A NONLINEAR PROBLEM IN THE THEORY OF A HYDROFOIL

![Fig. 1](figure)

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Fig. 1

Figure 1: Fig. 1

Abstract

Full Text

HYDROMECHANICS

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A NONLINEAR PROBLEM IN THE THEORY OF A HYDROFOIL

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1. We consider the steady motion of a hydrofoil (S) beneath the surface of an ideal incompressible heavy fluid of finite depth. It is assumed that far ahead the fluid is at rest. Without loss of generality one may take the velocity of the body and the depth of the fluid at infinity to be equal to unity. The motion may be reversed and the flow of the fluid past the body (S) considered. The coordinate axes are chosen according to Fig. 1. The sharp edge of the wing is at the point $P(0,\beta)$. Let $y=Y(x)$ be the equation of the free boundary (L) of the fluid; $y=0$ the equation of the bottom (T); D the domain enclosed between (L), (S), and (T); $z=x+iy$ the complex coordinate of a point; $w(z)=\varphi(x,y)+i\psi(x,y)$ the complex potential.

Fig. 1

The general problem of flow past the wing reduces to the following mathematical problem: to find a function $w(z)$, analytic in D and satisfying the conditions

$$\frac{1}{2}\left|\frac{dw}{dz}\right|^2+\nu y=\text{const}\qquad \text{on } (L); \tag{1}$$

$$\psi=1\qquad \text{on } (L); \tag{2}$$

$$\psi=0\qquad \text{on } (T); \tag{3}$$

$$\psi=\psi_0\qquad \text{on } (S); \tag{4}$$

$$\int_{(C)}\frac{dw}{dz}\,dz=\gamma; \tag{5}$$

$$\lim_{x\to-\infty} Y(x) = 1; \tag{6}$$

$$\lim_{x\to-\infty} \frac{dw}{dz} = 1; \tag{7}$$

$$\left|\frac{dw}{dz}\right|\Big|_{z=i\beta} < \infty. \tag{8}$$

Here ν is a prescribed number, while γ and ψ_0 must be found in the course of the solution; (C) is an arbitrary contour enclosing (S) and lying entirely in D.

2. We map conformally the curvilinear strip bounded by (L) and (T) onto the rectilinear strip $0 \leq \eta \leq 1$ by means of a certain

of the analytic function $\zeta = \zeta(z) = \xi(x, y) + i\eta(x, y)$ such that the infinitely distant point of the z-plane corresponds to the infinitely distant point of the ζ-plane. In this transformation the contour (S) becomes a certain distorted contour (S'). Suppose that the point $z = i\beta$ corresponds to the point $\zeta = i\varepsilon$. In the plane of the parametric variable ζ, the boundary conditions for determining $w(\zeta)$ are obtained as

$$\begin{gathered}\psi = 0, \quad \eta = 0; \qquad \psi = 1, \quad \eta = 1; \qquad \psi = \psi_0 \quad \text{on } (S'); \\ \int_{(C)} \left(\frac{dw}{d\zeta}\right) d\zeta = \gamma; \qquad \left|\left(\frac{dw}{d\zeta}\right)_{\zeta=i\varepsilon}\right| < \infty.\end{gathered} \tag{9}$$

The conditions (9) determine the flow around the body (S') in the channel $\eta = 0,\ \eta = 1$. The problem will be solved by the "inverse method." Its essence is as follows. We prescribe the form of the wing in the parametric plane. Then, solving the problem of flow around the wing (S'), we find $w(\zeta)$. If $w(\zeta)$ is known, then conditions (1), (6), (7) and the condition $y(\xi, 0) = 0$ make it possible to find the relation between the physical plane z and the parametric plane ζ. A new dependent variable is introduced,

$$\tau(\zeta) = \theta + i\lambda = i \ln \frac{d\zeta}{dz}. \tag{10}$$

The problem is reduced to finding the function $\tau(\zeta)$, analytic in the strip $0 < \eta < 1$, from the boundary and asymptotic conditions

$$\frac{\partial\theta}{\partial\eta} - \nu\theta = F(\theta, \lambda), \quad \eta = 1; \qquad \theta = 0; \quad \eta = 0; \qquad \lim_{\xi\to-\infty} \tau(\zeta) = 0, \tag{11}$$

where

$$F(\theta,\lambda) = \frac{\nu}{\omega^2} e^{-3\lambda} \sin\theta - \nu\theta + \frac{d}{d\xi} \ln\omega(\xi), \qquad \omega(\xi) = \left|\frac{dw}{d\zeta}\right|_{\eta=1}. \tag{12}$$

The posed problem is solved for $\nu < 1$.

3. Let us consider the following linear problem: find a function $\tau(\zeta)$, analytic in the strip $0 < \eta < 1$, satisfying the boundary conditions

$$\frac{\partial\theta}{\partial\eta} - \nu\theta = f(\xi), \quad \eta = 1; \qquad \theta(\xi, 0) = 0, \quad \eta = 0, \tag{13}$$

where $f(\xi)$ is an absolutely integrable function.

