ON THE SPIN OF THE τ-MESON

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Abstract

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§ 1. As is known, spin 2^+ is the only value of the spin (under certain very plausible restrictions*) with which a particle can decay, with parity conserved, into both two and three π-mesons. An analysis of the experimental data on the decay of the τ^+-meson, carried out by the method proposed by Dalitz ([1]), showed that the τ-meson cannot possess this spin. In view of the importance of this result (which, however, has been called into question in ([2,3])), we have analyzed the decay of the τ-meson within a set of assumptions which, in our opinion, should be preferred to Dalitz' s method, as well as to its modifications proposed in ([2,3]).

§ 2. For a given spin S and parity P of the τ-meson, the state of the system of three π-mesons can be characterized by two angular-momentum vectors $\mathbf{l}_1$ and $\mathbf{l}_2$, satisfying the conditions

$$\mathbf{l}_1 + \mathbf{l}_2 = \mathbf{S}, \qquad (-1)^{l_1+l_2} = -P. \tag{1}$$

Generally speaking, in the decay any superposition of states satisfying the requirements (1) may be realized. Proceeding from the fact that the decay energy is relatively small (~ 70 MeV), Dalitz assumed that, in fact, only that one of these states is realized which is characterized by the smallest possible values of the momenta l_1 and l_2. This assumption makes it possible to calculate the angular and energy distributions of the π-mesons. In order to save the spin-2^+ hypothesis for the τ-meson, Marshak ([2]) proposed considering an arbitrary superposition of two states of the π-mesons corresponding to the two minimum values of the sum $|\mathbf{l}_1| + |\mathbf{l}_2|$.

In order to remove the arbitrariness that arises, we consider it most correct to examine the decay of the τ-meson starting from a reasonably fixed Lagrangian of the decay interaction. It turns out that in the case of spin 2^+ the form of this Lagrangian is determined uniquely if one requires that it, being relativistically and gauge invariant, contain no second or higher derivatives of the wave functions of the τ- and π-mesons. The calculation is performed by perturbation theory, whose applicability to weak interactions is beyond doubt.

It is not difficult to carry out analogous calculations also for spins 2^- and 0^-. However, in these cases one can construct several different Lagrangians contain-

Fig. 1

Figure 1: Fig. 1

ing no second derivatives, and we have restricted ourselves, for each of these spins, to consideration of the "simplest" Lagrangian (in the sense that it contains the smallest number of derivatives of the wave functions). In the case of spin 0^- (as also for spin 2^+), the results of our calculations differ very little from Dalitz' s calculations; in the case of spin 2^-, however, the "simplest" Lagrangian contains a free parameter, the value of which has a very substantial effect on the result of the calculations.

* First, spin values $\geqslant 4$ are discarded; second, the possibility of decay of a neutral particle into two π^0-mesons is assumed to be proven. Otherwise, for the θ-meson the possibility opens up of spins 1^-, 3^-, etc., which, however, likewise do not agree with the decay of the τ-meson.

§ 3. Charged particles with spin 2 are quanta of the complex tensor field

$$\Phi_{\alpha\beta}(x) = \Phi_{\beta\alpha}(x), \tag{2}$$

satisfying the equations ([4])

$$(\Box - \mu^2)\Phi_{\alpha\beta} = 0, \qquad \partial\Phi^{\alpha\beta}/\partial x^\beta = 0, \qquad \Phi^\alpha_\alpha = 0. \tag{3}$$

If these particles are even (spin 2^+), then the only possible invariant Lagrangian corresponding to the decay of such a particle into three charged π-mesons and containing no second or higher derivatives of the fields is the Lagrangian

$$L'(2^+) = g(2^+)\varepsilon^{\alpha\beta\gamma\delta}\varphi^*_\alpha\varphi_\beta\varphi^{*\lambda} \times \frac{\partial}{\partial x^\gamma}\Phi_{\lambda\delta} + \text{c.c.}, \tag{4}$$

where $\varepsilon^{\alpha\beta\gamma\delta}$ is the completely antisymmetric unit tensor, and by $\varphi_\alpha \equiv \dfrac{\partial}{\partial x^\alpha}\varphi$ are denoted the derivatives of the field $\varphi(x)$ of charged pseudoscalar π-mesons.

