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Abstract

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FARADAY EFFECT ON MOTT EXCITONS

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Along with optical ([1]), electrical ([2]), and magnetic ([3]) studies of excitons in semiconductors, the study of the magneto-optical properties of excitons can provide information about the properties of these quasiparticles independent of the methods indicated above. Below, the Faraday effect is investigated for Mott excitons of not too large radii ($d < 10^{-5}$ cm). Verdet' s constant can be determined on the basis of its relation to the gyration vector ([4]); therefore the problem reduces to calculating the complex polarizability of an exciton subjected to the action of a constant magnetic field $\mathbf{H}$, under the influence of a monochromatic electromagnetic wave of frequency ω, whose vector potential we choose in the form

$$\mathbf{A}(\xi)\,\mathrm{Re}\,\mathbf{A}_0 e^{i[\omega t-(\mathbf{k}\xi)]}, \tag{1}$$

where $|\mathbf{k}| = \omega/c$ is the wave vector. We introduce the notation: $m_e^*, \mathbf{r}_e$; $m_h^*, \mathbf{r}_h$ are, respectively, the effective masses and radius vectors of the electron and the hole; ΔE is the activation energy of an electron-hole pair; $V(|\mathbf{r}_e - \mathbf{r}_h|)$ is the potential energy of their interaction (assumed spherically symmetric);

$$\mathbf{R} = \frac{m_e^*\mathbf{r}_e + m_h^*\mathbf{r}_h}{M}; \qquad \mathbf{r} = \mathbf{r}_e - \mathbf{r}_h;$$

$$M = m_e^* + m_h^*; \qquad m = \frac{m_e^* m_h^*}{M}; \tag{2}$$

$$\mu_{1,2} = \frac{m_{e,h}^*}{M}; \qquad \Delta = \frac{m_e^* - m_h^*}{M};$$

$\mathbf{P}$ and $\mathbf{p}$ are generalized kinetic momenta conjugate respectively to the coordinates $\mathbf{R}$ (the radius vector of the exciton "center of gravity") and $\mathbf{r}$ (the relative coordinate). We shall assume the medium to be isotropic.

With the aid of the introduced notation the exciton Hamiltonian can be represented in the form

$$\hat{\mathcal{H}} = \frac{1}{2M}\left\{\mathbf{P} - \frac{e}{2c}[\mathbf{Hr}] + \frac{e}{c}\mathbf{A}(\mathbf{R})\left(e^{i\mu_1(\mathbf{kr})} - e^{-i\mu_2(\mathbf{kr})}\right)\right\}^2 +$$

$$+\frac{1}{2m}\left\{\mathbf{p} - \frac{e}{2c}[\mathbf{HR}] + \frac{e\Delta}{2c}[\mathbf{Hr}] - \frac{e}{c}\mathbf{A}(\mathbf{R})\left(\mu_1 e^{i\mu_1(\mathbf{kr})} + \mu_2 e^{-i\mu_2(\mathbf{kr})}\right)\right\}^2 +$$

$$+\Delta E + V(r).$$

In the dipole approximation, assuming $(2\pi d/\lambda) < 1$ (λ is the wavelength of the electromagnetic field), we have $(\mathbf{kr}) \ll 1$, and the Hamiltonian $\hat{\mathcal{H}}$ can be signifi-considerably simplified. In the approximation linear in the electromagnetic field it takes the form

$$\hat{\mathcal{H}} = \hat{\mathcal{H}}_0 + \hat{\mathcal{H}}_1,$$

where

$$\hat{\mathcal{H}}_0 = \frac{1}{2M}\left(\mathbf{P} - \frac{e}{2c}[\mathbf{Hr}]\right)^2 + \frac{1}{2m}\left(\mathbf{p} - \frac{e}{2c}[\mathbf{HR}] + \frac{e\Delta}{2c}[\mathbf{Hr}]\right)^2 + \Delta E + V(r), \quad (3)$$

$$\hat{\mathcal{H}}_1 = -\frac{e}{2mc}\left\{(\mathbf{Ap}) - \frac{e}{2c}(\mathbf{A}[\mathbf{HR}]) + \frac{e\Delta}{2c}(\mathbf{A}[\mathbf{Hr}])\right\}. \quad (4)$$

$\hat{\mathcal{H}}_1$ can evidently be treated as a perturbation. We shall calculate the polarizability of the exciton according to the usual scheme.