The Green' s function for this problem has the form

$$H(\zeta,\zeta') = \frac{i}{2\pi} \int_{-\infty}^{\infty} \frac{\sin k(\zeta-\zeta')}{k} \, \frac{k \operatorname{ch} k(1-\eta) - \nu \operatorname{sh} k(1-\eta)}{k \operatorname{ch} k - \nu \operatorname{sh} k} \, dk. \tag{14}$$

The solution of problem (13) is expressed in terms of $H(\zeta,\zeta')$ in the following way (1):

$$\tau(\zeta) = \int_{-\infty}^{\infty} f(\xi') \, H(\zeta, \xi' + i) \, d\xi' + ic, \tag{15}$$

where c is an arbitrary real constant.

If in (15) we substitute $F(\theta,\lambda)$ for $f(\xi)$, set $\zeta = \xi + i$, and separate the real and imaginary parts, then we obtain a system of two nonlinear integral equations with respect to θ and λ, to which the problem posed in the preceding section is reduced.

Denote by H the linear operator that assigns to a function $f(x)$ the function

$$\int_{-\infty}^{+\infty} H(x+i, \xi+i) f(\xi) \, d\xi.$$

Let a be a real-

positive number; B^a is the set of functions continuous on the line $-\infty < x < +\infty$ for which the quantities $e^{ax}|f(x)|$ and $e^{-ax}|f(x)|$ are bounded. As is known, every continuous function can be represented in the form $f(x) = f^+(x) + f^-(x)$, where $f^-(x)$ is an odd function and $f^+(x)$ is an even one. We turn B^a into a Banach function space by introducing the norm

$$\|f(x)\|_{B^a} = \sup e^{ax}|f^-(x)| + \sup e^{ax}|f^+(x)|.$$

The properties of the operator H are given by the following theorems:

Theorem 1. *The operator* $\operatorname{Re} H$ *acts from* B^δ *into* B^δ*, and the operator* $\operatorname{Im} H$ *from* B^δ *into* B^δ_*.

Here 2δ is the absolute value of the first root of the equation $k \operatorname{ch} k - \nu \operatorname{sh} k = 0$; B^δ_* is the space of continuous bounded functions on the line $-\infty < x < +\infty$.

Let T^δ be the space of pairs of functions $q(\theta, \lambda)$, with $\theta \in B^\delta$, $\lambda \in B^\delta_*$, and

$$\|q\|_{T^\delta} = \|\theta\|_{B^\delta} + \|\lambda\|_{B^\delta_*}.$$

Denote by F the nonlinear operator which assigns to an element $q \in T^\delta$ the function

$$F(\theta, \lambda) = \frac{\nu}{\omega^2} e^{-3\lambda} \sin\theta - \nu\theta + \frac{d}{d\xi} \ln \omega(\xi).$$

It is known that the potential in the flow past a profile in a tube behaves at infinity like the potential of a source. Therefore $\omega(\xi) = 1 + g(\xi)$, where $g(\xi) \in B^{\pi/2}$ and $dg(\xi)/d\xi \in B^{\pi/2}$, and, since $\delta < \pi/2$, both $g(\xi)$ and $dg(\xi)/d\xi$ belong to B^δ.

Theorem 2. *The operator* F *acts from* T^δ *into* B^δ.

Denote by Ω the pair of operators $\{\operatorname{Re} HF, \operatorname{Im} HF\}$.

Theorem 3. *The operator* Ω *acts from* T^δ *into* T^δ.

The problem has been reduced to the integral equation $q = \Omega q$.

Let $\varepsilon = \max\{\|g(\xi)\|, \|dg/d\xi\|\}$.

Theorem 4. *For any* $\nu < 1$ *one can choose an* ε_0 *such that, if* $\varepsilon < \varepsilon_0$*, then the operator* Ω *maps some ball* M *in the space* T^δ *into itself.*

Theorem 5. *If* ε *is sufficiently small, then the operator* Ω *gives a contraction mapping in* M.

Theorems 4 and 5 prove the existence and uniqueness of a solution of the integral equation $q = \Omega q$ for small ε.

The results obtained may be summarized as follows:

Theorem 6. *If the contour of the wing is prescribed in the parametric plane, then, for a Froude number greater than unity, there exists a unique solution of the problem of flow past a hydrofoil, tending to a plane-parallel flow as* $\varepsilon \to 0$.

In the first approximation the contours (S) and (S') coincide. If $\max|\lambda| < \ln 2$, then the correspondence between the physical and parametric planes will be one-to-one.

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REFERENCES

1. A. M. Ter-Krikorov, *Izv. AN SSSR, Ser. Mat.*, **22**, No. 2 (1958).

Note: Figure translations are in progress. See original paper for figures.

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