Fig. 1

In the case of spin 2^-, the simplest Lagrangian, containing only two derivatives of the fields, depends essentially on the arbitrary constant ν:

$$L'(2^-) = g(2^-)\varphi^*_\alpha\Phi^{\alpha\beta}\{\varphi_\beta\varphi^* + (1-\nu)\varphi^*_\beta\varphi\} + \text{c.c.} \tag{5}$$

Let us note that a τ-meson with spin 0^- corresponds to the pseudoscalar field $\Phi(x)$, and that in this case the simplest decay Lagrangian has the form

$$L'(0^-) = g(0^-)\Phi\varphi^*\varphi^*\varphi + \text{c.c.} \tag{6}$$

Carrying out the quantization of the field $\Phi^{\alpha\beta}$ in the usual way, we may represent it in the form

$$\Phi^{\alpha\beta}(x) = \frac{1}{(2\pi)^{3/2}} \int \frac{d^3k}{\sqrt{\varepsilon_k}} \sum_{m=-2}^{2} U_m^{\alpha\beta}(\mathbf{k})\{a_{m+}(\mathbf{k})e^{-ikx} + a_{m-}^*(\mathbf{k})e^{ikx}\}, \tag{7}$$

where $\varepsilon_k = \sqrt{\mu^2 + k^2}$; the matrices $U_m^{\alpha\beta}(\mathbf{k})$ satisfy the conditions

$$g_{\alpha\beta}U_m^{\alpha\beta} = 0, \qquad U_m^{\alpha\beta}K_\beta = 0, \qquad U_m^{*\alpha\beta}U_{m',\alpha\beta} = \delta_{mm'},$$

$$i\left(U_m^{*\alpha 1}U_{m',\alpha 2} - U_m^{*\alpha 2}U_{m',\alpha 1}\right) = \frac{1}{2}m\delta_{mm'}, \tag{8}$$

and the operators $a_{m\pm}(\mathbf{k})$ obey the commutation relations

$$[a_{m+}(\mathbf{k}),\, a_{m'+}^*(\mathbf{k}')] = [a_{m-}(\mathbf{k}),\, a_{m'-}^*(\mathbf{k}')] = \delta_{mm'}\delta(\mathbf{k} - \mathbf{k}'). \tag{9}$$

Substituting the expansion (8) into the expressions for the energy-momentum tensor, for the current, and for the spin of the field $\Phi_{\alpha\beta}$, obtained canonically from the Lagrangian of the free field ([4,5]), we find that the operators $a_{m\pm}(\mathbf{k})$ are–

are the annihilation operators of particles with momentum $\hbar\mathbf{k}$, charge $\pm e$, and spin projection on the z axis equal to $\hbar m$ $(-2 \le m \le 2)$.

In the case of a particle at rest ($\mathbf{k} = 0$) the matrices $U_m^{\alpha\beta}(0)$ have only spatial components, and for these matrices one can verify the following spin-summation law:

$$S(M) = \frac{1}{5}\sum_{m=-2}^{2} \left|U_{mab}M_{ab}\right|^2 = \frac{1}{5}N_{ab}^*N_{ab}, \tag{10}$$

where M_{ab} is an arbitrary matrix and $N_{ab} = \frac{1}{2}(M_{ab} + M_{ba}) - \frac{1}{3}\delta_{ab}M_{cc}$.

§ 4. The structure of each three-prong decay "star" formed by π-mesons is determined by specifying two parameters. As such parameters Dalitz chose the energy ω_3 of the π^--meson in the rest system of the τ-meson and the angle ϑ between the corresponding momentum $\mathbf{k}_3$ and the vector $\mathbf{q}$, directed along the line of flight of the π^+-mesons in the center-of-inertia system of these π^+-mesons. This angle ϑ is expressed in terms of the energies ω_1 and ω_2 of the π^+-mesons in the rest system of the τ^+-meson by the relation

Fig. 2

Figure 2: Fig. 2

$$x = \cos\vartheta = \frac{\omega_1 - \omega_2}{k_3} \times \left(\frac{\mu_\tau^2 - 2\mu_\pi\omega_3 + \mu_\pi^2}{\mu_\tau^2 - 2\mu_\tau\omega_3 - 3\mu_\pi^2} \right)^{1/2}, \tag{11}$$

Fig. 2

where μ_τ and μ_π are, respectively, the masses of the τ- and π-mesons.