We shall seek the solution of the time-dependent Schrödinger equation

$$\left[i\hbar\frac{\partial}{\partial t} - \hat{\mathcal{H}}\right]\Phi_\alpha(\mathbf{R}, \mathbf{r}; t) = 0 \quad (5)$$

in the form

$$\Phi_\alpha(\mathbf{R}, \mathbf{r}; t) = e^{-i\omega_\alpha t}\left\{\Psi_\alpha(\mathbf{R}, \mathbf{r}) + \sum_{\alpha'}[a_{\alpha\alpha'}e^{i\omega t} + b_{\alpha\alpha'}e^{-i\omega t}]\Psi_{\alpha'}(\mathbf{R}, \mathbf{r})\right\}, \quad (6)$$

where

$$\hat{\mathcal{H}}_0\Psi_\alpha = E_\alpha\Psi_\alpha = \hbar\omega_\alpha\Psi_\alpha. \quad (7)$$

$\hbar\omega_0$ is the energy of the ground state of the exciton.

It follows from (3) that

$$\Psi_\alpha = \Psi_{K,s} = \frac{1}{\sqrt{V}} e^{i\left[\frac{e}{\hbar c} f + (\mathbf{KR})\right]} \varphi_{K,s}(\mathbf{r}), \tag{8}$$

where

$$\left\{ \frac{1}{2M}\left(\hbar\mathbf{K} - \frac{e}{c}[\mathbf{Hr}]\right)^2 + \frac{1}{2m}\left(\mathbf{p} + \frac{e\Delta}{2c}[\mathbf{Hr}]\right)^2 + V(r) \right\} \varphi_{K,s} = (E_{K,s} - \Delta E)\varphi_{K,s}, \tag{9}$$

and V denotes the volume of the crystal.

Thus, the state of the exciton (for $\mathbf{A} = 0$) can be described by a set of integrals of motion: $\hbar\mathbf{K}$ (the momentum of the "center of gravity") and s (for example, the quantum numbers n, l, m), if $V(r)$ is the Coulomb potential. Substituting (8) into (6) and then into (5), one readily obtains, after standard calculations, the values of the expansion coefficients $a_{\alpha\alpha'}$ and $b_{\alpha\alpha'}$.

Finally we have

$$\Phi_{K,0}(\mathbf{R}, \mathbf{r}; t) = \frac{1}{\sqrt{V}} e^{i\left[\frac{e}{\hbar c} f + (\mathbf{KR}) - \omega_0\right]} \chi_{K,0}(\mathbf{r}; t), \tag{10}$$

where

$$\chi_{K,0} = \varphi_{K,0} + \frac{e^{i\omega t}}{\hbar} \sum_s \frac{(\mathbf{K} - \mathbf{k}, s|\beta|\mathbf{K}, 0)}{\omega_{K-\mathbf{k},s;\mathbf{K},0} + \omega} \varphi_{K-\mathbf{k},s} + $$
$$+ \frac{e^{-i\omega t}}{\hbar} \sum_s \frac{(\mathbf{K} + \mathbf{k}, s|\beta|\mathbf{K}, 0)}{\omega_{K+\mathbf{k},s;\mathbf{K},0} - \omega} \varphi_{K+\mathbf{k},s}, \tag{11}$$

$$\omega_{K\pm\mathbf{k},s;\mathbf{K},0} = \omega_{K\pm\mathbf{k},s} - \omega_{K,0}, \tag{12}$$

$$(\mathbf{K} \pm \mathbf{k}, s|\beta|\mathbf{K}, 0) = \frac{e}{4mc} \int \varphi^*_{K\pm\mathbf{k},s}(\mathbf{r}) \left\{ (\mathbf{A}_0\mathbf{p}) + \frac{e\Delta}{2c}(\mathbf{A}_0[\mathbf{Hr}]) \right\} \varphi_{K,0}(\mathbf{r})\, d\tau. \tag{13}$$

