Reports of the Academy of Sciences of the USSR

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1958

SovietRxiv

Abstract

Full Text

Reports of the Academy of Sciences of the USSR

1958. Vol. 123, No. 4

HYDROMECHANICS

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FORMATION OF OSCILLATIONS IN THE WAKE BEHIND A BODY

(Presented by Academician G. I. Petrov on 1 VII 1958)

Let a two-dimensional disturbance with stream function be superposed on a plane-parallel flow of a viscous incompressible fluid such that the velocity component $v = 0,\ u = u(y)$:

$$\psi(x, y, t) = \varphi(y)e^{i\alpha(x-ct)}, \tag{1}$$

where $c = c_r + ic_i$; $c_r,\ c_i$ are real numbers; $\alpha = 2\pi/a$; a is the wavelength of the disturbance, and the amplitude $\varphi(y)$ is assumed small. The stability problem for such a flow, as is known ([1]), reduces to the study of the equation

$$(u - c)(\varphi'' - \alpha^2\varphi) - u''\varphi + \frac{i\nu}{\alpha}(\varphi^{\mathrm{IV}} - 2\alpha^2\varphi'' + \alpha^4\varphi) = 0, \tag{2}$$

where ν is the viscosity.

We shall assume that the distribution of velocities in the wake behind a body (the wake jet) is close to that computed by Tollmien ([2]):

$$u = u_\infty \left(1 - \frac{c_{xd}}{4\sqrt{\pi}}\sqrt{\frac{u_\infty}{\nu x}}\, e^{-\frac{1}{4}\frac{u_\infty}{\nu x}y^2}\right), \tag{3}$$

where u_∞ is the velocity of the flow incident on the body; d is a characteristic transverse dimension of the body; c_x is the drag coefficient.

Let us pass in equation (2) to dimensionless quantities, taking as the characteristic linear dimension the width l of the wake jet, defined as the distance between the inflection points of the velocity profile (3), and as the characteristic velocity the difference $u_\infty - u_0$, where $u_0 = u(0)$. Denoting

$$y_1 = \frac{y}{l}, \qquad u_1 = \frac{u - u_\infty}{u_\infty - u_0}, \qquad \mathrm{Re}_1 = \frac{(u_\infty - u_0)l}{\nu}, \qquad c_2 = \frac{c}{u_\infty},$$

$$c_1 = \frac{c_2 - 1}{1 - (u_0/u_\infty)}, \qquad \alpha_1 = \alpha l, \qquad \alpha_2 = \alpha d, \qquad \varphi_1 = \varphi(y_1), \tag{4}$$

we obtain

$$(u_1 - c_1)(\varphi_1'' - \alpha_1^2\varphi_1) - u_1''\varphi_1 + \frac{i}{\alpha_1 \mathrm{Re}_1}(\varphi_1^{\mathrm{IV}} - 2\alpha_1^2\varphi_1'' + \alpha_1^4\varphi_1) \equiv L(\varphi_1) = 0. \tag{5}$$

For the velocity profile (3),

$$u_1 = -e^{-2y_1^2}, \qquad c_1 = \frac{4\sqrt{\pi}}{c_x}(c_2 - 1)\sqrt{\frac{x/d}{\mathrm{Re}}}, \qquad \mathrm{Re}_1 = \frac{\mathrm{Re}\, c_x}{\sqrt{2\pi}}, \tag{6}$$

where $\mathrm{Re} = u_\infty d/\nu$.

Boundary conditions:

$$\varphi_1'(0) = \varphi_1'''(0) = 0; \qquad \varphi_1 \text{ and } \varphi_1' \to 0 \quad \text{as } y_1 \to \infty. \tag{7}$$

for antisymmetric disturbances and

$$\varphi_1(0) = \varphi_1''(0) = 0; \qquad \varphi_1 \text{ and } \varphi_1' \to 0 \quad \text{as } y_1 \to \infty \tag{8}$$

for symmetric disturbances.

To solve equation (5) with boundary conditions (7) or (8), we apply the Galerkin method. To this end we study the equation

$$|D_{ik}| = 0, \tag{9}$$

where $|D_{ik}|$ is the determinant of the system of equations

$$\sum_{k=1}^{m} e_k \int_0^\infty L(\psi_k)\psi_i \, dy = 0, \qquad i, k = 1, 2, \dots, m; \tag{10}$$

$\{\psi_k(y_1)\}$ is a system of "approximating" functions, taken in the form

$$\psi_k = e^{-y_1^2} \cos k y_1 \tag{11}$$

for antisymmetric disturbances and

$$\psi_k = e^{-y_1^2} \sin k y_1 \tag{12}$$

for symmetric disturbances; e_k are constants.