Instead of the parameter ω_3, it is customary to introduce the kinetic energy of the π^--meson, expressed in fractions of its maximum possible value:

$$\varepsilon = \frac{(\omega_3 - \mu_\pi)2\mu_\tau}{\mu_\tau^2 - 2\mu_\tau\mu_\pi - 2\mu_\pi^2}. \tag{12}$$

Carrying out the calculation of the probability of the decay $\tau^+ \to \pi^+ + \pi^+ + \pi^-$ by perturbation theory, we obtain the following expressions for the relative probability of different configurations of the decay "star" :

$$dW = W(\varepsilon, x)\, d\varepsilon\, dx = \left[\frac{\varepsilon(1-\varepsilon)(\varepsilon+a)}{1-\varepsilon+b} \right]^{1/2} F(\varepsilon, x)\, d\varepsilon\, dx, \tag{13}$$

where $F(\varepsilon, x)$ depends on the spin of the τ-meson:

spin 0^-

$$F = 1;$$

spin 2^+

$$F = (1-x^2)\varepsilon(\varepsilon-1)^2(\varepsilon+a)\left[1 + \frac{cx^2\varepsilon(\varepsilon+a)}{1-\varepsilon+b}\right];$$

spin 2^-

$$F = \left\{ \frac{1-\nu}{2}(1-\varepsilon)l\left(h + \frac{\varepsilon(\varepsilon+a)}{1-\varepsilon+b}x^2\right) + \frac{1+\nu}{2}\varepsilon(\varepsilon+a) \right\}^2 + \frac{3}{4}hl^2(\nu^2-1)\varepsilon(\varepsilon+a)(1-\varepsilon)(1-x^3). \tag{14}$$

In these formulas the following notation has been introduced:

$$a = \frac{4\mu_\tau\mu_\pi}{\mu_\tau^2 - 2\mu_\tau\mu_\pi - 3\mu_\pi^2} = 5.75, \qquad b = \frac{a\mu_\pi}{\mu_\tau} = 1.62, \tag{15}$$

Fig. 3

Figure 3: Fig. 3

$$c = \frac{\mu_\pi}{a\mu_\tau} = 96.17, \qquad h = \frac{a\mu_\tau}{\mu_\pi} = 20.41, \qquad l = \frac{1}{4}\left(1 - \frac{3\mu_\tau^2}{\mu_\pi^2}\right) = -9.20.$$

The Dalitz formulas for spin 2^+ are obtained from (13) and (14) if in them one sets $a = b = \infty,\ c = 0$.

For comparison with experiment it is convenient to compute from (13) the distributions of stars over energies ε, normalized to unity, and their angular distribution in x:

$$\frac{dN(\varepsilon)}{d\varepsilon} = \frac{1}{A}\int_0^1 W(\varepsilon, x)\,dx, \qquad \frac{dN(x)}{dx} = \frac{1}{A}\int_0^1 W(\varepsilon, x)\,d\varepsilon, \qquad A = \int_0^1\int_0^1 d\varepsilon\,dx\,W(x, \varepsilon). \tag{16}$$

In Figs. 1-3 the calculated distributions are compared with the experimental ones; the latter (broken dashed lines) were taken by us from Marshak' s paper [(2)]. In Fig. 1, $dN(\varepsilon)/d\varepsilon$ is plotted for spins 2^+ and 0^-, the solid curves being calculated from our formulas and the dashed ones from Dalitz. In Fig. 2 only one curve is plotted for each of the spins 2^+ and 0^-, since the difference between our curves for $dN(x)/dx$ is less than the accuracy of the graph in the case 2^+ and is identically zero in the case 0^-. Fig. 3 illustrates the dependence of the distributions in the case 2^- on the value of the parameter ν.

Fig. 3

For spins 2^+ and 0^- our curves practically coincide with the Dalitz curves, so that spin 2^+ is clearly not in agreement with experiment, whereas spin 0^- agrees well with it. In the case of spin 2^-, however, our curves depend very sharply on the arbitrary parameter ν and, with a proper choice of its value ($\nu = -1/4$), agree excellently with experiment. This possibility is missed when the Dalitz method is used, which should be kept in mind in all cases where the "simplest" Lagrangian contains an arbitrary parameter.

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[1] R. Dalitz, Proc. Phys. Soc., **64**, 710 (1953); Phil. Mag., **44**, 1068 (1953); Phys. Rev., **94**, 1046 (1954); E. Fabri, Nuovo Cim., **11**, 479 (1954).
[2] R. E. Marshak, Proceedings of the Rochester Conference, 1956.
[3] I. S. Shapiro, E. I. Dolinskii, A. P. Mishakov; ZhETF, **32**, 173 (1957).
[4] M. Fierz, Helv. Phys. Acta, **12**, 3 (1939).
[5] M. Fierz, W. Pauli, Proc. Roy. Soc., **173**, 211 (1939).

Note: Figure translations are in progress. See original paper for figures.