In what follows we shall neglect the difference between $\mathbf{K} + \mathbf{k}$ and $\mathbf{K} - \mathbf{k}$ and replace them by $\mathbf{K}$, since for the visible part of the spectrum $|\mathbf{k}| \ll |\mathbf{K}|$.

To determine the polarizability tensor we proceed as follows. Using (11) and (12), we calculate the quantum-mechanical current in the exciton in the state $(\mathbf{K}, 0)$ from the formula

$$\mathbf{j}_{K,0}(t) = -\frac{e}{m}\int \Phi^*_{K,0}\left(\mathbf{p} - \frac{e}{2c}[\mathbf{HR}] + \frac{e\Delta}{2c}[\mathbf{Hr}]\right)\Phi_{K,0}\,(d\tau_R)(d\tau_r) \tag{14}$$

and then find the vector of complex polarization $\vec{\Pi}_{K,0}$ according to the equality

$$\vec{\Pi}_{K,0}(t) = \int_0^t \mathbf{j}_{K,0}(\tau)\,d\tau = \alpha\vec{\mathcal{E}}, \tag{15}$$

where α is the (complex) polarizability tensor, and $\vec{\mathcal{E}}$ is the complex intensity of the electric field:

$$\vec{\mathcal{E}} = -i\,\frac{\omega}{c}\,\mathbf{A}_0 e^{i\omega t}. \tag{16}$$

Simple calculations lead to the following result:

$$\alpha_{ij} = \alpha^{(0)}_{ij} + \alpha^{(1)}_{ij},$$

where

$$\alpha^{(0)}_{ij} = \alpha_{ij}(\mathbf{H}=0) = -\frac{e^2}{2\hbar(m\omega)^2}\sum_s\left\{\frac{(s|p^i|0)^*(s|p^j|0)}{\omega_{s,0}+\omega} + \frac{(s|p^i|0)(s|p^i|0)^*}{\omega_{s,0}-\omega}\right\} \tag{17}$$

is the polarizability of the exciton in the absence of a magnetic field (in the case of an isotropic crystal considered by us this tensor is diagonal: $\alpha^{(0)}_{ij} = \alpha\delta_{ij}$) and

$$\alpha^{(1)}_{ij} = -\frac{e^3\Delta}{4m^2\omega^2 c}\sum_s\left\{\frac{(0|p^i|s)(s|[\mathbf{H}r]^j|0) + (s|p^j|0)(0|[\mathbf{H}r]^i|s)}{\omega_{s,0}+\omega} + \right.$$
$$\left. + \frac{(s|p^i|0)(0|[\mathbf{H}r]^j|s) + (0|q^j|s)(s|[\mathbf{H}r]^i|0)}{\omega_{s,0}-\omega}\right\}, \tag{18}$$

where, for convenience of notation, the indices $\mathbf{K}$ have everywhere been omitted.

By definition of the z-component, the gyration vector is

$$\gamma_z = \frac{1}{2i}\left(\alpha^{(1)}_{xy} - \alpha^{(1)}_{yx}\right). \tag{19}$$

Using (18) and (19), it is easy to obtain

$$\vec{\gamma} = \frac{e^3\Delta}{8m^2\omega\hbar c\, i}\sum_s \frac{[(s|\mathbf{p}|0)(0|[\mathbf{H}r]|s)] - [(0|\mathbf{p}|s)(s|[\mathbf{H}r]|0)]}{\omega_{s,0}^2 - \omega^2}. \tag{20}$$