Putting c_{1i} = const, equating to zero the real part and the coefficient of the imaginary part of the left-hand side of (9), and eliminating c_{1r} from the two equations obtained, we obtain, for each c_{1i}, an equation of the form

$$f(\alpha_1, \mathrm{Re}_1) = 0, \tag{13}$$

which in the $(\alpha_1, \mathrm{Re}_1)$ plane determines a certain curve. The curve corresponding to the value $c_{1i} = 0$ separates, in the $(\alpha_1, \mathrm{Re}_1)$ plane, the region where $c_{1i} > 0$ from the region where $c_{1i} < 0$. The smallest value $\mathrm{Re}_1 = R_{1\mathrm{cr}}$ on it characterizes the transition of the flow to an unstable state.

The calculations were carried out in the second approximation. It was found that antisymmetric disturbances begin to develop at considerably smaller values of Re_1 ($\mathrm{Re}_{1\mathrm{cr}} \approx 19$) than symmetric ones ($\mathrm{Re}_{1\mathrm{cr}} \approx 55$). This confirms the result obtained earlier by G. I. Petrov[3] and agrees with experiment[4]. Therefore, in the subsequent calculations only the system of functions (11) was used.

It was established that the minimum values of Re_1 on the curves (13) correspond to one and the same value $\alpha_1 = 1.2$, and that c_{1i} increases comparatively rapidly with increasing Re_1; the latter circumstance, naturally, limited the possibility of applying equation (5) to the study of the development of disturbances to comparatively small values of Re_1 (of the order of several tens).

The results mentioned apply to wakes behind bodies of any shape, since the velocity distribution in these wakes is similar to that calculated by Tollmien. To obtain results for the wake behind a body of a definite shape, it is necessary to know the dependence of c_x on Re for this body and to apply formulas (4) and (6).

Experimentally, the phenomena in the wake behind a circular cylinder have been studied most thoroughly[5–9]. At small Re, the flow past the cylinder is laminar in character. At Re ≈ 30, periodic disturbances arise in the wake, which, with increasing Re, lead to the formation of a "staggered" vortex street. For each Re, the frequency of vortex formation behind the body and the relative distance a/d between vortices of one row of the street are constant (varying with Re).

G. I. Petrov (~3) suggested that the system of vortices in a wake is formed as a result of the development of disturbances after the flow has passed into an unstable state, and that for each Re in the wake only that disturbance develops which tends to grow faster than the others. This means that for each Re we must consider only that disturbance to which the largest c_i corresponds. Hence, from what was set forth above, it follows that $l/a = 0.191$ for all Re.

Fig. 1

Figure 1: Fig. 1

Fig. 2

Figure 2: Fig. 2

In studying the stability of the flow in the wake behind a cylinder, the section of the wake was considered in which the velocity gradient is greatest, i.e., where $u(0) = 0$.

The Strouhal number, characterizing the frequencies of disturbances which, for each Re, tend to develop faster than the others, was calculated from the formula

$$\mathrm{Sh} = \frac{a_2 c r_2}{2\pi}. \tag{14}$$

The graph of the dependence of Sh on Re is shown in Fig. 1; there, for comparison, are also plotted experimental results (^{6}-9) for the dependence of the quantity Nd/u_∞ on Re, where N is the frequency of vortex formation. For the same Re (at $a_1 = 1.2$) the values a/d were calculated (a is the wavelength of the disturbance). The results are seen from Fig. 2, where, for comparison, experimental data (^5) on the dependence on Re of the relative distance a/d between vortices of one row of the street are also given.

Fig. 1

Fig. 2

We see that the results obtained on the basis of the linear hydrodynamic theory of stability concerning the development of disturbances in the wake agree fairly well with the experimental data concerning the formation of vortex streets. This confirms the validity of the hypothesis formulated above.

In conclusion I express my deep gratitude to Academician G. I. Petrov for valuable advice.

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Received
25 VI 1958

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Note: Figure translations are in progress. See original paper for figures.