric type: $\mathrm{TE}_{R\theta\, nm}$ ($m \geqslant 2$) in circular waveguides, $\mathrm{TE}_{nm}$ ($n \geqslant 2,\ m \geqslant 2$) in rectangular waveguides, etc.,* and, for certain values of the phase velocities, also in waves of the transverse-magnetic type.

It is possible to use waveguides with an inhomogeneous dielectric filling, as well as with periodically repeated boundary outlines.

**Fig. 1**

**Fig. 2**

At the same time, if the phase velocity of the wave proves to be substantially less than the speed of light, then, as indicated above, the corresponding system may be regarded as a trivial generalization of a static system with periodic focusing.

We note that, in principle, it is permissible to use not only traveling waves but also standing waves, if, when passing through the nodes of the field, the particles do not manage to leave the potential well averaged over the spatial period.

Finally, one may use simultaneously fields of several different frequencies. In this case the equation for the average motion, obtained by averaging (1) over a time exceeding the periods of all the difference frequencies, is written in the form $\ddot{\mathbf{r}}_0 = -\nabla\Phi,\ \Phi = \sum_i \Phi_i(\omega_i)$, where $\Phi_i(\omega_i)$ is the potential of the force field produced by oscillations of frequency $\omega_i$.

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* Of special interest is the case of a four-conductor line composed of hyperbolic cylindrical conductors ([1]). Since the field structure of one of the waves of the TEM type is described by an electrostatic potential of the form $\varphi_{\text{el. st.}} = \text{const}\,(x^2 - y^2)$, the original equation (1), for $Q = 0$, reduces to Mathieu' s equation, which permits investigation not only of the averaged trajectory, but also of the "instantaneous" trajectory of the particle and, in particular, refinement of condition (3).

*Note: Figure translations are in progress. See original paper for figures.*