On the basis of the equality

$$(s|\mathbf{p}|0) = -im\omega_{s,0}(s|\mathbf{r}|0) \tag{21}$$

and assuming the z-axis to be directed along the magnetic field $\mathbf{H}$, we have

$$\gamma_z = \frac{e^3\Delta H_z}{4m\omega\hbar c}\sum_s \frac{\omega_{s,0}}{\omega_{s,0}^2 - \omega^2}\left\{|(s|x|0)|^2 + |(s|y|0)|^2\right\}, \tag{22}$$

where the matrix elements of the coordinates are taken in the functions $\varphi_{K,s}$ and depend, in general (as do the frequencies $\omega_{s,0}$), on $\mathbf{K}$. We note, however, that if

if one restricts oneself in the γ_z terms to $\mathbf{H}$ to no higher than first order, then the matrix elements and frequencies indicated above cease to depend on $\mathbf{K}$. We shall restrict ourselves here precisely to this approximation. From comparison of (22) and (17) it is easy to see that, in the case considered by us,

$$\vec{\gamma} = \frac{e\Delta\alpha_e}{4mc}\,\mathbf{H}, \tag{23}$$

where α_e is the scalar polarizability of the exciton in an isotropic crystal in the absence of a magnetic field.

The gyration vector calculated per unit volume is $\vec{\Gamma} = n_e\vec{\gamma}$, where n_e is the density of the exciton gas:

$$\vec{\Gamma} = \frac{\Delta n_e\alpha_e}{4\omega}\,\frac{e\mathbf{H}}{mc}. \tag{24}$$

A special feature of this result is the proportionality of the gyration vector to the difference Δ of the electron and hole masses. This is a consequence of the fact that the gyration vector is proportional to the Larmor frequency. The proportionality of the gyration vector (or of the Verdet constant) to the magnetic field is a consequence of the proportionality of the Larmor frequency to the magnetic field.

As is known, the angle of rotation θ of the plane of polarization in the Faraday effect in a specimen of thickness l obeys the relation

$$\theta = \eta H l, \tag{25}$$

where η is the Verdet constant. On the other hand, the following relation[4] exists between θ and Γ_z:

$$\theta \approx \frac{\pi l}{\varepsilon_0 \lambda} \Gamma_z, \tag{26}$$

where ε_0 is the dielectric constant of the medium.

From comparison of (25) and (26), on the basis of (24), we have

$$\theta \approx \frac{\pi}{4} \Delta \frac{eH}{mc\omega} \frac{n_e \alpha_e}{\varepsilon_0} \frac{l}{\lambda}, \qquad \eta = \frac{\pi}{4} \Delta \frac{e}{mc\omega} \frac{n_e \alpha_e}{\varepsilon_0} \frac{1}{\lambda}. \tag{27}$$

Thus, the Faraday effect on excitons is possible only in the case when $m_e^* \neq m_h^*$; otherwise the rotations of the plane of polarization caused separately by the electron and the hole are completely compensated. The formula for θ contains the ratio Δ/m and can serve as an independent method for determining this ratio. To estimate the expected magnitude of the effect, let us note that $n_e \alpha_e$, in order of magnitude, is $\sim \alpha_{\text{at}} n_e a_0^3$. Put $H \sim 3 \cdot 10^3$ oersted, $m \sim 0.1\, m_0$, $\Delta \sim 0.5$, $\omega \sim 10^{16}$ sec^{-1}, $l \sim 10^{-1}$ cm, $\lambda \sim 10^{-5}$ cm, $\varepsilon_0 \sim 10$, $n_e \sim 10^{18}$ cm^{-3}, $\alpha_{\text{at}} \sim 10^{-23}$ abs. units. Then $eH/mc \sim 10^{11}$ sec^{-1}, $eH/mc\omega \sim 10^{-5}$, $n_e \alpha_e / \varepsilon_0 \sim 10^{-2}$, $l/\lambda \sim 10^4$, $\theta \sim 7'$, $\eta \sim 0.02'$, i.e., the effect may attain the usual order of magnitude (for nonferromagnetic bodies).

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Note: Figure translations are in progress. See original paper for figures.